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1890 †Bearman, Harry, <i>Gresham Life Assur. Soc., Ltd.,</i> St. Mildred's-house, Poultry, E.C.	1866 †Bumsted, David Alexander, <i>Sparnon, Blyth-road, Bromley,</i> Kent.
1889 †Bell, Frederick, <i>Alliance Assurance Co., Limited</i> (Imperial Life Assurance Fund), 47 Chancery-lane, w.c.	1894 †Burn, Joseph, P.A.S.I., <i>Prudential Assurance Company,</i> Holborn-bars, E.C.
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1894 †Blackadar, Alfred Kimball, M.A., F.A.S., <i>Government Insur. Department,</i> Ottawa, Canada.	1907 †Carter, Norman John, <i>Eagle Insurance Company,</i> 79 Pall-mall, s.w.
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1907 †Blanchard, Norman, M.A., <i>Equity & Law Life Assur. Soc.,</i> 18 Lincoln's-inn-fields, w.c.	1905 †Chandler, Thomas Richard, 113 Powis-street, Woolwich.
1897 †Bradshaw, Thomas, F.A.S., <i>Imperial Life Assurance Co. of Canada,</i> Toronto, Canada.	1889 †Chatham, James, F.F.A., F.S.S., <i>North British and Mercantile Insurance Co.,</i> 64 Princes-street, Edinburgh.
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1894	†McDonald, John, <i>Prudential Assurance Company, Holborn-bars</i> , E.C.	1905	†Milligan, Charles Livingstone, <i>Alliance Ass. Co., Ltd. (Provident Life Fund)</i> , 50 <i>Regent-street</i> , w.
1883	†McGowan, James, B.A., <i>The Treasury</i> , <i>Cape Town, South Africa</i> .	1893	†Milner, John William, <i>North British & Mercantile Insur. Co.</i> , 61 <i>Threadneedle-street</i> , E.C.
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1907	†Mackenzie, Michael Alexander, <i>University of Toronto, Toronto, Canada</i> .	1899	†Moir, Henry, F.F.A., F.A.S., <i>Home Life Insurance Co.</i> , 256 <i>Broadway</i> , <i>New York, U.S.A.</i>
1900	†Macnaghten, Steuart Edye, A.C.A., (TUTOR, Part II), <i>Equitable Life Assurance Soc.</i> , <i>Mansion House-street</i> , E.C.	1890	†Molyneux, Arthur Ernest, <i>Provident Clerks' and General Mutual Life Assurance Assoc.</i> , 27 & 29 <i>Moorgate-street</i> , E.C.
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1908	†Malthy, Charles Hugh, <i>North British and Mercantile Insurance Co.</i> , 61 <i>Threadneedle-street</i> , E.C.	1897	†Moors, Elphinstone McMahon, M.A., <i>University of Sydney, Australia</i> .
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1888 †Nash, Willie Oscar, <i>Law Reversionary Interest Soc., Limited, Thanet-house, 231 & 232 Strand (opposite the Law Courts), w.c.</i>	1909 †Raynes, Harold Ernest, <i>Legal and General Life Assurance Society, 10 Fleet-street, E.C.</i>
1906 †Neill, Samuel Bennett, <i>China Mutual Life Insurance Co., Shanghai, China.</i>	1897 †Rees, Martin, <i>Law Reversionary Interest Soc., Limited, Thanet-house, 231 & 232 Strand (opposite the Law Courts), w.c.</i>
1883 Neison, Francis G. P., F.S.S., <i>19 Abingdon-st., Westminster, s.w.</i>	1901 †Reeve, Charles Ernest, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>
1888 †Newman, Philip Lewin, B.A., <i>Yorkshire Insur. Co., Ltd., York.</i>	1902 †Richmond, George William, <i>Scottish Widows' Fund and Life Assur. Society, 9 St. Andrew-square, Edinburgh.</i>
1865 Newton, Algernon, M.A., <i>The Norfolk Hotel, Brighton, Sussex.</i>	1904 †Rietschel, Hermann Julius, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1887 †Nightingale, Harry Ethelston, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>	1898 †Robinson, George Frederick, <i>Legal and General Life Assur. Society, 10 Fleet-street, E.C.</i>
1903 †Norris, Charles Arthur, <i>National Mutual Life Association of Australasia, Limited, Melbourne, Australia.</i>	1905 †Robinson, Hugh Thomas Kay, <i>Clergy Mutual Assur. Soc., 2 & 3 The Sanctuary, s.w.</i>
1901 †Norton, William Ernest, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1888 †Rusher, Edward Arthur, F.S.S., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1905 †Oakley, Henry John Percy, <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>	1882 †Ryan, Gerald Hemmington, <i>Phoenix Assurance Co., Ltd., 19 Lombard-street, E.C.</i>
1864 Pearson, Arthur, <i>Betchworth-house, The Bank, Highgate, n.</i>	1898 †Salmon, Richard George, F.S.S., <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1905 †Penman, William, Jr., <i>Atlas Assurance Company, Ltd., 92 Cheapside, E.C.</i>	1883 Saunders, Harris Charter Lindon, <i>F.R.A.S., "Marquise," Twickenham.</i>
1891 †Phelps, William Peyton, M.A., (HON. SEC.), <i>Equity and Law Life Assur. Soc., 18 Lincoln's-inn-fields, w.c.</i>	1886 †Schooling, Frederick, F.A.S., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
Under the Charter. Priestley, John George, 44 St. German's-road, Forest-hill, s.e.	1901 †Searle, George Morley, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1891 †Pulley, William Pritchard, <i>Norwich Union Life Insur. Soc., 71 & 72 King William-st., E.C.</i>	1901 †Sharman, William Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1903 †Rae, Joseph, <i>Finance Department, Town-hall, Upper-street, n.</i>	1905 †Sherriff, Francis Henry, <i>Provident Clerks' and General Mutual Life Assurance Assoc., 27 & 29 Moorgate-street, E.C.</i>
1899 †Raisin, Arthur Herbert, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>	

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	Date of becoming a Fellow.
1896 †Sim, William Abernethy, F.F.A., <i>Scottish Union and National Insurance Co., 35 St. Andrew-square, Edinburgh.</i>	1902 †Strong, William Richard, <i>London Guarantee & Accident Co., 42-45 New Broad-street, E.C.</i>
1875 †Smither, Arthur, <i>Green Bank, Lewes.</i>	1884 †Stuart, John Moody, F.F.A., <i>Leeds Permanent Benefit Building Society, Victoria-buildings, Park-lane, Leeds.</i>
1881 †Somerville, William Finlay, <i>Liverpool and London and Globe Insurance Co., 1 Dale-street, Liverpool.</i>	1900 †Sutherland, John, M.A., <i>Australasian Temperance and General Mutual Life Assurance Society, Swanston-street, Melbourne, Australia.</i>
1877 †Sorley, James, F.S.S., F.R.S.E., <i>82 Onslow-gardens, s.w.</i>	1906 †Symmons, Frank Percy, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1898 †Spencer, John, <i>English and Scottish Law Life Assurance Assoc., 12 Waterloo-place, s.w.</i>	1889 †Tarn, Arthur Wyndham, <i>Guardian Assurance Company, 28 King-street, Covent-garden, w.c.</i>
1894 †Sprague, Alfred Ernest, D.Sc., M.A., F.F.A., <i>Edinburgh Life Assurance Co., 26 George-street, Edinburgh.</i>	1887 Teece, Richard, F.F.A., F.A.S., F.S.S., <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1857 Sprague, Thomas Bond, M.A., LL.D., Hon. F.F.A., F.S.S., F.R.S.E. (PAST-PRESIDENT, 1882-86), <i>29 Buckingham-ter, Edinburgh.</i>	1864 †Terry, James, <i>56 Kidbrook Park-road, Blackheath, s.e.</i>
1906 †Spurgeon, Ernest Frank, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1889 †Thiselton, Herbert Cecil, F.F.A., F.A.S., <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1896 †Stahlschmidt, Louis, <i>St. John's College, Agra, India.</i>	1901 †Thodey, Robert, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
Under the Charter Stevens, Charles, <i>Aberdeen Ho., Preston, Brighton.</i>	1893 †Thomas, Ernest Charles, <i>Gresham Life Assurance Society, Limited, St. Mildred's-house, Poultry, E.C.</i>
1888 Stewart, John, F.F.A., <i>City of Glasgow Life Assur. Co., 30 Renfield-street, Glasgow.</i>	1899 †Thomas, Robert Arthur Caradoc, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>
1906 †Stewart, Lionel William, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1905 †Thompson, Thomas Percy, B.A., (TUTOR, Part 1), <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>
1898 Stirling, Robert, F.F.A., <i>Law Union & Rock Insur. Co., Ltd., 126 Chancery-Lane, w.c.</i>	1895 †Thomson, Herbert Archer, M.A., <i>3 Kings-bench-walk, Temple, E.C.</i>
1892 †Straker, Edward Robert, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>	1893 †Thorne, Alfred Charles, <i>Equity & Law Life Assur. Soc., 18 Lincoln's-inn-fields, w.c.</i>
1878 †Straker, Frank Arthur, <i>Legal and General Life Assur. Society, 10 Fleet-street, E.C.</i>	

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.	Date of becoming a Fellow.
1891 †Tilt, Robert Ruthven, <i>General Reversionary & Investment Co., Ltd., 26 Pall-mall, s.w.</i>	1905 †Wilson, John Sydney, <i>Australian Widows' Fund Life Assurance Society, Launceston, Tasmania.</i>
1902 †Tinner, Thomas, <i>Comptroller's Depart., London County Council, Spring-gardens, s.w.</i>	1864 Wilson, Robert, <i>44 Talfourd-rd., Camberwell, s.e.</i>
1881 †Todd, George, M.A., (VICE-PRESIDENT), <i>Economic Life Assurance Society, 6 New Bridge-street, e.c.</i>	1888 †Wilson, Robert, Jr., <i>General Assurance Company, 103 Cannon-street, e.c.</i>
1894 †Todhunter, Ralph, M.A., <i>University Life Assur. Soc., 25 Pall-mall, s.w.</i>	Under the Charter. Winser, Thomas Boorman, F.R.G.S., F.R.N.S., <i>81 Shooter's-hill-road, Blackheath, s.e.</i>
1899 †Trounce, Harold Moltke, M.A., <i>London Life Association, Ltd., 81 King William-street, e.c.</i>	1899 †Winter, Arthur Thomas, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, e.c.</i>
1878 Turnbull, Andrew Hugh, F.F.A., F.R.S.E., <i>18 Whitehouse-loan, Edinburgh.</i>	1897 †Wintle, Lancelot Andrewes, <i>Economic Life Assurance Soc., 6 New Bridge-street, e.c.</i>
1909 †Turner, Sidney, B.A., <i>20 Minster-road, Cricklewood, n.w.</i>	1904 †Wood, Arthur Barton, B.A., F.A.S., <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1889 Wallace, Thomas, F.F.A., <i>North British & Mercantile Insurance Co., 64 Princes-street, Edinburgh.</i>	1884 †Woods, Ernest, F.A.S., <i>Guardian Assurance Company, 11 Lombard-street, e.c.</i>
1905 †Wandless, John Robert, <i>Canada Life Assurance Co., 14 King William-street, e.c.</i>	1902 †Woolmer, Alfred Henry, <i>Star Life Assurance Society, 32 Moorgate-street, e.c.</i>
1906 †Wares, Harold Wallace, <i>Yorkshire Insurance Co., Ltd., Bank-buildings, Princes-street, e.c.</i>	1902 †Workman, William Arthur, <i>Legal and General Life Assur. Soc., 10 Fleet-street, e.c.</i>
1888 †Warner, Samuel George, (VICE-PRESIDENT), <i>Law Union & Rock Insur. Co., Ltd., 126 Chancery-lane, w.c.</i>	1902 †Worthington, William, <i>Royal Insur. Co., Ltd., Liverpool.</i>
1893 †Watson, Alfred William, F.S.S., <i>Wenhamston, Ebers-road, Nottingham.</i>	1875 †Wyatt, Frank Bertrand, F.A.S., (EX-PRESIDENT), <i>Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, s.w.</i>
1895 †Watson, James Douglas, F.A.S., <i>Star Life Assurance Society, 32 Moorgate-street, e.c.</i>	1906 †Young, Arthur Stanley, <i>H.H. The Nizam's Service Family Pension Fund, Hyderabad, The Deccan, India.</i>
1904 †Weatherill, Henry, <i>National Debt Office, e.c.</i>	1874 Young, Thomas Emley, B.A., F.R.A.S., (PAST-PRESIDENT, 1896-S.), <i>108 Evering-road, Stoke Newington, n.</i>
1880 †Whittall, Wm. Joseph Hutchings, F.A.S., <i>18 Airlie-gardens, Campden-hill, w.</i>	

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate	Date of becoming an Associate.
1900 2 Adams, Cecil Francis, <i>New Zealand Insurance Co., Ltd., Accident Branch, Palmerston North, New Zealand.</i>	1894 3 Barton, Robert Whitechurh, 48 William-street, Montreal, Canada.
1908 2 Addey, Leonard, <i>Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, s.w.</i>	1908 2 Beatty, Samuel, M.A., <i>University of Toronto, Toronto, Canada.</i>
1869 2 Adey, Theodore Henry, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>	1901 2 Benjamin, Stanley O., <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1908 2 Alder, Milton Cromwell, <i>Mutual Life & Citizens' Assur- ance Co., Limited, Sydney, Australia.</i>	1908 2 Bennett, Samuel, <i>Registrar of Friendly Societies, and Government Actuary, Perth, West Australia.</i>
1908 2 Anderson, Robert Duncan, 45 Southbrook-road, Lee, S.E.	1881 Birks, Edmund Alfred, <i>Yorkshire Ins. Co., Ltd., York.</i>
1899 2 Ansell, George Frederic, <i>National Debt Office, E.C.</i>	1906 2 Blake, Francis Seymour, <i>London County Council, Spring- gardens, s.w.</i>
1904 2 Ashley, Charles Henry, <i>British Widows' Assurance Co., 1 Old-street, E.C.</i>	1906 2 Blehl, Ernest M., A.M., A.A.S., <i>Philadelphia Life Insurance Co., North American Building, Philadelphia, Pa., U.S.A.</i>
1883 2 Ashley, John Geo., M.A., <i>War Office, s.w.</i>	1898 (2) Blount, Edward Thos. Joseph, F.F.A., F.S.S., <i>Standard Life Assurance Co., 3 George-street, Edinburgh.</i>
1881 2 Ayling, Charles Stephen, <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>	1906 2 Boag, Harold, 33 Albert-drive, Low Fell, Gateshead.
1905 2 Bain, William Algernon, <i>Manufacturers Life Insurance Co., Toronto, Canada.</i>	1873 2 Boon, Gerald Inglis, <i>Omnium Insurance Corp., Ltd., 18 New Bridge-street, E.C.</i>
1909 2 Baker, Sydney Harry, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1906 2 Borrajo, Edward Joseph William, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1903 3 Ball, Sidney Robertson, <i>English and Scottish Law Life Assurance Association, 12 Waterloo-place, s.w.</i>	1908 2 Bradbury, Algernon Charles, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1905 2 Barford, Frederick William, M.A., <i>Australasian Temperance and General Mutual Life Assurance Society, Swanston-street, Mel- bourne, Australia.</i>	1889 (2) Bremner, Thomas William, F.F.A., <i>Mutual Life of New York Building, Martin-place, Sydney, Australia.</i>
1903 2 Barnett, Isaac, <i>North British and Mercantile Insurance Co., 61 Threadneedle- street, E.C.</i>	1905 (2) Brodie, Robert Raynal, F.F.A., <i>Scottish Provident Institution, 6 St. Andrew-sq., Edinburgh.</i>
1904 2 Barrett, William Goodman, <i>United Kingdom Temperance and General Provident Institu- tion, 196 Strand, w.c.</i>	1907 2 Brown, Arthur Ewart, <i>Metropolitan Life Assurance Society, 13 Moorgate-street, E.C.</i>
1885 Barton, Arthur, <i>Royal Insurance Company, Ltd., Maidstone.</i>	1896 (2) Brown, George Andrew, <i>Clerical, Medical & General Life Assurance Society, 1 King William-street, E.C.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.	Date of becoming an Associate.
1899 2 Brown, Harold, <i>Scottish Union and National Insurance Co., 3 King William- street, E.C.</i>	1897 2 Coop, Charles Rowland, <i>United Kingdom Temperance and General Provident Institution, 28 High-street, Birmingham.</i>
1908 2 Brown, James, B.A., <i>Friendly Societies Office, Young- street, Sydney, Australia.</i>	1905 2 Cooper, John James, <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1886 (2) Buckley, Thomas John Wesley, <i>9 St. Andrew-street, Holborn- circus, E.C.</i>	1891 2 Coote, Ernest Charles, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>
1882 Burke, David, F.S.S., <i>Royal Victoria Life Insur. Co., Montreal, Canada.</i>	1871 Coutts, Edwin Arthur, <i>North British and Mercantile Insurance Company, 12 Low- pavement, Nottingham.</i>
1906 2 Burrows, George Eastoe, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1900 2 Covington, Oliver Henry, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1895 3 Butterfield, William Thos., A.C.A., <i>9 Market-street, Bradford, Yorkshire.</i>	1908 2 Coward, Charles Ernest, B.A., <i>Estate Duty Office, Somerset House, W.C.</i>
1905 3 Cameron, Finlay James, F.F.A., <i>Friends' Provident Institution, Bradford, Yorkshire.</i>	1907 (2) Cowan, Hugh Francis, F.F.A., <i>Edinburgh Life Assurance Co., 26 George-street, Edinburgh.</i>
1908 2 Carpenter, Thomas B. Boyd, <i>Clergy Mutual Assur. Society, 2 & 3 The Sanctuary, S.W.</i>	1884 Craig, Robert Alexander, <i>Abstainers' and General Assur. Co., Edmund-street, Birmingham.</i>
1876 Carter, Eric Mackay, <i>33 Waterloo-street, Birmingham.</i>	1908 2 Dark, Thomas Arthur, <i>Excelsior Life Insurance Co., Toronto, Canada.</i>
1906 3 Carter, George Stanley, <i>Life Association of Scotland, 18 Bishopsgate-street-Within, E.C.</i>	1909 (2) Davidson, William, F.F.A., <i>Scottish Amicable Life Assur. Society, 35 St. Vincent-place, Glasgow.</i>
1904 (2) Cathles, Lawrence MacLagan, F.F.A., <i>South Western Life Insurance Co., Dallas, Texas, U.S.A.</i>	1906 2 Davis, Mervyn, B.A., <i>American Home Life Ins. Co., Fort Worth, Texas, U.S.A. (Reinstated, 1909.)</i>
1905 2 Chubb, William, <i>Superintendent of Insurance, Treasury Dept., Quebec, Canada.</i>	1908 2 Dawson, Herbert John, B.A., <i>Royal Military College, King- ston, Ontario, Canada.</i>
1908 2 Clemens, Frederic Broadbent, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1906 2 Defries, Frederick, <i>China Mutual Life Insurance Co., Ltd., Shanghai, China.</i>
1898 2 Coates, Thomas Linnaeus, <i>Mutual Life Insur. Co. of New York, 16, 17 & 18 Cornhill, E.C.</i>	1901 2 Diamond, George Frederick, <i>City Mutual Life Assur. Society, Hunter-st., Sydney, Australia.</i>
1904 2 Collier, Charles Aubrey, 6 Old Palace-yard, S.W.	1901 2 Donald, Alexander Graham, M.A., F.F.A., <i>Scottish Provident Institution, 6 St. Andrew-square, Edinburgh.</i>
1871 Cook, Arthur Janes, M.J.I., 6 Dorset-square, Regent's-park, N.W.	
1899 3 Cook, William Playfair, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>	

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.	Date of becoming an Associate
1881 Donaldson, John, <i>Australian Widows' Fund Life Assurance Society, Collins-street- west, Melbourne, Australia.</i>	1896 2 Featherstonehaugh, William Irwin, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1899 2 Dougherty, Harold, F.S.S., F.C.I.S., (AUDITOR), <i>London and Lancashire Life Assurance Company, 66 & 67 Cornhill, E.C.</i>	1903 2 Ferguson, Colin C., B.A., <i>Great West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>
1902 2 Doust-Smith, Ernest Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1906 3 Fielder, William Crowhurst, <i>Atlas Assurance Company, Ltd., 92 Cheapside, E.C.</i>
1881 Dovey, William Roadly, F.F.A., F.A.S., <i>c/o Bank of New South Wales, Sydney, Australia.</i>	1897 2 Findlay, Alexander Wynaud, LL.D., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1906 2 Downes, Sidney Cecil, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1902 2 FitzGerald, Charles Reginald, <i>State Mutual Life Assur. Co., Worcester, Mass., U.S.A.</i>
1870 Dowson, John, <i>Royal Insur. Co., Ltd., Liverpool.</i>	1901 2 FitzGerald, William George, B.A., <i>95 Charles-street East, Toronto, Canada.</i>
1908 2 Duffell, James Henry, <i>Royal London Mutual Insurance Society, Ltd., Finsbury Square, E.C.</i>	1890 (2) Fox, Charles Edward, F.F.A., <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>
1901 2 Earle, Arthur Percival, <i>Columbian National Life Insur. Co., 176-180 Federal-st., Boston, Mass., U.S.A.</i>	1886 (2) Fox, Morris, F.A.S., <i>New Zealand Government Life Insurance Dept., Wellington, New Zealand.</i>
1868 Eaton, Henry William, <i>Liverpool & London & Globe Insurance Company, William- street, New York, U.S.A.</i>	1894 2 Fraser, Thomas John, <i>Australian Alliance Assurance Company, Melbourne, Australia.</i>
1904 2 Ecroyd, Cuthbert W., <i>Friends' Provident Institution, Ocean Chambers, 44 Waterloo- street, Birmingham.</i>	1907 2 Fulford, William John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1905 2 Elderton, Robert Lapidge, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1909 (2) Fyfe, Aystyn James Claude, F.F.A., <i>Northern Assurance Co., Ltd., 1 Union-terrace, Aberdeen.</i>
1907 2 Eldridge, Ernest Edward Booth, <i>National General Insurance Co., King's House, King-street, E.C.</i>	1901 (2) Gaff, William Robertson, C.A., F.F.A., <i>54 New Broad-street, E.C.</i>
1872 2 Evans, William, F.F.A., F.R.S.E., 38 Morningside-park, Edinburgh.	1873 2 Gage, Uriah Woodard, <i>North British & Mercantile Insur. Co., 61 Threadneedle-st., E.C.</i>
1905 3 Farmer, Ernest Chattock, <i>London, Edinburgh & Glasgow Insurance Co., Ltd., Euston- square, N.W.</i>	1895 2 Galwey, Charles Edmund, <i>New Zealand Government Life Insurance Dept., Wellington, New Zealand.</i>
	1893 2 Gardiner, Robert Edward, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
	1885 2 Gayford, Herbert Stannard, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.	Date of becoming an Associate.
1899 3 Gibb, James Burnett, F.F.A., <i>Penn Mutual Life Insce. Co. of Philadelphia, 923 Chestnut-st., Philadelphia, Pa., U.S.A.</i>	1905 2 Hammond, Reginald, <i>British Equitable Assur. Co., Ltd., 1, 2 & 3 Queen-street-place, E.C.</i>
1909 2 Gilliland, William Henry, B.A., <i>Government Insurance Department, Ottawa, Canada.</i>	1869 Hann, Robert George, F.A.S., <i>Equitable Life Assur. Soc. of the United States, 120 Broadway, New York.</i>
1871 2 Glennie, William Gordon, <i>Scottish Union & National Insur. Co., 3 King William-street, E.C.</i>	1894 2 Hardeastle, Edward Edgington, M.A., F.A.S., <i>Union Central Life Office, Cincinnati, Ohio, U.S.A.</i>
1897 2 Goggs, Frank Sidney, <i>Scottish Metropolitan Assurance Co., Ltd., 25 St. Andrew-sq., Edinburgh.</i>	1900 2 Harding, Harry Burnard, <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>
1882 Goldman, Leopold, F.S.S., <i>North American Life Assurance Co., Toronto, Canada.</i>	1909 2 Harley, Brian, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>
1897 2 Goodwyn, John, <i>Ocean Accident and Guarantee Corporation, Ltd., 131 Pitt-st., Sydney, Australia.</i>	1908 2 Harnack, Frederick William, <i>Sceptre Life Association, Ltd., 40 Finsbury-parement, E.C.</i>
1905 2 Gould, W. H., M.A., <i>Volunteer State Life Insurance Co., Chattanooga, Tennessee, U.S.A.</i>	1909 2 Harrington, Eustace Woods, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>
1908 (2) Graham, George, Jr., F.F.A., <i>Actuary, State Insur. Depart., Springfield, Illinois, U.S.A.</i>	1896 3 Harris, Frederick Joseph, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1908 (2) Granger, Charles Keith, F.F.A., <i>City of Glasgow Life Assurance Co., 30 Renfield-street, Glasgow.</i>	1909 2 Harvey, Percy Norman, <i>Atlas Assurance Company, Ltd., 92 Cheapside, E.C.</i>
1902 2 Gray, Robert Alexander, B.A., 324 Markham-street, Toronto, Canada.	1897 2 Haycraft, William Melluish, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1868 Greig, John Andrew, <i>Sun Life Assurance Society, 60 Charing-cross, S.W.</i>	1897 2 Hazell, Stanley, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>
1907 3 Gunningham, Sidney Joseph, B.Sc., <i>Ecclesiastical Commission, Millbank, S.W.</i>	1895 2 Heness, Leonard Thomas, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1909 2 Hall, Arthur F., <i>North American Life Assurance Co., Toronto, Canada.</i>	1878 Henry, Alfred, F.C.A., <i>Throgmorton-house, Copthall-avenue, E.C.</i>
1903 2 Hall, John Bertram, A.A.S., <i>Dominion Life Assurance Co., Waterloo, Ontario, Canada.</i>	1909 2 Hines, Walter Robert, <i>Norwich Union Life Insurance Society, Norwich.</i>
1905 2 Hallman, M. S., F.A.S., <i>Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.</i>	1894 2 Hollingworth, Albert Charles, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
	1907 2 Holness, Archibald Stephen, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute. Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.	
Date of becoming an Associate.	Date of becoming an Associate
1883 Holt, Edward Hallett, <i>Law Life Assurance Society,</i> 187 Fleet-street, E.C.	1871 Jellicoe, George Rogers, <i>Eagle Insurance Company,</i> 79 Pall-mall, s.w.
1894 ² Home, Noel Charles Minehin, M.A., LL.B., Barrister-at-Law, 6 King's Bench-walk, Temple, e.c. (<i>Reinstated</i> , 1909).	1883 Jerman, Richard, <i>Commercial Union Assurance</i> <i>Company, Exeter.</i>
1909 ⁽²⁾ Hope, Francis Moffat, F.F.A., <i>Caledonian Insurance Company,</i> 19 George-street, Edinburgh.	1908 ² Jerrold, Allan Laman, 64 Rue de la Tour, Paris (XV ^e), France.
1898 ² Howell, Chas. Edward, M.A., LL.D., <i>Standard Life Assurance Compy.,</i> 59 Dawson-street, Dublin.	1896 ² Jobson, Alexander, <i>Challis House, Martin Place,</i> Sydney, Australia.
1899 ³ Hudson, Alfred James, <i>Northern Assurance Company,</i> Ltd., 1 Moorgate-street, E.C.	1894 ² Johnston, Frederick H., F.A.S., <i>Prudential Life Insurance Co.</i> of America, Newark, N.J., U.S.A.
1908 ² Humphreys, John Alfred, <i>National Mutual Life Assurance</i> <i>Society, 39 King-street, Cheap-</i> <i>side, E.C.</i>	1903 ² Jones, Leonard Alexander Mouat, <i>Commercial Union Assur. Co.,</i> 24, 25 & 26 Cornhill, E.C.
1907 ² Humphry, Edmund William, <i>Life Association of Scotland,</i> 18 Bishopsgate-st.-Within, E.C.	1903 ² Jones, Wallace Monat, <i>General Reversionary & Invest-</i> <i>ment Company, Limited,</i> 26 Pall- mall, s.w.
1875 Hunt, Richard Aldington, F.S.S., <i>Wesleyan & General Assur. Soc.,</i> Steelhouse-lane, Birmingham.	1898 ² Kaufman, Henry N., A.A.S., <i>Phoenix Mutual Life Insurance</i> Co., Hartford, Connecticut, U.S.A.
1893 ⁽²⁾ Hunter, Arthur, F.F.A., F.A.S., F.S.S., <i>New York Life Insurance Co.,</i> 346 & 348 Broadway, New York, U.S.A.	1876 Kearry, Joseph, 44 Charlwood-street, Belgrave- road, s.w.
1902 ² Hunter, Robertson G., F.A.S., 161 Devonshire-street, Boston, U.S.A.	1899 ³ Kelly, John Joseph, <i>Mutual Life & Citizens' Assur.</i> Co., Ltd., Sydney, Australia.
1887 ² Hunter, Samuel, 66 St. Lawrence-road, Clontarf, Dublin.	1897 ² Kemp, Julian Ernest Sandford, <i>Eagle Insurance Company,</i> 79 Pall-mall, s.w.
1909 ² Hustwitt, William Edmund, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.	1902 ² Kilgour, David Errett, M.A., F.A.S., <i>North American Life Assurance</i> Co., Toronto, Canada.
1889 ⁽²⁾ Jacobs, Frederick Job, <i>Post Office, Queen Victoria</i> <i>Markets, Sydney, Australia.</i>	1909 ² King, Albert Edward, <i>Provident Clerks' and General</i> <i>Mutual Life Assur. Association,</i> 27 & 29 Moorgate-street, E.C.
1876 ² James, George Trevelyan, 12 Waterloo-place, s.w.	1882 ² King, William Alfred, <i>Northern Assurance Co., Ltd.,</i> 1 Moorgate-street, E.C.
1905 ⁽²⁾ Jamieson, Charles William Steele, F.F.A., <i>Scottish Amicable Life Assur.</i> Society, 1 Threadneedle-st., E.C.	1908 ² Kyd, James Gray, F.F.A., <i>Northern Assurance Co., Ltd.,</i> 1 Union-terrace, Aberdeen.
1905 ³ Jefferson, John Arthur, <i>Britannic Assurance Co., Ltd.,</i> Broad-st.-corner, Birmingham.	1909 ² Lafford, Harry George, <i>Legal and General Life Assur.</i> Society, 10 Fleet-street, E.C.

ASSOCIATES.

Date of becoming an Associate.	Date of becoming an Associate.
1907 ² Laing, John Morrison, <i>Mutual Life Assurance Co. of Canada, Waterloo, Ontario, Canada.</i>	1909 ² Lohan, John Joseph, <i>National Mutual Life Associa- tion of Australasia, Melbourne, Australia.</i>
1893 ² Laing, William Cland, <i>North British and Mercantile Insurance Company, 61 Thread- needle-street, E.C.</i>	1876 ² Lucey, Herbert, <i>General Assurance Company, 103 Cannon-street, E.C.</i>
1908 ² Laird, John Melvin, B.A., <i>Connecticut General Life In- surance Co., Hartford, Conn., U.S.A.</i>	1890 ⁽²⁾ Lugton, Hugh, F.F.A., <i>North British and Mercantile Insurance Co., 61 Threadneedle- street, E.C.</i>
1897 ² Lane, Arthur Vere, B.A., <i>Legal & General Life Assur- ance Society, 217 West George- street, Glasgow.</i>	1900 ³ McArthur, Harry de C., <i>Box 282, Dunedin, New Zealand.</i>
1907 ² Langstaff, Milton Palmer, <i>Continental Life Insurance Co., Toronto, Canada.</i>	1867 McDonald, William Rae, F.F.A., <i>Scottish Metropolitan Assurance Co., Limited, 25 St. Andrew- square, Edinburgh.</i>
1905 ² Latham, Bertrand, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1882 ³ McDougald, Alfred, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>
1906 ⁽²⁾ Latta, Alexander, F.F.A., <i>Guardian Assurance Company, 28 King-st., Covent-garden, w.c.</i>	1905 ² Macfarlane, James Allan, <i>Monarch Life Assurance Co., Winnipeg, Manitoba, Canada.</i>
1899 ² Lawton, George Herbert, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's- square, s.w.</i>	1884 Mackay, Alexander, <i>Law Union & Rock Insur. Co., Ltd., 126 Chancery-lane, w.c.</i>
1905 ³ Leigh, Samuel George, <i>Refuge Assurance Co., Oxford- street, Manchester.</i>	1905 ³ McKechnie, James Baldwin, M.A., F.A.S., <i>Manufacturers Life Insurance Company, Toronto, Canada.</i>
1879 Leitch, Alexander, <i>Scottish Prudent Institution, 3 Lombard-street, E.C.</i>	1896 ² Macmillan, John Campbell, <i>Apartado Postal 827, Mexico D.F.</i>
1897 ² Le Maitre, Frank William, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1905 ² McPhail, Frederick Charles, <i>Colonial Mutual Life Assurance Soc., Ltd., Melbourne, Australia.</i>
1885 Leveaux, Arthur Michael, F.S.S., <i>Registry of Friendly Societies, Central Office, 28 Abingdon- street, Westminster, s.w.</i>	1883 ² Makeham, William Reed, <i>Alliance Assurance Co., Ltd. (Imperial Life Assurance Fund), 47 Chancery-lane, w.c.</i>
1907 ³ Levey, Ralph, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1905 ³ Makepeace, Francis Lucas, B.A., 229 Norwood-rd., Herne-hill, s.e.
1907 ² Ley, James, <i>Office of the Actuary for Friendly Societies, Melbourne, Australia.</i>	1880 Manwaring, Henry, <i>National Debt Office, E.C.</i>
1868 Litchfield, Edward, <i>c/o Messrs. Knox & Service, 41 St. Vincent-place, Glasgow.</i>	1909 ² Marples, Percy Morris, M.A., B.Sc., 57 Oval-road, Gravelly-hill, Birmingham.

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.	Date of becoming an Associate.
1896 2 Martin, Sidney George, <i>National Mutual Life Assoc. of Australasia, Ltd., Melbourne, Australia.</i>	1905 2 Moore, George Edward, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>
1897 2 Mascall, Alfred John, <i>Standard Life Assurance Co., 3 Pall-mall East, s.w.</i>	1898 2 Moore, Joseph Patrick, <i>Mutual Life & Citizens' Assur- ance Co., Ltd., Sydney, Australia.</i>
1900 2 Maund, George Harvard, <i>National Mutual Life Assur. Society, 39 King-st., Cheapside, E.C.</i>	1871 2 Moore, Roderick Mackenzie, <i>United Kingdom Temperance and General Provident Institution, 196 Strand, w.c.</i>
1902 2 Maxwell Benjamin Bell, F.F.A., <i>Scottish Equitable Life Assur. Society, 28 St. Andrew-square, Edinburgh.</i>	1900 2 Nash, Alfred Charles, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-s-square, s.w.</i>
1899 2 Meade, Gerald Willoughby, <i>North British & Mercantile Insurance Company, 61 Thread- needle-street, E.C.</i>	1909 2 Nathan, Eric Burnett, <i>London & Lancashire Life Assur. Co., 66 & 67 Cornhill, E.C.</i>
1896 2 Merfield, Percy Henry, <i>Law Life Assurance Society, 187 Fleet-street, E.C.</i>	1897 2 Newling, Sidney Wallis, B.A., <i>Woodleigh, South Woodford, Essex.</i>
1909 2 Mol. Wilhelmus Johannes Bartholomeus, <i>Algemeene Maatschappij van Levensverzekering en Lijfrente, Soerabaia, Dutch East Indies.</i>	1905 2 Newnham, Ernest Whiffin, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1905 2 Monilaws, William Barrington, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>	1907 2 Nicholl, Charles Carlyon, B.A., F.F.A., <i>Royal Exchange Assoc. Corp., Royal Exchange, E.C.</i>
1879 Monilaws, William Macgeorge, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>	1903 2 Nicholls, Arthur William, <i>Australian Mutual Provident Society, Brisbane, Australia.</i>
1905 2 Monkhouse, Charles Cosmo, B.A., <i>Clerical, Medical and General Life Assurance Sociely, 15 St. James's-s-square, s.w.</i>	1884 Nicoll, John, F.F.A., <i>Life Association of Scotland, 82 Princes-street, Edinburgh.</i>
1877 Moon, James, J.P., <i>Prudential Assurance Company, 30 Dale-street, Liverpool.</i>	1883 Orr, Lewis Potter, F.F.A., <i>Scottish Life Assur. Co., Ltd., 19 St. Andrew-sq., Edinburgh.</i>
1877 Moon, John, <i>Parkhurst, Didsbury, Man- chester.</i>	1908 2 Osborne, William Arthur, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>
1879 Moon, Sidney Norman Laming, <i>Columbian National Life Insur- ance Company, 176-180 Federal- street, Boston, Mass., U.S.A.</i>	1908 2 Owen, David John, B.A., <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>
1903 2 Moore, George Cecil, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1906 2 Padday, Percy King, F.F.A., <i>Scottish Metropolitan Assurance Co., Ltd., 8 King-st., Cheapside, E.C.</i>
	1895 2 Pagden, Lionel King, <i>Union Life Branch of the Com- mercial Union Assur. Co., 1 & 2 Royal Exchange-buildings, E.C.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.	Date of becoming an Associate.
1864 Panton, Edward Henry, 50 Wood-vale, Forest Hill, s.e.	1906 2 Portch, Albert Garfield, F.A.S., <i>Franklin Life Insurance Co.,</i> <i>Springfield, Illinois, U.S.A.</i>
1901 3 Papps, Percy Charles Herbert, F.A.S., <i>Mutual Benefit Life Insur. Co.,</i> <i>Newark, New Jersey, U.S.A.</i>	1890 3 Powell, Alfred, <i>Alliance Assurance Company,</i> <i>Limited, Bartholomew-lane, E.C.</i>
1895 2 Paradice, William Henry, <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>	1869 Pringle, James, C.A., F.F.A., 42 Drumshengh-gardens, Edinburgh.
1869 Park, David Francis, C.A., F.F.A., Kenly Green, Boarhills, Fife, N.B.	1884 Pullar, James, F.F.A., <i>Colonial Mutual Life Assurance</i> <i>Society, Melbourne, Australia.</i>
1907 2 Parker, John Gowans, <i>Imperial Life Assurance Co. of</i> <i>Canada, Toronto, Canada.</i>	1881 Purves, Thomas Peter, <i>New York Life Insurance Com-</i> <i>pany, Sydney, Australia.</i>
1905 2 Paton, Albert George, <i>London Assurance Corporation,</i> 7 Royal Exchange, E.C.	1904 (2) Rankin, John Adam, F.F.A., <i>Edinburgh Life Assurance Co.,</i> 26 George-street, Edinburgh.
1898 (2) Pearce, Henry John, F.F.A., <i>Scottish Amicable Life Assurance</i> <i>Society, 1 Threadneedle-st., E.C.</i>	1867 Ratray, Patrick, C.A., Karishee, Dunblane.
1899 2 Peele, Thomas, <i>Universal Insur. Loan & Invest-</i> <i>ment Co., Ltd., New Briggate,</i> Leeds.	1885 Rea, Charles Herbert Edmund, F.R.A.S., F.S.S., <i>National Standard Assur. Cor-</i> <i>poration, 149 Leadenhall-st., E.C.</i>
1909 2 Perry, Sidney James, <i>Northern Assurance Co., Ltd.,</i> 1 Moorgate-street, E.C.	1898 2 Reid, Edward E., B.A., <i>London Life Insurance Co.,</i> London, Ontario, Canada.
1909 2 Peter, James Calthorpe, <i>London and Lancashire Life</i> <i>Assurance Company, 66 & 67</i> <i>Cornhill, E.C.</i>	1901 2 Rhodes, Francis, B.A., <i>Royal Insur. Co., Ltd., Liverpool.</i>
1900 2 Peters, Charles Furness, <i>L'pool, Victoria Legal Friendly</i> <i>Society, St. Andrew-street, E.C.</i>	1887 (2) Richardson, Josephus Hargreaves, F.F.A., F.A.S., <i>New Zealand Government Life</i> <i>Insurance Department, Wellington,</i> New Zealand.
1907 2 Phillips, Thomas Ashley, <i>Minnesota Mutual Life Insur.</i> <i>Co., St. Paul, Minn., U.S.A.</i>	1879 Roberts, Thomas B., <i>Australian Alliance Assurance</i> <i>Company, Collins-street, Mel-</i> bourne, Australia.
1908 2 Pickup, John Richardson, <i>National Provident Institution,</i> 48 Gracechurch-street, E.C.	1908 2 Robertson, Bernard, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.
1902 2 Pigrome, George Davey, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.	1904 3 Robertson, Frederick William, F.F.A., <i>Yorkshire Insur. Co., Ltd., York.</i>
1899 2 Pipe, Sidney Herbert, F.A.S., 5 Temple Building, Toronto, Canada.	1904 3 Robertson, James Leask, F.F.A., <i>Edinburgh Life Assurance Co.,</i> 26 George-street, Edinburgh.
1909 2 Pollard, Edward Cecil, <i>General Accid., Fire and Life</i> <i>Assurance Corporation, Ltd.,</i> 59-62 Chancery-lane, w.c.	1909 (2) Robertson, John Howard, M.A., F.F.A., <i>University Life Assurance Soc.,</i> 219 St. Vincent-street, Glasgow.

ASSOCIATES.

*Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.*

Date of becoming an Associate.	Date of becoming an Associate.
1878 Robertson, William, F.F.A., <i>29 Stafford-street, Edinburgh.</i>	1861 ² Searle, Thomas John, <i>Mansion - house - chambers, Bucklersbury, E.C.</i>
1876 Robinson, Andrew, <i>Sunningdale-park, Sunningdale, Berks.</i>	1900 ² Searls, Edwin Richard, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>
1885 Ronald, Thomas Robert, <i>Law Guarantee, Trust and Acci- dent Soc., Ltd., 49 Chancery- lane, w.c.</i>	1909 ⁽²⁾ Sellar, Alexander Smith, F.F.A., <i>95 Culverley-road, Catford, S.E.</i>
1909 ² Rowland, Stanley Jackson, <i>Equitable Life Assurance Soc., Mansion House-street, E.C.</i>	1909 ² Sen, Jogesh Chandra, M.A., B.L., <i>15 Sitaram Ghose's-st., Calcutta, India.</i>
1904 ² Rudd, Alfred James, <i>Australian Widows' Fund Life Assurance Society, Grenfell- street, Adelaide, South Australia.</i>	1909 ² Sharp, Harold Gregory, <i>Friends' Provident Institution, Bradford, Yorkshire.</i>
1897 ² Ryley, Edmund, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1900 ² Sharpe, Edgar Cecil Engledue, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>
1896 ² Sanderson, Frank, M.A., F.F.A., F.A.S., LL.D., <i>Canada Life Assurance Company, Toronto, Canada.</i>	1907 ⁽²⁾ Shearer, Gilbert Edward, F.F.A., <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>
1904 ² Sare, Thomas Henry, <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1894 ³ Sheppard, Herbert Norman, B.A., F.A.S., <i>Home Life Insurance Company, 256 Broadway, New York, U.S.A.</i>
1905 ³ Savery, Robert S. B., <i>Gresham Life Assurance Society, Ltd., 30 Rue du Provence, Paris, France.</i>	1897 ² Shinnell, James Edward, F.S.S., <i>United Provident Assurance Co., Ltd., 96 Oxford-rd., Manchester.</i>
1909 ² Savory, Donald Stuart, B.A., <i>37 Churchill-road, Boscombe.</i>	1896 ² Shlager, Joseph, <i>Equitable Life Assurance Society of the United States, Mansion- house-chambers, Adderley-street, Cape Town, South Africa.</i>
1884 Schooling, John Holt, <i>Fotheringay-house, Montpelier- row, Twickenham.</i>	1903 ² Shovelton, Sydney Taverner, M.A., <i>Polruan, Stanhope-av., Finchley, N.</i>
1899 ² Schouten, Pieter, <i>Verzekering Maatschappij, "Arnhem," Stations-plein, 17, Arnhem, Holland.</i>	1905 ² Shute, Oxenham Bent, <i>National Provincial Bank of England, 53 Baker-street, w.</i>
1906 ⁽²⁾ Scott, Albert George, (AUDITOR), <i>English and Scottish Law Life Assur. Association, 12 Waterloo- place, s.w.</i>	1864 Smith, Howard Samuel, F.F.A., F.C.A., F.S.S., <i>Bank-chambers, 11 Waterloo- street, Birmingham.</i>
1873 Scott, Ernest Willem, F.A.S., <i>Algemeene Maatschappij van Lerensverzekering en Lijfrente, Damrak, 74, Amsterdam.</i>	1898 ² Smith, Robert Parker, <i>Royal Insurance Company, Ltd., Liverpool.</i>
1904 ³ Searle, Arthur Joseph, <i>English & Scottish Law Life Assurance Association, 12 Waterloo-place, s.w.</i>	1906 ² Smithier, Herbert Buxton, <i>University Life Assurance Soc., 25 Pall-mall, s.w.</i>
	1905 ² Somerville, Walter Harold, <i>Mutual Life Assur. Co. of Canada, Waterloo, Ontario, Canada.</i>
	1871 Spencer, Robert James, F.S.S., <i>75 King's-road, Southsea.</i>

ASSOCIATES.

*Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.*

Date of becoming an Associate.	Date of becoming an Associate.
1868 Spens, William George, <i>44 Albany-street, Edinburgh.</i>	1906 ³ Thomson, Frederick Robert T., <i>Law Union & Rock Insur. Co., Ltd., 126 Chancery-lane, w.c.</i>
1866 Stark, William Emery, <i>Safe Deposit, Chapel-walks, Manchester.</i>	1909 ⁽²⁾ Thomson, Gordon William, F.F.A., <i>18 Comiston-terrace, Edinburgh.</i>
1878 Stevenson, Charles, <i>9 Albert-square, Manchester.</i>	1904 ⁽²⁾ Thomson, John Walter, F.F.A., <i>F.S.S., Scottish Life Assur. Co., 19 St. Andrew-square, Edinburgh.</i>
1880 Stock, Edward James, <i>National Mutual Life Assoc. of Australasia, Melbourne, Australia.</i>	1883 ² Titmuss, Walter George, <i>Alliance Assur. Co., Ltd. (Provident Life Fund), 50 Regent-street, w.</i>
1909 ² Stocks, Joseph, <i>Law Union and Rock Insur. Co., Ltd., 126 Chancery-lane, w.c.</i>	1905 ² Touzel, Philip Duncan, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1906 ² Story, Cyril Lionel William Steane, <i>Norwich Union Life Insurance Society, 71 & 72 King William-street, E.C.</i>	1905 ³ Townley, E. William, <i>National Mutual Life Assurance Soc., 39 King-st., Cheapside, E.C.</i>
1905 ² Strong, Allan Wilmot, <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>	1902 ³ Traversi, Antonio Thomas, <i>Friendly Societies' Department, Wellington, New Zealand.</i>
1909 ² Strong, William Boughton, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1905 ² Tully, Arthur Patrick Thomas, <i>Gresham Life Assurance Society, Ltd., Sharia Suleiman Pacha, Cairo, Egypt.</i>
1896 ² Stuckey, Jos. James, M.A., <i>Salisbury Chambers, 49a King William-street, Adelaide, South Australia.</i>	1891 ² Turnbull, A. D. Lindsay, F.F.A., C.A., F.C.I.S., <i>18 St. Swithin's-lane, E.C.</i>
1905 ² Stuckey, Reginald Robert, <i>Australian Mutual Provident Soc., Adelaide, South Australia.</i>	1907 ² Underwood, Reginald Edward, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, s.w.</i>
1905 ³ Sturt, Herbert Rothsay, <i>Phoenix Assurance Co., Ltd., 70 Lombard-street, E.C.</i>	1884 Vian, William Collett, <i>Railway Passengers' Assurance Company, 64 Cornhill, E.C.</i>
1904 ² Sugars, Robert Morrison, B.A., <i>Calle de Corrie, 23, San Gervasio, Barcelona, Spain. (Re-instituted, 1909.)</i>	1884 Vincent, Frederick James, F.S.S., <i>London, Edinburgh & Glasgow Assurance Co., Ltd., Euston-square, N.W.</i>
1904 ⁽²⁾ Tatlock, John, M.A., F.R.A.S., F.A.S., <i>141 Broadway, New York, U.S.A.</i>	1899 ² Vokins, George Alfred, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1893 ² Taylor, Arthur, <i>Guardian Assurance Company, 28 King-street, Covent-garden, w.c.</i>	1905 ² Walker, Dwight A., <i>Equitable Life Assurance Society of the United States, 120 Broadway, New York, U.S.A.</i>
1908 ² Thompson, John Henry Reginald, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1879 Wall, Walter George, <i>34 Kingsland-road, Birkenhead.</i>
1906 ³ Thompson, John Spence, <i>Mutual Life Insurance Co. of New York, New York, U.S.A.</i>	

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.	Date of becoming an Associate.
1905 2 Wansbrough, Thomas Percival, <i>English and Scottish Law Life Assurance Assoc., and British Law Fire Insurance Co., 37 Queen Victoria-street, E.C.</i>	1896 2 Wilkinson, Edward Berkeley, <i>24 Maxilla-gardens, N. Kensington, W.</i>
1906 2 Wardrop, James Charles, <i>(AUDITOR), Life Association of Scotland, 18 Bishopsgate-st.-Within, E.C.</i>	1903 2 Wilkinson, William Magnay, <i>Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.</i>
1907 2 Warren, Cyril Ferdinand, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1904 2 Williams, Frederick Alfred, F.S.S., F.A.S., <i>"La Nacional" Compania de Seguros sobre la Vida, Apartado, 1420, Mexico D.F.</i>
1903 2 Watherston, Charles F., B.A., <i>War Office, S.W.</i>	1900 2 Wilson, George, <i>Standard Life Assurance Com- pany, 3 George-st., Edinburgh.</i>
1909 2 Watson, Andrew Daniel, <i>Government Insurance Depart- ment, Ottawa, Canada.</i>	1870 2 Wilson, Henry Edward, <i>Northern Assurance Co., Ltd., 1 Moorgate-street, E.C.</i>
1883 2 Watson, John Robertson, <i>British Law Fire Insurance Co., 105 West George-st., Glasgow.</i>	1873 2 Windett, Charles, <i>Profits & Income Insurance Co., Ltd., 9 Fleet-street, E.C.</i>
1908 2 Watt, Arthur William, <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>	1905 2 Winstanley, Charles William, <i>North British & Mercantile Insurance Co., 61 Threadneedle- street, E.C.</i>
1894 2 Watt, George, <i>Royal Insur. Co., Ltd., Liverpool.</i>	1903 2 Wood, William Archibald Porter, B.A., <i>Canada Life Assurance Co., Toronto, Canada.</i>
1900 2 Watt, James, W.S., F.F.A., <i>24 Rothesay-terrace, Edinburgh.</i>	1909 2 Woodall, Edward Arthur, <i>National Mutual Life Assurance Soc., 39 King-st., Cheapside, E.C.</i>
1902 2 Weatherill, Charles, <i>Scottish Office, S.W.</i>	1883 Woodhouse, Lister, A.C.A., F.S.S., <i>City Comptroller, Westminster City-hall, Charing Cross-road, W.C.</i>
1894 2 Weeks, Rufus Wells, F.A.S., <i>New York Life Insurance Co., 346 & 348 Broadway, New York, U.S.A.</i>	1877 2 Woods, Arthur Biddle, <i>Law Union & Rock Insur. Co., Ltd., 15 New Bridge-street, E.C.</i>
1909 2 Wenn, Albert Edward <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1866 Woods, Bernard, <i>Metropolitan Life Assurance Society, 13 Moorgate-street, E.C.</i>
1909 2 Wenyon, Herbert John, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1879 Wornum, Thornton Selden, <i>Law Union & Rock Insur. Co., Ltd., 15 New Bridge-street, E.C.</i>
1898 3 Whigham, Charles Frederick, F.F.A., C.A., <i>22 Old Broad-street, E.C.</i>	1903 2 Worth, Bertram Oliver, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>
1908 2 White, Osborn Deuyer, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>	
1909 2 White, Wilfred Clare, B.A., A.A.S., <i>Federal Life Assurance Co., Hamilton, Ontario, Canada.</i>	
1897 2 Wiekens, Charles H., <i>Commonwealth Bureau of Census and Statistics, Melbourne, Vic- toria, Australia.</i>	

STUDENTS.

*Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute.
Those marked (1) have been exempted under the Bye-laws from Sections (1) and (2) of Part I of the
current Syllabus.*

Date of becoming a Student.	Date of becoming a Student.
1892 ¹ Aaron, David Hyam, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1899 ¹ Baber, Walter Crosbie, A.A.S., <i>Royal Victoria Life Insur. Co. of Canada, Montreal, Canada.</i>
1903 ¹ Aeum, Wilfred Harry, <i>15 Lordship-lane, Wood Green, N.</i>	1903 ¹ Baggs, Henry Ernest, <i>English and Scottish Law Life Assurance Association, 12 Water- loo-place, s.w.</i>
1905 ¹ Agutter, William John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1907 ¹ Bailey, Frank Arthur, <i>General Reversionary & Invest- ment Co., Ltd., 26 Pall Mall, s.w.</i>
1908 ¹ Aldridge, Wilfred Henry, <i>Liverpool Victoria Insurance Corporation, Ltd., Atlantic House, 45-49 Holborn-bridge, E.C.</i>	1907 ¹ Bannatyne, Arthur Gordon, B.A., <i>19 Westbourne-square, W.</i>
1906 ² Allen, Arthur Ormiston, M.A., B.Sc., <i>2 St. Michael's Villas, Cardigan- road, Leeds.</i>	1908 ¹ Bazell, Harry, <i>Sun Life Assurance Company of Canada, 93 Queen Victoria- street, E.C.</i>
1907 ¹ Allen, Sidney, A.C.A., <i>1 Walbrook, E.C.</i>	1907 ¹ Beaven, Cecil Livingstone, B.A., <i>Royal Military Academy, Wool- wich, s.e.</i>
1904 ¹ Allison, Sinclair E., A.A.S., <i>New York Life Insurance Co., 346 & 348 Broadway, New York, U.S.A.</i>	1907 ¹ Beeston, Harold Lewis, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1908 ¹ Armon, Thomas, A.C.I.S., <i>Pearl Life Assurance Co., Ltd., London-bridge, E.C.</i>	1908 ¹ Bell, William Francis, <i>Universal Insurance Loan and Investment Co., Ltd., New Brig- gate, Leeds.</i>
1904 ¹ Armstrong, Charles Henry, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1898 ¹ Bennell, Samuel Thomas, <i>25 Meath-road, Ilford.</i>
1886 Arnold, Thomas, Jr., <i>British Equitable Assur. Co., Ltd., 1, 2 & 3 Queen-street-place, E.C.</i>	1906 ¹ Bennett, Henry Gordon, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1902 ¹ Askwith, Thomas Nowell, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1902 ¹ Biden, Norman Frederick, <i>11 Lower Wycombe-rd., Central Bay, Sydney, Australia.</i>
1905 ¹ Atkins, Francis Cuthbert, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1895 ¹ Bigby, Robert Frederick Mitchell, <i>General Assurance Company, 103 Cannon-street, E.C.</i>
1904 ¹ Ayseough, Ivan, <i>Equity and Law Life Assurance Soc., 18 Lincoln's-inn-fields, W.C.</i>	1900 ¹ Bingeman, Milton H., <i>Great West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>
	1903 ¹ Binney, Charles Eardley Wilmot, <i>Royal Exchange Assurance Cor- poration, Royal Exchange, E.C.</i>

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute.
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Date of becoming a student.		Date of becoming a Student.	
1905	¹ Blackadar, E. Gordon, B.A., A.A.S., <i>Government Annuities Branch, Department of Trade and Commerce, Ottawa, Canada.</i>	1891	¹ Brown, William Heron, <i>Gresham Life Assur. Soc., Ltd., St. Mildred's-house, Poultry, E.C.</i>
1908	¹ Blake, Leslie Sarjant, <i>10 Onslow-road, Richmond, Surrey.</i>	1907	² Bullwinkle, Leonard Albert, <i>c/o T. G. Ackland, Esq., 5 & 6 Clement's Inn, Strand, w.c.</i>
1887	Blossom, James, <i>24 Grange-crescent, Sheffield.</i>	1905	³ Burrows, Victor Albert, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1892	¹ Boddy, Henry Mitchell, F.S.S., <i>Manufacturers Life Insurance Co., Cape Town, South Africa.</i>	1908	¹ Cammack, Edmund Ernest, A.A.S., <i>African Life Assurance Society, Ltd., Hatfield House, President and Elöff-streets, Johannesburg, South Africa.</i>
1906	¹ Bolt, Jan Cornelis, <i>Middelland-straat 102, Rotterdam, Holland.</i>	1903	¹ Capon, Frank Christopher, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1902	¹ Bowerman, Judah Philip, <i>Southern States Mutual Life Insur. Co., Charleston, Kanawha County, West Virginia, U.S.A.</i>	1902	¹ Capon, Geoffrey William, <i>Norwich Union Life Insurance Society, Norwich.</i>
1897	¹ Bowles, Francis Marsh, <i>Pearl Life Assurance Co., Ltd., London-bridge, E.C.</i>	1907	¹ Carey, Norman Lewis, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, s.w.</i>
1891	¹ Boyd, Henry Norris, F.F.A., <i>City of Glasgow Life Assurance Company, 21 St. Andrew-square, Edinburgh.</i>	1908	¹ Carmichael, Charlton, <i>The Limes, Ingatestone, Essex.</i>
1905	² Bradshaw, Frank Law, <i>Royal Exchange Assur. Corporation, Royal Exchange, E.C.</i>	1907	¹ Cashman, Thomas, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>
1899	¹ Brady, John Francis, <i>Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.</i>	1900	¹ Chambers, John Joseph, <i>17 Albion-street, Leeds.</i>
1906	¹ Breeds, Arthur Heywood, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1907	¹ Chandler, Francis Philip, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>
1908	¹ Brenton, William Percy, <i>Radlett, Herts.</i>	1902	¹ Chandler, Frederick Joseph, <i>Eagle Insurance Co., 79 Pall-mall, s.w.</i>
1894	¹ Brough, Frank, <i>Federal Life Assurance Company, Hamilton, Ontario, Canada.</i>	1907	¹ Charles, Ashley Hyde, <i>13 South Moulton-street, w.</i>
1906	¹ Brown, B. G. H., <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>	1908	¹ Chase, Harold Philip, <i>National Mutual Life Assur. Soc., 39 King-st., Cheapside, E.C.</i>
1906	¹ Brown, Peter Gordon, <i>Ecclesiastical Commission, Millbank, s.w.</i>	1897	¹ Cherry, John Arnold, Barrister-at-Law, <i>Bombay Port Trust, Bombay, India. (Reinstated, 1905.)</i>

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute.
Those marked (1) have been exempted under the Bye-laws from Sections (1) and (2) of Part I of the current Syllabus.

Date of becoming a Student.	Date becoming a Student.
1903 ¹ Cheshire, Harold Frank, <i>9 Wellington-place, Hastings.</i>	1894 Cox, Edward William, <i>Canada Life Assurance Co., Toronto, Canada.</i>
1905 ¹ Clarke, Herbert George, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>	1907 ¹ Cox, Harry, <i>London, Edinburgh and Glasgow Assurance Company, Euston- square, N.W.</i>
1907 ⁽¹⁾ Clarke, Harold Thomas, B.A., <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>	1894 Cox, Herbert Coplin, <i>Canada Life Assurance Co., Toronto, Canada.</i>
1897 ¹ Clinton, George, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1905 ¹ Cox, Stanley Nelson, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1902 ³ Clinton, Louis Ernest, <i>Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.</i>	1907 ¹ Crang, James Simon, <i>77 Blenheim-road, Waltham- stow, E.</i>
1901 ¹ Cockerton, John Leonard, <i>Pioneer Life Assurance Co., Ltd., 67 Dale-street, Liverpool.</i>	1887 ¹ Cross, Henry John, <i>240 Trinity-rd., Wandsworth- common, S.W.</i>
1895 ¹ Cogar, William Edward, <i>New York Life Insurance Co., Trafalgar-square, W.C.</i>	1907 ¹ Currie, James Thorn, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1899 ¹ Collins, Patrick A., <i>Mutual Life & Citizens' Assur. Co., Ltd., Sydney, Australia.</i>	1907 ¹ Curtis, Augustus Thomas George, <i>London, Edinburgh and Glasgow Assurance Company, Euston- square, N.W.</i>
1902 ¹ Collins, William Ernest, <i>Fell. Inst. Accts. S.A.</i> , 49a King William-street, Adelaide, South Australia.	1904 ¹ Cushing, Robertson Macaulay, <i>Sun Life Assurance Company of Canada, Montreal, Canada.</i>
1896 ¹ Cook, Henry Milton, <i>Standard Life Assurance Co., Dalhousie-sq., Calcutta, India.</i>	1904 ¹ Dalrymple, Alfred George, <i>Canada Life Assurance Company, Toronto, Canada.</i>
1903 ¹ Cook, H. Milton, B.A., <i>Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.</i>	1897 ¹ Dalton, John, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>
1906 ¹ Cooper, John Lewis, <i>Liverpool and London and Globe Insur. Co., 1 Dale-st., Liverpool.</i>	1889 ¹ Davies, Hugh Myddleton, <i>Royal Insur. Co., Ltd., Liverpool.</i>
1905 ¹ Contts, Kenneth Vawdrey, <i>Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, S.W.</i>	1900 ¹ Davies, William Allison, <i>Assistant Borough Treasurer, Town Hall, Birkenhead.</i>
1904 ² Cowdy, Henry Leslie, <i>Scottish Union & National Insur. Co., 3 King William-street, E.C.</i>	1891 ¹ Dawson, Frank Aubrey, <i>Ecclesiastical Insurance Office, Limited, 11 Norfolk-street, Strand, W.C.</i>
	1902 ² Deck, James Gilbert, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Exams. of the Institute. Those marked (1) have been exempted under the Bachelors from Sections (1) & (2) of Part I of the current Syllabus.

Date of becoming a Student.	Date of becoming a Student.
1902 ¹ Denmark, Robert John, <i>Norwich Union Life Insurance Society, Norwich.</i>	1905 ² Edwards, Herbert Horace, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1901 ¹ Dent, Ernest Edward, <i>London and Lancashire Life Assurance Company, 66 & 67 Cornhill, E.C.</i>	1892 ¹ Eddy, Arthur Malcolm, <i>Mutual Life & Citizens' Assur- ance Co., Ltd., Sydney, Australia.</i>
1905 ² Derrick, Victor Percival Augustine, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1901 ¹ Egerton, Harold Edward, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1896 ¹ de Ville, Francis, <i>Clergy Pensions Institution, 11 Norfolk-street, Strand, W.C.</i>	1907 ¹ Emery, Charles Grover, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1906 ¹ Dobbie, John Albert, <i>Provincial Normal School, Ottawa, Canada.</i>	1906 ² Emery, Walter Sydney, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>
1890 ¹ Doeker, Leslie, <i>North British and Mercantile Insurance Co., 61 Threadneedle- street, E.C.</i>	1906 ¹ Emmerson, Walter Hector Ross, <i>London and Lancashire Life Assurance Company, 66 & 67 Cornhill, E.C.</i>
1906 ² Doucet, Gerald Danby, <i>Law Union & Rock Insurance Company, Ltd., 15 New Bridge- street, E.C.</i>	1907 ² Epps, George Selby Washington, B.A., <i>English and Scottish Law Life Assur. Association, 12 Waterloo- place, S.W.</i>
1906 ² Doyle, Joseph Patrick, <i>Mutual Life & Citizens' Assur- ance Co., Ltd., Sydney, Australia.</i>	1908 ¹ Evans, Cyril Ormond, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1904 ¹ Drake, Charles Clifford Hall, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1906 ¹ Fairlie, James, M.A., <i>Franklin Life Insurance Co., Springfield, Illinois, U.S.A.</i>
1908 ¹ Eames, George Stanley, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1892 ¹ Farrell, John, <i>Mutual Life & Citizens' Assur- ance Co., Ltd., Sydney, Australia.</i>
1905 ¹ Eastcott, William Merrill, <i>Sun Life Assur. Co. of Canada, Ottawa, Canada.</i>	1906 ¹ Fender, William Martin, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1892 ¹ Edwards, Edward Samuel, M.A., <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1907 ¹ Fidler, William Edward, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>
1905 ¹ Edwards, Herbert Alfred, <i>28 Plashet-rd., Upton Manor, E.</i>	1908 ¹ Fielder, Tom Lionel, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>

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Date of becoming a student.	Date of becoming a student.
1908 1 Finlayson, G. D., <i>Government Insurance Department, Ottawa, Canada.</i>	1908 1 Gawler, Oswald, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>
1904 3 Fippard, Richard Clift, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1901 1 Gerrish, Frank Wilfred, B.A., <i>Minerva-ville, Albert-rd.-south, Buckhurst-hill, Essex.</i>
1901 1 Fisher, John William, B.A., A.A.S., <i>Crown Life Insurance Co., Toronto, Canada.</i>	1901 1 Glassford, David Murray, <i>Mutual Life & Citizen's Assur. Co., Ltd., Sydney, Australia.</i>
1896 1 Fisk, George William Victor, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1893 Glasson, George Cornish, <i>Economic Life Assurance Soc., 4 St. Stephen's-chbrs., Baldwin-street, Bristol.</i>
1904 1 Fletcher, Andrew W. A. C., <i>Standard Life Assurance Co., 3 George-street, Edinburgh.</i>	1902 1 Gleave, Charles Sheldon, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1905 1 Flynn, Benedict Devine, F.A.S., <i>Travelers Insurance Company, Hartford, Conn., U.S.A.</i>	1899 1 Goddard, Egbert, <i>Coombe Brook, Coombe, Surrey. (Reinstated, 1908).</i>
1905 1 Forbes, James, A.A.S., <i>Great West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>	1909 1 Golden, Charles Albert, <i>Norwich Union Life Insur. Soc., Norwich.</i>
1906 1 Foster, Joseph, <i>33 Westwood-street, Moss Side, Manchester.</i>	1894 1 Golding, Arthur, <i>40 Allerton-road, Stoke Newington, N.</i>
1906 1 Foster, Wilfred Justus, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1905 1 Goodall, Ernest Victor, <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>
1901 1 Franklin, Herbert Dare, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1903 1 Gopp, John Eve, <i>14 Church-hill-road, Walthamstow, E.</i>
1908 1 Frisby, Herbert Rowell, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1907 1 Grant, Frederick John, <i>Star Life Assur. Soc., 30 Cross-street, Manchester.</i>
1906 1 Frost, Charles Frederick, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1909 1 Gravatt, Hubert Charles Alfred, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1900 1 Garner, James, <i>9 Arlington-gardens, Gunnersbury, W.</i>	1907 1 Green, John Spence, <i>Omnium Insurance Corporation, Ltd., 18 New Bridge-street, E.C.</i>

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Date of becoming a Student.	Date of becoming a Student.
1907 1 Green, William James, <i>Australian Metropolitan Life Assurance Company, Sydney, Australia.</i>	1889 1 Harris, Henry, <i>Friends' Provident Institution, 17 Gracechurch-street, E.C.</i>
1886 Greening, Herbert Joseph, <i>Abstainers' & General Insurance Company, Edmund-street, Birmingham.</i>	1908 1 Harris, Sydney Ewart, <i>41 Houston-road, Forest-hill, s.e.</i>
1907 1 Guthrie, Isles Hampden, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>	1908 1 Harrison, Alfred Lowther, <i>Northern Assurance Company, Ltd., 1 Moorgate-street, E.C.</i>
1901 1 Hamilton, George Powell, <i>North American Life Assurance Company, McLean Block, 10 Douglas-street, Guelph, Ontario, Canada.</i>	1908 1 Harrison, Wilfrid, <i>Eastburn, Hexham-on-Tyne.</i>
1905 1 Hamley, Ernest Fountain, <i>Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.</i>	1896 Haskins, George Frederick, A.C.A., <i>18 Walbrook, E.C.</i>
1902 1 Hammant, Francis Clive, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1894 1 Hatten, David Leslie, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>
1905 1 Hammond, Harry Pierson, B.A., A.A.S., <i>Insurance Dept. of the State of Connecticut, Hartford, Conn., U.S.A.</i>	1908 1 Hawes, Ernest Edward, <i>North British & Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>
1892 Hanecock, Arthur Tom, <i>Clerical, Medical & General Life Assurance Society, 15 St.James's-square, S.W.</i>	1907 2 Henry, Alfred, <i>5 Branstone-road, Kew-gardens.</i>
1906 1 Handford, John James William, <i>Scottish Office, S.W.</i>	1905 1) Heron, David, M.A., <i>Viewbank, New Sccone, Perth, N.B.</i>
1901 1 Harpell, James John, <i>Policyholders' Association, 14 Classic-avenue, Toronto, Canada. (Reinstated, 1908.)</i>	1906 1 Hilbery, Reginald William, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>
1901 1 Harper, Henry, <i>83 Waverley-road, Small Heath, Birmingham.</i>	1902 1 Hodge, Cecil Wilfred, <i>Star Life Assurance Society, 32 Moorgate-street, E.C.</i>
	1907 1 Holgate, Benjamin, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
	1908 2 Holgate, Thomas, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
	1898 2 Hooper, George Duncan, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>

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Date of becoming a Student.	Date of becoming a Student.
1902 1 Houston, Charles Cornelius, <i>Metropolitan Asylums Board,</i> <i>Victoria-embankment, E.C.</i>	1890 2 Jackson, Samuel, F.F.A., <i>Scottish Widows' Fund and Life</i> <i>Assurance Society, Liverpool.</i>
1901 1 Howell, Archibald Rennie, B.A., <i>Royal Insurance Company, Ltd.,</i> <i>Montreal, Canada.</i>	1896 1 Jepps, John Blacklee, <i>Star Life Assurance Society,</i> <i>32 Moorgate-street, E.C.</i>
1907 1 Howell, Percy, <i>34 Marlow House, Calvert-</i> <i>avenue, E.</i>	1905 1 Johns, Arthur Humphreys, <i>Colonial Mutual Life Assurance</i> <i>Society, Melbourne, Australia.</i>
1907 1 Hudson, Claude Hamilton, <i>Australasian Temperance and</i> <i>General Mutual Life Assurance</i> <i>Society, Melbourne, Australia.</i>	1904 1 Johnson, Frank Henry, <i>Law Life Assurance Society,</i> <i>187 Fleet-street, E.C.</i>
1898 Hughes, Arthur J., <i>China Mutual Life Insur. Co.,</i> <i>Shanghai, China.</i>	1898 1 Johnston, Arthur Edward, <i>3 Cumnor-road, Sutton.</i>
1902 1 Hughes, Charles, A.A.S., <i>State of New York Insurance</i> <i>Dept., 165 Broadway, New York,</i> <i>U.S.A.</i>	1908 1 Johnston, Henry Frieborn, <i>University of Toronto, Toronto,</i> <i>Canada.</i>
1902 1 Hugill, Herbert, <i>"Briarfield," Keighley.</i>	1908 1 Johnstone, William Darnley, <i>Phoenix Assurance Co., Ltd.,</i> <i>70 Lombard-street, E.C.</i>
1908 1 Hull, Edgar Penrose, <i>67 Beauval-road, Dulwich, S.E.</i>	1903 2 Jones, Ernest Stephens, <i>National Debt Office, E.C.</i>
1904 1 Humphreys, Harry Lewis, <i>Phoenix Assurance Company,</i> <i>Limited, 70 Lombard-street, E.C.</i>	1908 1 Jones, Ernest Washington, <i>30 Bertram-road, Hendon, N.W.</i>
1891 Hunt, Arthur Leonard, <i>Bryns, Somerville-road, Sutton</i> <i>Coldfield.</i>	1896 1 Jones, Richard Foxley, A.C.I.S., <i>Refuge Assurance Co., Oxford-</i> <i>street, Manchester.</i>
1909 1 Hurley, John Cromwell, <i>Australian Mutual Provident</i> <i>Society, Melbourne, Australia.</i>	1909 1 Jones, Richard McNair, B.A., <i>210 Putney-bridge-road, s.w.</i>
1907 1 Hutchings, Leonard Hollinworth, <i>Phoenix Assurance Company,</i> <i>Limited, 70 Lombard-street, E.C.</i>	1907 1 Keable, Henry Batten, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1902 ① Jackson, Charles William, M.A., F.A.S., <i>Greensboro Life Insurance Co.,</i> <i>Greensboro, N.C., U.S.A.</i>	1907 1 Keachie, Morton M., <i>34 Isabella-street, Toronto,</i> <i>Canada.</i>
1902 2 Jackson, Herbert Moore, <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>	1906 1 Kearns, William Norman, <i>Royal Insur. Co., Ltd., Liverpool.</i>
	1905 1 Keevil, Norman Alexander Clement, <i>Blagdon, Station-road, New</i> <i>Barnet.</i>

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current Syllabus.*

Date of becoming a Student.	Date of becoming a Student.
1905 ¹ Kenchington, Frank, B.A., <i>North British and Mercantile Insurance Company, 61 Thread- needle-street, E.C.</i>	1906 Lewis, David Hugh, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>
1906 ¹ Kime, Virgil Morrison, <i>Missouri State Life Insurance Co., Chemical Building, St. Louis, Missouri, U.S.A.</i>	1904 ² Lewty, Francis Arthur, <i>Equity and Law Life Assurance Society, 18 Lincoln's-inn-fields, w.c.</i>
1894 ² Kingsbury, James William, <i>Australian Mutual Provident Society, South Sea House, 37 Threadneedle-street, E.C.</i>	1889 ¹ Lighton, Harold John, <i>Law Union & Rock Insurance Co., Ltd., 126 Chancery-lane, w.c.</i>
1903 ¹ Kirsopp, Frederick, <i>Liverpool Victoria Legal Friendly Society, St. Andrew- street, E.C.</i>	1908 ¹ Lithgow, James Hector Farn- combe, <i>Manufacturers Life Insurance Co., Toronto, Canada.</i>
1895 ¹ Knight, Alfred Murray, <i>Bank-house, Chapel-st., Devon- port.</i>	1895 ¹ Littell, Lewis Lloyd, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>
1908 ¹ Kubota, Takajiro, <i>82b Portsdown-rd., Maida-vale, W.</i>	1904 ¹ Littlefair, James Taylor, <i>Refuge Assurance Co., Oxford- street, Manchester.</i>
1902 ¹ Lang, Frederick John, <i>Royal London Mutual Insur. Soc., Ltd., Finsbury-square, E.C.</i>	1906 ¹ Lolley, Clement Francis, <i>17 Frankland-terr., Leeds.</i>
1907 ¹ Latham, Fergus Norman Wilkinson, <i>3 Wyresdale-rd., Bolton, Lancs.</i>	1908 ¹ Long, Walter Meriton, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1907 ¹ Ledger, Robert John, <i>Grove-lodge, Grove-rd., Epsom.</i>	1890 Love, Robert, <i>Ecclesiastical Insurance Office, Ltd., 11 Norfolk-street, Strand, w.c.</i>
1894 Leonard, Maurice, <i>Frith Hill Cottage, Great Missenden, Bucks.</i>	1906 ¹ McCall, Robert, <i>58 Lichfield-road, Bow, E.</i>
1906 ¹ Le Rossignol, Leonard F., <i>English and Scottish Law Life Assur. Association, 12 Waterloo- place, S.W.</i>	1888 ¹ McConway, James Robert, <i>15 Henthorn-road, New Ferry, Cheshire.</i>
1908 ¹ Lever, Ernest H., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1906 ¹ McCulloch, James Arthur, <i>Ecclesiastical Commission, Mill- bank, S.W.</i>

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Date of becoming a Student.	Date of becoming a Student.
1903 1 Macdonald, Charles Strange, M.A., <i>Confederation Life Association, Toronto, Canada.</i>	1906 1 Martin, William Alexander, <i>National Mutual Life Association of Australasia, Melbourne, Australia.</i>
1907 1 Maeleod, John, <i>Yorkshire Insurance Co., Ltd., 2 Bank-buildings, Princes-st., E.C.</i>	1904 1 Matheson, Donald, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>
1908 1 McMullen, William Albert, <i>32 Woodberry-grove, Finsbury-park, N.</i>	1906 1 Maund, Henry Ernest, <i>69 Tyrwhitt-road, St. John's, S.E.</i>
1907 1 Macorqudale, F. D., <i>Manufacturers Life Ins. Co., Toronto, Canada.</i>	1908 1 Meakin, William Lionel, <i>National General Ins. Co., King's House, King Street, Cheapside, E.C.</i>
1908 1 MacTavish, Archie Neil, B.A., <i>Government Insurance Department, Ottawa, Canada.</i>	1890 1 Meikle, Henry George Watson, F.F.A., <i>Oriental Government Security Life Assurance Co., Limited, Bombay, India.</i>
1903 1 Manly, George William, B.A., <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>	1908 1 Meredith, Charles Edward Buchanan, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1908 1 Mann, Frederick Christmas, <i>Liverpool & London & Globe Insurance Co., 1 Cornhill, E.C.</i>	1892 1 Meyers, Henry Wilson, <i>National Mutual Life Association of Australasia, 295 Queen-street, Brisbane, Australia.</i>
1904 1 Marlin, James Harold, <i>Ocean Accident and Guarantee Corporation, 36-44 Moorgate-street, E.C.</i>	1907 1 Miller, Arthur Axel, <i>32 Kyrerdale-road, N.</i>
1905 1 Marshall, Arthur William, <i>Consolidated Assur. Co., Ltd., Temple-bar-house, 23 Fleet-street, E.C.</i>	1899 2 Minns, Ernest Edwin, <i>Norwich Union Life Insurance Society, Norwich.</i>
1908 1 Marshall, Cecil George, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1907 1 Monilaws, Stanley Hope, <i>Scottish Provident Institution, 3 Lombard-street, E.C.</i>
1905 1 Marshall, John Edwin, <i>Prudential Assurance Company, 47 Earl-street, Coventry.</i>	1902 2 Moore, Hubert Fred, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>
1903 1 Martin, Frederick Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1902 1 Morton, Francis, <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>

STUDENTS.

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 Those marked (1) have been exempted under the Bye-laws from Sections (1) and (2) of Part I of the current Syllabus.

Date of becoming a Student.	Date of becoming a Student.
1907 ¹ Morton, Frederick William, <i>British Widows' Assurance Co.,</i> <i>1 Old-street, E.C.</i>	1897 ¹ Patrick, James, <i>Audit Office, Town-hall, Birkenhead.</i> (<i>Reinstated, 1905.</i>)
1902 ¹ Muckle, Charles P., <i>Union Life Assur. Co., Toronto,</i> <i>Canada. (Reinstated, 1907.)</i>	1906 ¹ Patrick, Walter S., <i>North Cliff, Cambridge-road,</i> <i>King's-heath, Birmingham.</i>
1903 ¹ Myers, Harry Duxbury, A.S.A.A., <i>Burlington-chambers, North-st.,</i> <i>Keighley.</i>	1907 ¹ Pattison, George Benjamin, <i>Manufacturers Life Insurance</i> <i>Company, Toronto, Canada.</i>
1907 ¹ Nash, Kenneth Oscar, <i>The Baru, Thames Ditton.</i>	1896 ² Penny, Charles Augustus, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1906 ¹ Needell, Brian, <i>Alliance Assurance Co., Ltd.</i> (<i>Provident Life Fund</i>), 50 <i>Regent-street, w.</i>	1901 ¹ Petter, Herbert, <i>Britannic Assurance Co., Ltd.,</i> <i>Broad-st.-corner, Birmingham.</i>
1903 ¹ Neill, William Adam Hoyes, <i>Scottish Widows' Fund & Life</i> <i>Assur. Soc., 28 Cornhill, E.C.</i>	1908 ¹ Phillips, Ernest William, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1907 ¹ Newland, Edward Albert, <i>City Equitable Fire Insur. Co.,</i> <i>38 Old Jewry, E.C.</i>	1904 ¹ Phillips, Walter, A.C.I.S., A.S.A.A., <i>c/o Messrs. Arthur Guinness,</i> <i>Son & Co., Ltd., St. James's-</i> <i>gate, Dublin.</i>
1902 ⁽¹⁾ O'Connor, William, M.A., M.D., <i>Mutual Life Insurance Co. of</i> <i>New York, Toronto, Canada.</i>	1908 ¹ Pickworth, Edgar Broughton, <i>65 Hindes-road, Harrow.</i>
1892 ¹ O'Reilly, Anthony James, <i>Government Insurance Depart-</i> <i>ment, Ottawa, Canada.</i>	1907 ¹ Pocock, Horace George Grooby, <i>Alliance Assurance Company,</i> <i>Ltd. (Imperial Life Assurance</i> <i>Fund), 47 Chancery-lane, w.c.</i>
1897 ¹ Osborn, Nathaniel Banner Francis, <i>11 Bruce-grove, Tottenham, N.</i>	1898 Poort, Willem Anthomie, Phil. Nat. Doct., <i>Algemeene Friesche Levens-</i> <i>verzekerings Maatschappij Leeu-</i> <i>warden, Leeuwarden, Holland.</i>
1901 ¹ Papworth, Frederick William, A.S.A.A., 54 Elderton-road, Westclif- on-Sea.	1903 ⁽¹⁾ Porter, Frank, M.A., <i>Mansfield House, Canning Town, E.</i>
1904 ¹ Parker, Walter Montgomery, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1907 ⁽¹⁾ Preston, John Edwin, B.A., <i>Yorkshire Insur. Co., Ltd., York.</i>
1895 ¹ Pascoe, William Yeoman Bennett, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1906 ¹ Priestman, Basil, 23 Highfield-road, Edgbaston, Birmingham.

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Date of becoming a Student.	Date of becoming a Student.
1908 1 Proddow, William Norman, <i>Pearl Life Assurance Company, Ltd., London Bridge, E.C.</i>	1896 1 Robinson, Frederick Charles, <i>Royal Exchange Assur. Corporation, Royal Exchange, E.C.</i>
1907 1 Prout, Herbert John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1893 1 Roll, Frederick James, <i>Pearl Life Assurance Co., Ltd., London-bridge, E.C.</i>
1908 1 Purry, William Baldwin, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1893 1 Roodenburgh, Bartholomeus Adrianus, <i>Terzekeringsbank Victoria, 126 Keizersgracht, Amsterdam.</i>
1901 1 Ramsay, Cecil Byron, <i>Mutual Life Insur. Co. of New York, 16, 17 & 18 Cornhill, E.C.</i>	1895 1 Ross, Christopher Watson, <i>c/o Messrs. M. Moss & Co., Flinders-lane, Melbourne, Australia.</i>
1905 3 Reeve, Gilfrid Mentier, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>	1909 1 Rossetti, Gabriel Arthur Madox, <i>30 Harper's-lane, Bolton, Lancashire.</i>
1898 1 Reynell, Guy Courtenay, <i>Eagle Insurance Company, 79 Pall-mall, s.w.</i>	1901 1 Rountree, Arthur FitzGerald, <i>c/o Messrs. Ralli Bros., Calcutta, India.</i>
1904 1 Reyner, Harry Fane, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>	1895 Rowley, James Edward, A.C.A., <i>30 Waterloo-street, Birmingham.</i>
1894 1 Richards, Gilbert P. A., <i>Oak Cottage, Bulwer-road, New Barnet.</i>	1906 1 Ruddell, Francis, <i>Consolidated Assur. Co., Temple-bar-house, 23 Fleet-street, E.C.</i>
1908 1 Richardson, George Rowley, Jr., "Dawmunkuttie," Anglesea-road, Kingston.	1907 1 Rushton, Thomas Arthur, <i>85 Fleet-street, E.C.</i>
1904 1 Ridgway, Wulfrie, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1899 1 Rutter, Edward Valentine, <i>Phoenix Assurance Company, Limited, 70 Lombard-street, E.C.</i>
1902 1 Robertson, Aubrey Charles, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1907 1 Sanders, Bertram G. T., <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>
1908 1 Robinson, Archie, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>	1897 1 Scott, Alexander Lewis, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1903 1 Robinson, Ernest William, <i>Colonial Mutual Life Assurance Society, Ltd., 105 Pitt-street, Sydney, Australia.</i>	

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Date of becoming a Student.	Date of becoming a Student.
1906 ¹ Selmer, Carl Frederick, <i>c/o C. J. Hambro & Sons, 70 Old Broad-street, E.C. (Reinstated, 1909.)</i>	1907 ¹ Smith, Reginald Thomas, <i>7 Ashmount-rd., Hornsey-lane, N.</i>
1908 ⁽¹⁾ Shepherdson, Herbert Jepson, B.A., B.Sc., <i>Royal Insurance Company, Limited, Liverpool.</i>	1907 ¹ Smith, Sydney Arthur George, <i>Fire Offices Committee, Appliances Department, 3 York-street, Manchester.</i>
1907 ¹ Shine, John Nugent, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1903 ¹ Smith, William, B.A., <i>Standard Life Association, Ltd., 28 Elizabeth-street, Sydney, Australia.</i>
1907 ¹ Shinmi, Shoji, <i>Manufacturers Life Insurance Company, Toronto, Canada.</i>	1903 ² Sneddon, Andrew William, <i>Australian Mutual Provident Society, South Sea House, 37 Threadneedle-st., E.C.</i>
1908 ¹ Shurrock, Christopher William, <i>London, Edinburgh & Glasgow Assurance Co., Euston-square, N.W.</i>	1907 ¹ Spiegel, Ellis William Ralfs, <i>70 Coniston-rd., Muswell-hill, N.</i>
1906 ² Simmonds, Reginald Claud, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1904 ¹ Spring, Stanley Harold, <i>London Guarantee and Accident Company, 42-45 New Broad-street, E.C.</i>
1892 ¹ Simpson, William Murray, <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>	1901 ¹ Steffensen, Johan F., <i>Forsikringsraadet, 1 Christians-gade, Copenhagen.</i>
1905 ¹ Sinclair, Coll Claude, B.A., <i>Great West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>	1906 ¹ Stephenson, Herbert Roy, <i>Manufacturers Life Insurance Company, Toronto, Canada.</i>
1891 ¹ Sindall, Alfred John, <i>London and Lancashire Life Assur. Co., 66 & 67 Cornhill, E.C.</i>	1886 ² Stirling, James, F.F.A., <i>Law Union and Rock Insur. Co., Ltd., 126 Chancery-lane, W.C.</i>
1907 ¹ Singer, Charles Paul, <i>84 Grove-park-ter., Chiswick, w.</i>	1908 ¹ Stiver, Claud Frank, <i>Manufacturers Life Insurance Co., Toronto, Canada.</i>
1888 ² Slimon, William James, F.F.A., <i>10 Mayfield-terrace, Edinburgh.</i>	1888 ¹ Stott, Walter, <i>Royal Insur. Co., Ltd., Liverpool.</i>
1905 ¹ Sloan, Joseph James Eastwood, <i>Royal Insur. Co., Ltd., Liverpool.</i>	1893 ¹ Streeter, Revd. Theodore Edward, <i>St. John's College, Winnipeg, Manitoba, Canada.</i>
1907 ¹ Smith, Frederick James, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>	

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current Syllabus.*

Date of becoming a Student.	Date of becoming a Student.
1904 ² Strong, Gordon Gilbert, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1902 ¹ Thwaites, Frederick George, <i>Norwich Union Life Insurance Society, Norwich.</i>
1904 ¹ Stuart, Arthur William, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1907 ¹ Tomlinson, Benjamin, <i>London, Edinburgh and Glasgow Assurance Company, Euston- square, N.W.</i>
1907 ¹ Sturgeon, Robert W., <i>Royal Insur. Co., Ltd., Liverpool.</i>	1897 ¹ Townshend, Edward Villiers, <i>Scottish Widows' Fund and Life Assurance Soc., 28 Cornhill, E.C.</i>
1904 ¹ Sturt, Arthur James, <i>Phoenix Assurance Company, Limited, 70 Lombard-street, E.C.</i>	1907 ¹ Trembach, Allan Edward, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1907 ¹ Stutfield, Martin, <i>Consolidated Assur. Company, Temple-bar-hse., 23 Fleet-st., E.C.</i>	1907 ¹ Turner, John Gilmour, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1906 ¹ Sutton, Maurice William, <i>93 Grove-end-road, St. John's- wood, N.W.</i>	1905 ¹ Tutill, Hubert Linzee, <i>English & Scottish Law Life Assur. Association, 12 Waterloo- place, S.W.</i>
1907 ¹ Tayler, Harold H., <i>London, Edinburgh & Glasgow Assurance Company, Euston- square, N.W.</i>	1908 ¹ Tyler, David Kinnear, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1907 ¹ Taylor, Frederick George, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1891 Tyler, Edgar Alfred, F.S.A.A., F.C.I.S., F.S.S., <i>9 Old Jewry-chambers, Bank, E.C.</i>
1907 ¹ Taylor, Frederick Rowland Stallard, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>	1906 ¹ Tyler, Victor William, <i>Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.</i>
1907 ¹ Taylor, Herbert George Brooks, <i>National Mutual Life Assoc. of Australasia, 5 Cheapside, E.C.</i>	1909 ¹ Van Homrigh, Gerald Mayaffre, <i>British Widows' Assurance Co., 1 Old-street, E.C.</i>
1908 ¹ Thomlinson, Harry, <i>Yorkshire Insur. Co., Ltd., York.</i>	1906 ³ Vaughan, Hubert, <i>Mutual Life & Citizens' Assurance Co., Ltd., Sydney, Australia.</i>
1905 ¹ Thompson, Joseph William, <i>Norwich Union Life Insurance Society, Norwich.</i>	1907 ¹ Vineberg, Harris Elias, <i>Security Life and Annuity Co., Greensboro, N.C., U.S.A.</i>
1904 ¹ Thompson, William George, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>	
1906 ¹ Thomson, Ernest H. W., <i>London and Lancashire Fire Insuree. Co., Dale-st., Liverpool.</i>	

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Date of becoming a Student.	Date of becoming a Student.
1908 1 Walters, Arthur Hawksley, A.C.A., 15 George-street, Mansion-house, E.C.	1908 1 Williams, Caradoc, Alliance Assurance Co., Ltd., St. James's-street, s.w.
1906 1 Warhurst, James, Alliance Assur. Co., Ltd. (<i>Pro- vident Life Fund</i>), 68 Fountain- street, Manchester.	1895 1 Williams, Henry Samuel Walter, Liverpool & London & Globe Insur. Co., Melbourne, Australia.
1908 1 Warner, Arthur Joseph, Equitable Life Assurance Soc. of the United States, 6 Princes- street, Bank, E.C.	1900 (1) Williams, Lewis, B.A., Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.
1904 1 Warnock-Fielden, Francis Hugh, Prudential Assurance Company, Holborn-bars, E.C.	1907 1 Williams, Thomas Walter, London, Edinburgh and Glasgow Assurance Company, Euston- square, N.W.
1903 1 Watson, Alexander R. D., 89 Queen-street, Auckland, New Zealand.	1906 1 Williamson, Wallace White, Norwich Union Life Insurance Society, Norwich.
1906 1 Watson, John A., Law Guarantee, Trust and Ac- cident Soc., Ltd., 49 Chancery- lane, w.c.	1907 1 Wilson, William Clement, Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.
1906 (1) Webb, Herbert Anthony, M.A., Trinity College, Cambridge.	1901 2 Wilton, Herbert George, Norwich Union Life Insurance Society, Norwich.
1898 1 Webb, Lloyd, Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.	1894 1 Windett, Sydney V., Eagle Ins. Co., 79 Pall-mall, s.w.
1907 1 Welch, Leslie Gordon, Phoenix Assurance Company, Limited, 76 Lombard-street, E.C.	1905 1 Wisdom, Sidney Herbert, Estate Duty Office, Somerset- house, w.c.
1905 1 Wellington, Frank, Australian Mutual Provident Society, Melbourne, Australia.	1903 2 Wolfenden, Edgar Sydney, Australian Mutual Provident Society, Sydney, Australia.
1893 1 Welman, Arthur Joseph, Legal & General Life Assurance Soc., 15 Tithebarn-st., Liverpool.	1908 2 Wolfenden, Hugh Herbert, The Grange, Sidcup, Kent.
1905 1 Welsh, Willis, Prudential Assurance Company, Holborn-bars, E.C.	1895 1 Wood, David James, Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.
	1901 1 Wood, Roland Stuart, Liverpool & London & Globe Insurance Co., 1 Cornhill, E.C.

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1902	¹ Woodhouse, David Alfred, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>	1906	¹ Yeomans, Ernest Charles, <i>Australian Widows' Fund Life Assur.Soc, Melbourne, Australia.</i>
1900	¹ Woolston, Paul Livingston, B.S., <i>50 Maine-avenue, Ocean-grove, New Jersey, U.S.A.</i>	1903	¹ Young, Henry J., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1907	¹ Wright, Alexander William, <i>Dereholm, Oakleigh-road, New Southgate.</i>	1897	¹ Younger, R. H., <i>Liverpool & London & Globe Insur. Co., 1 Dale-st., Liverpool.</i>
1886	Yeatman, Alexander Alfred, <i>2 Coleman-street, E.C.</i>	1904	¹ Zumstein, Herbert Christian, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1895	¹ Yeldham, William James, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>		

CORRESPONDING MEMBERS.

Austria.

TRIEST.

Dr. Julius Graf,
Secretary of the "Assicurazioni Generali." Member of the Mathematisch-Statistischen Vereinigung, and of the Deutscher Verein für Versicherungs-Wissenschaft.

VIENNA.

Dr. Ernest Blaschke,
Professor at the "Technischen Hochschuler." Member of the Mathematisch-Statistischen Vereinigung; Judenplatz, 3-4.

Dr. James Klang,
General Director of the "Oesterreichischer Phœnix" Life Assurance Company. Member of the Mathematisch-Statistischen Vereinigung; Reichsgasse, 3.

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M. George H. Adan,
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Belgium—contd.

BRUSSELS—contd.

M. Am. Bégault, F.A.S.,
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M. Léon Hamoir,
Directeur-Général de la Cie. des Propriétaires Réunis; 15 Rue de Loxum.

M. Fl. Hankar,
Directeur-Général de la Caisse Générale d'Epargne et de Retraite; 50 Rue du Fossé-aux-Loups.

M. Omer Lepreux, F.A.S.,
Directeur-Général Honoraire de la Caisse Générale d'Epargne et de Retraite. Directeur de la Banque Nationale de Belgique. Président du Comité Permanent des Congrès Internationaux d'Actuaires. Président de l'Association des Actuaires Belges. Membre Correspondant de l'Institut des Actuaires Français. Membre de la Commission Permanente des Sociétés Mutualistes, de la Commission Centrale de Statistique,

Belgium—contd.**BRUSSELS—contd.**

et de la Commission des Accidents du Travail; 39 Rue de Turin.

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Prof. Thorvald Nicolai Thiele, Dr. Phil.,
Stockholmsgade 25.

France.**PARIS.**

M. Paul Guieysse, F.A.S.,
Président de l'Institut des Actuaires Français. Député du Morbihan; 2 Rue Dante.

M. Léon Marie, F.A.S.,
Sous-Directeur, Le Phénix Compagnie d'Assurances sur la vie. Membre Correspondant de l'Association des Actuaires Belges. Membre Agréé de l'Institut des Actuaires Français; 28 Rue de Châteaudun (IX^e).

M. Albert Quiquet, F.A.S.,
Actuaire, La Nationale Compagnie d'Assurances sur la vie. Membre Correspondant de l'Association des Actuaires Belges, et de l'Association des Actuaires Suisses. Vice-Président de l'Institut des Actuaires Français; 17 Rue Lafitte (IX^e).

Germany.**GOTHA.**

Dr. Johannes Karup,
Actuary of the Gotha Life Assurance Company.

Dr. Karl Samwer,
Manager of the Gotha Life Assurance Company, Ohrdrufferstrasse, 5.

Hungary.**BUDAPEST.**

M. Julius Altenburger,
Consulting Actuary. Corresponding Member of the "Institut des Actuaires Français," and of the "Association des Actuaires Belges." Member of the "Versicherungs-Wissenschaftliche Vereinigung of Vienna"; vii. Hajtsár ut, 20.

* * It is requested that any inaccuracy in the foregoing list may be pointed out to the ASSISTANT SECRETARY.

Italy.**FLORENCE.**

M. Guido Toja,
Ingénieur. Directeur Général des Compagnies Italienne d'Assurances sur la Vie et Contre L'Incendie "La Fondiaria." Membre du Comité permanent des Congrès Internationaux d'Actuaires. Membre Correspondant de l'Association des Actuaires Belges. Membre de la "Commission Reale per la Valutazione dei disavanzi delle Casse Pensioni peroriarie Italiane." Membre du "Circolo Matematico di Palermo." Ancien Professeur chargé du Cours de "Matematica Attuariale à l'Università Bocconi di Milano." Piazza Vittorio Emanuele.

Russia.**ST. PETERSBURG.**

M. Serge de Savitch,
Professor of Mathematics in the Electro-Technical Institute and in the University of St. Petersburg. Manager of the Tariffs' Service of the United Russian Insurance Companies; Nikolaevskaia, 35.

Spain.**MADRID.**

Dr. José Maluquer y Salvador,
General Manager, National Provident Institution (Spain), Member of the Institute of Social Reforms; calle de Campomanes, 10.

Sweden.**STOCKHOLM.**

M. Anders Lindstedt, Dr. Phil.,
Professor of Mathematics and Director of the Polytechnic. Counsellor of the Regency for Insurance Questions. Member of the Royal Swedish Academy of Science; etc.

Switzerland.**ZURICH.**

Dr. Gottfried Schaertlin,
Manager of the "Schweizerischen Lebensversicherungs-und Rentenanstalt."

United States.**NEW YORK.**

Mr. David Parks Fackler, M.A., F.A.S.,
Past-President of the Actuarial Soc. of America (1891-93). Consulting Actuary; 35 Nassau-street.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Assurance Companies Act, 1909.

A.D.1909.

[9 Edward VII. ch. 49.]

An Act to consolidate and amend and extend to other Companies carrying on Assurance or Insurance business the Law relating to Life Assurance Companies, and for other purposes connected therewith.

[3 December 1909.]

BE it enacted by the King's most Excellent Majesty, by and with the advice and consent of the Lords Spiritual and Temporal, and Commons, in this present Parliament assembled, and by the authority of the same, as follows :

Companies to which Act applies.

1.—This Act shall apply to all persons or bodies of persons, whether corporate or unincorporate, not being registered under the Acts relating to friendly societies or to trade unions (which persons and bodies of persons are hereinafter referred to as assurance companies), whether established before or after the commencement of this Act and whether established

Companies to
which Act
applies.

within or without the United Kingdom, who carry on within the United Kingdom assurance business of all or any of the following classes :

- (a) Life assurance business ; that is to say, the issue of, or the undertaking of liability under, policies of assurance upon human life, or the granting of annuities upon human life ;
- (b) Fire insurance business ; that is to say, the issue of, or the undertaking of liability under, policies of insurance against loss by or incidental to fire ;
- (c) Accident insurance business ; that is to say, the issue of, or the undertaking of liability under, policies of insurance upon the happening of personal accidents, whether fatal or not, disease, or sickness, or any class of personal accidents, disease, or sickness ;
- (d) Employers' liability insurance business ; that is to say, the issue of, or the undertaking of liability under, policies insuring employers against liability to pay compensation or damages to workmen in their employment ;
- (e) Bond investment business ; that is to say, the business of issuing bonds or endowment certificates by which the company, in return for subscriptions payable at periodical intervals of two months or less, contract to pay the bond holder a sum at a future date, and not being life assurance business as hereinbefore defined ;

subject as respects any class of assurance business to the special provisions of this Act relating to business of that class.

A company registered under the Companies Acts which transacts assurance business of any such class as aforesaid in any part of the world shall for the purpose of this provision be deemed to be a company transacting such business within the United Kingdom.

General.

Deposit.

2.—(1) Every assurance company shall deposit and keep deposited with the Paymaster-General for

and on behalf of the Supreme Court the sum of twenty thousand pounds.

(2) The sum so deposited shall be invested by the Paymaster-General in such of the securities usually accepted by the Court for the investment of funds placed under its administration as the company may select, and the interest accruing due on any such securities shall be paid to the company.

(3) The deposit may be made by the subscribers of the memorandum of association of the company, or any of them, in the name of the proposed company, and, upon the incorporation of the company, shall be deemed to have been made by, and to be part of the assets of, the company, and the registrar shall not issue a certificate of incorporation of the company until the deposit has been made.

(4) Where a company carries on, or intends to carry on, assurance business of more than one class, a separate sum of twenty thousand pounds shall be deposited and kept deposited under this section as respects each class of business, and the deposit made in respect of any class of business in respect of which a separate assurance fund is required to be kept shall be deemed to form part of that fund, and all interest accruing due on any such deposit or the securities in which it is for the time being invested shall be carried by the company to that fund.

(5) The Paymaster-General shall not accept a deposit except on a warrant of the Board of Trade.

(6) The Board of Trade may make rules with respect to applications for warrants, the payment of deposits, and the investment thereof or dealing therewith, the deposit of stocks or other securities in lieu of money, the payment of the interest or dividends from time to time accruing due on any securities in which deposits are for the time being invested, and the withdrawal and transfer of deposits, and the rules so made shall have effect as if they were enacted in this Act, and shall be laid before Parliament as soon as may be after they are made.

(7) This section shall apply to an assurance

company registered or having its head office in Ireland, subject to the following modifications :

References to the Supreme Court shall be construed as references to the Supreme Court of Judicature in Ireland, and references to the Paymaster-General shall be construed as references to the Accountant-General of the last-mentioned Court.

Separation of funds.

3.—(1) In the case of an assurance company transacting other business besides that of assurance or transacting more than one class of assurance business, a separate account shall be kept of all receipts in respect of the assurance business or of each class of assurance business, and the receipts in respect of the assurance business, or, in the case of a company carrying on more than one class of assurance business, of each class of business, shall be carried to and form a separate assurance fund with an appropriate name : Provided that nothing in this section shall require the investments of any such fund to be kept separate from the investments of any other fund.

(2) A fund of any particular class shall be as absolutely the security of the policy holders of that class as though it belonged to a company carrying on no other business than assurance business of that class, and shall not be liable for any contracts of the company for which it would not have been liable had the business of the company been only that of assurance of that class, and shall not be applied, directly or indirectly, for any purposes other than those of the class of business to which the fund is applicable.

Accounts and balance sheets

4.— Every assurance company shall, at the expiration of each financial year of the company, prepare :

- (a) A revenue account for the year in the form or forms set forth in the First Schedule to this Act and applicable to the class or classes of assurance business carried on by the company ;
- (b) A profit and loss account in the form set forth in the Second Schedule to this Act, except where the company carries on assurance business of one class only and no other business ;
- (c) A balance sheet in the form set forth in the Third Schedule to this Act.

5.—(1) Every assurance company shall, once in every five years, or at such shorter intervals as may be prescribed by the instrument constituting the company, or by its regulations or byelaws, cause an investigation to be made into its financial condition, including a valuation of its liabilities, by an actuary, and shall cause an abstract of the report of such actuary to be made in the form or forms set forth in the Fourth Schedule to this Act and applicable to the class or classes of assurance business carried on by the company.

(2) The foregoing provisions of this section shall also apply whenever at any other time an investigation into the financial condition of an assurance company is made with a view to the distribution of profits, or the results of which are made public.

6.—Every assurance company shall prepare a statement of its assurance business at the date to which the accounts of the company are made up for the purposes of any such investigation as aforesaid in the form or forms set forth in the Fifth Schedule to this Act and applicable to the class or classes of assurance business carried on by the company: Provided that, if the investigation is made annually by any company, the company may prepare such a statement at any time, so that it be made at least once in every five years.

7.—(1) Every account, balance sheet, abstract, or statement hereinbefore required to be made shall be printed, and four copies thereof, one of which shall be signed by the chairman and two directors of the company and by the principal officer of the company and, if the company has a managing director, by the managing director, shall be deposited at the Board of Trade within six months after the close of the period to which the account, balance sheet, abstract, or statement relates: Provided that, if in any case it is made to appear to the Board of Trade that the circumstances are such that a longer period than six months should be allowed, the Board may extend that period by such period not exceeding three months as they think fit.

Actuarial
report and
abstract.

Statement of
assurance
business.

Deposit of
accounts, &c.,
with Board
of Trade.

(2) The Board of Trade shall consider the accounts, balance sheets, abstracts, and statements so deposited, and, if any such account, balance sheet, abstract, or statement appears to the Board to be inaccurate or incomplete in any respect, the Board shall communicate with the company with a view to the correction of any such inaccuracies and the supply of deficiencies.

(3) There shall be deposited with every revenue account and balance sheet of a company any report on the affairs of the company submitted to the shareholders or policy holders of the company in respect of the financial year to which the account and balance sheet relates.

(4) Where an assurance company registered under the Companies Acts in any year deposits its accounts and balance sheet in accordance with the provisions of this section, the company may, at the same time, send to the registrar a copy of such accounts and balance sheet; and, where such copy is so sent, it shall not be necessary for the company to send to the registrar a statement in the form of a balance sheet as required by subsection (3) of section twenty-six of the Companies (Consolidation) Act, 1908, and the copy of the accounts and balance sheet so sent shall be dealt with in all respects as if it were a statement sent in accordance with that subsection.

8 Edw. 7.
c. 69.

Right of
shareholders,
&c., to copies
of accounts,
&c.

Audit of
accounts.

8 & 9 Vict.
c. 16.

8.—A printed copy of the last-deposited accounts, balance sheet, abstract, or statement, shall on the application of any shareholder or policy holder of the company be forwarded to him by the company by post or otherwise.

9.—Where the accounts of an assurance company are not subject to audit in accordance with the provisions of the Companies (Consolidation) Act, 1908, or the Companies Clauses Consolidation Act, 1845, relating to audit, the accounts of the company shall be audited annually in such manner as the Board of Trade may prescribe, and the regulations made for the purpose may apply to any such company the provisions of the Companies (Consolidation) Act, 1908, relating to audit, subject to such adaptations

and modifications as may appear necessary or expedient.

10.—Every assurance company which is not registered under the Companies Acts, or which has not incorporated in its deed of settlement section ten of the Companies Clauses Consolidation Act, 1845, shall keep a “Shareholders’ Address Book,” in accordance with the provisions of that section, and shall, on the application of any shareholder or policy holder of the company, furnish to him a copy of such book, on payment of a sum not exceeding sixpence for every hundred words required to be copied.

11.—Every assurance company which is not registered under the Companies Acts shall cause a sufficient number of copies of its deed of settlement or other instrument constituting the company to be printed, and shall, on the application of any shareholder or policy holder of the company, furnish to him a copy of such deed of settlement or other instrument on payment of a sum not exceeding one shilling.

12.—Where any notice, advertisement, or other official publication of an assurance company contains a statement of the amount of the authorised capital of the company, the publication shall also contain a statement of the amount of the capital which has been subscribed and the amount paid up.

13.—(1) Where it is intended to amalgamate two or more assurance companies, or to transfer the assurance business of any class from one assurance company to another company, the directors of any one or more of such companies may apply to the Court, by petition, to sanction the proposed arrangement.

(2) The Court, after hearing the directors and other persons whom it considers entitled to be heard upon the petition, may sanction the arrangement if it is satisfied that no sufficient objection to the arrangement has been established.

(3) Before any such application is made to the Court :

(a) notice of the intention to make the application shall be published in the *Gazette*; and

List of share-holders.

Deed of settlement.

Publication of authorised, subscribed, and paid-up capital.

Amalgamation or transfer.

- (b) a statement of the nature of the amalgamation or transfer, as the case may be, together with an abstract containing the material facts embodied in the agreement or deed under which the amalgamation or transfer is proposed to be effected, and copies of the actuarial or other reports upon which the agreement or deed is founded, including a report by an independent actuary, shall, unless the Court otherwise directs, be transmitted to each policy holder of each company in manner provided by section one hundred and thirty-six of the Companies Clauses Consolidation Act, 1845, for the transmission to shareholders of notices not requiring to be served personally: Provided that it shall not be necessary to transmit such statement and other documents to policy holders other than life, endowment, sinking fund, or bond investment policy holders, nor in the case of a transfer to such policy holders if the business transferred is not life assurance business or bond investment business; and
- (c) the agreement or deed under which the amalgamation or transfer is effected shall be open for the inspection of the policy holders and shareholders at the offices of the companies for a period of fifteen days after the publication of the notice in the *Gazette*.

(4) No assurance company shall amalgamate with another, or transfer its business to another, unless the amalgamation or transfer is sanctioned by the Court in accordance with this section.

14.—Where an amalgamation takes place between any assurance companies, or where any assurance business of one such company is transferred to another company, the combined company or the purchasing company, as the case may be, shall, within ten days from the date of the completion of the amalgamation or transfer, deposit with the Board of Trade:

- (a) certified copies of statements of the assets and liabilities of the companies concerned in

Statements in
in case of
amalgamation
or transfer.

such amalgamation or transfer, together with a statement of the nature and terms of the amalgamation or transfer; and

- (b) a certified copy of the agreement or deed under which the amalgamation or transfer is effected; and
- (c) certified copies of the actuarial or other reports upon which that agreement or deed is founded; and
- (d) a declaration under the hand of the chairman of each company, and the principal officer of each company, that to the best of their belief every payment made or to be made to any person whatsoever on account of the amalgamation or transfer is therein fully set forth, and that no other payments beyond those set forth have been made or are to be made either in money, policies, bonds, valuable securities, or other property by or with the knowledge of any parties to the amalgamation or transfer.

15.—The Court may order the winding up of an assurance company, in accordance with the Companies (Consolidation) Act, 1908, and the provisions of that Act shall apply accordingly, subject, however, to the following modification :—

Special pro-
visions as to
winding up
of assurance
companies.

The company may be ordered to be wound up on the petition of ten or more policy holders owning policies of an aggregate value of not less than ten thousand pounds:

Provided that such a petition shall not be presented except by the leave of the Court, and leave shall not be granted until a *prima facie* case has been established to the satisfaction of the Court and until security for costs for such amount as the Court may think reasonable has been given.

16.—(1) Where the assurance business or any part of the assurance business of an assurance company has been transferred to another company under an arrangement in pursuance of which the first-mentioned company (in this section called the subsidiary company) or the creditors thereof has or

Winding up
of subsidiary
companies.

have claims against the company to which such transfer was made (in this section called the principal company), then, if the principal company is being wound up by or under the supervision of the Court, the Court shall (subject as hereinafter mentioned) order the subsidiary company to be wound up in conjunction with the principal company, and may by the same or any subsequent order appoint the same person to be liquidator for the two companies, and make provision for such other matters as may seem to the Court necessary, with a view to the companies being wound up as if they were one company.

(2) The commencement of the winding up of the principal company shall, save as otherwise ordered by the Court, be the commencement of the winding up of the subsidiary company.

(3) In adjusting the rights and liabilities of the members of the several companies between themselves, the Court shall have regard to the constitution of the companies, and to the arrangements entered into between the companies, in the same manner as the Court has regard to the rights and liabilities of different classes of contributories in the case of the winding up of a single company, or as near thereto as circumstances admit.

(4) Where any company alleged to be subsidiary is not in process of being wound up at the same time as the principal company to which it is subsidiary, the Court shall not direct the subsidiary company to be wound up unless, after hearing all objections (if any) that may be urged by or on behalf of the company against its being wound up, the Court is of opinion that the company is subsidiary to the principal company, and that the winding up of the company in conjunction with the principal company is just and equitable.

(5) An application may be made in relation to the winding up of any subsidiary company in conjunction with a principal company by any creditor of, or person interested in, the principal or subsidiary company.

(6) Where a company stands in the relation of a principal company to one company, and in the

relation of a subsidiary company to some other company, or where there are several companies standing in the relation of subsidiary companies to one principal company, the Court may deal with any number of such companies together or in separate groups, as it thinks most expedient, upon the principles laid down in this section.

17.—(1) Where an assurance company is being wound up by the Court, or subject to the supervision of the Court, or voluntarily, the value of a policy of any class or of a liability under such a policy requiring to be valued in such winding up shall be estimated in manner applicable to policies and liabilities of that class provided by the Sixth Schedule to this Act.

Valuations of annuities and policies.

(2) The rules in the Sixth and Seventh Schedules to this Act shall be of the same force, and may be repealed, altered, or amended, as if they were rules made in pursuance of section two hundred and thirty-eight of the Companies (Consolidation) Act, 1908, and rules may be made under that section for the purpose of carrying into effect the provisions of this Act with respect to the winding up of assurance companies.

18.—The Court, in the case of an assurance company which has been proved to be unable to pay its debts, may, if it thinks fit, reduce the amount of the contracts of the company upon such terms and subject to such conditions as the Court thinks just, in place of making a winding-up order.

Power to Court to reduce contracts.

19.—Section two hundred and seventy-four of the Companies (Consolidation) Act, 1908 (which contains provisions as to companies incorporated outside the United Kingdom), shall apply to every assurance company constituted outside the United Kingdom which carries on assurance business within the United Kingdom, whether incorporated or not.

Extension of § Edw. 7 c. 69 s. 274, to all assurance companies established outside the United Kingdom.

20.—The Board of Trade may direct any documents deposited with them under this Act, or certified copies thereof, to be kept by the registrar or by any other officer of the Board of Trade; and any such documents and copies shall be open to inspection, and copies thereof may be procured by any person on

Custody and inspection of documents deposited with Board of Trade.

payment of such fees as the Board of Trade may direct.

Evidence of documents.

21.—(1) Every document deposited under this Act with the Board of Trade, and certified by the registrar or by any person appointed in that behalf by the President of the Board of Trade to be a document so deposited, shall be deemed to be a document so deposited.

(2) Every document purporting to be certified by the registrar, or by any person appointed in that behalf by the President of the Board of Trade, to be a copy of a document so deposited shall be deemed to be a copy of that document, and shall be received in evidence as if it were the original document, unless some variation between it and the original document be proved.

Alteration of forms.

22.—The Board of Trade may, on the application or with the consent of an assurance company, alter the forms contained in the schedules to this Act as respects that company, for the purpose of adapting them to the circumstances of that company.

Penalty for non-compliance with Act.

23.—Any assurance company which makes default in complying with any of the requirements of this Act shall be liable to a penalty not exceeding one hundred pounds, or, in case of a continuing default, to a penalty not exceeding fifty pounds for every day during which the default continues, and every director, manager, or secretary, or other officer or agent of the company who is knowingly a party to the default shall be liable to a like penalty; and, if default continue for a period of three months after notice of default by the Board of Trade (which notice shall be published in one or more newspapers as the Board of Trade may, upon the application of one or more policyholders or shareholders, direct), the default shall be a ground on which the Court may order the winding up of the company, in accordance with the Companies (Consolidation) Act, 1908.

Penalty for falsifying statements, &c.

24.—If any account, balance sheet, abstract, statement, or other document required by this Act is false in any particular to the knowledge of any person who signs it, that person shall be guilty of a misdemeanour

and shall be liable on conviction on indictment to fine and imprisonment, or on summary conviction to a fine not exceeding fifty pounds.

25.—Every penalty imposed by this Act shall be recovered and applied in the same manner as penalties imposed by the Companies (Consolidation) Act, 1908, are recoverable and applicable.

26.—Any notice which is by this Act required to be sent to any policy holder may be addressed and sent to the person to whom notices respecting such policy are usually sent, and any notice so addressed and sent shall be deemed and taken to be notice to the holder of such policy :

Provided that where any person claiming to be interested in a policy has given to the company notice in writing of his interest, any notice which is by this Act required to be sent to policy holders shall also be sent to such person at the address specified by him in his notice.

27.—The Board of Trade shall lay annually before Parliament the accounts, balance sheets, abstracts, statements, and other documents under this Act, or purporting to be under this Act, deposited with them during the preceding year, except reports on the affairs of assurance companies submitted to the shareholders or policy holders thereof, and may append to such accounts, balance sheets, abstracts, statements, or other documents any note of the Board of Trade thereon, and any correspondence in relation thereto.

28.—(1) This Act shall not affect the National Savings, Debt Commissioners or the Postmaster-General, acting under the authorities vested in them respectively by the Government Annuities Acts, 1829 to 1888, and the Post Office Savings Bank Acts, 1861 to 1908.

(2) This Act shall not apply to a member of Lloyd's, or of any other association of underwriters approved by the Board of Trade, who carries on assurance business of any class, provided that he complies with the requirements set forth in the Eighth Schedule to this Act, and applicable to business of that class.

Recovery and application of penalties.

Service of notices.

Accounts, &c., to be laid before Parliament.

National Savings.

(3) Save as otherwise expressly provided by this Act, nothing in this Act shall apply to assurance business of any class other than one of the classes specified in section one of this Act, and a policy shall not be deemed to be a policy of fire insurance by reason only that loss by fire is one of the various risks covered by the policy.

Interpreta-
tion.

29.—In this Act, unless the context otherwise requires :

The expression “chairman” means the person for the time being presiding over the board of directors or other governing body of the assurance company;

The expression “annuities on human life” does not include superannuation allowances and annuities payable out of any fund applicable solely to the relief and maintenance of persons engaged or who have been engaged in any particular profession, trade, or employment, or of the dependants of such persons;

The expression “policy holder” means the person who for the time being is the legal holder of the policy for securing the contract with the assurance company;

The expression “underwriter” includes any person named in a policy or other contract of insurance as liable to pay or contribute towards the payment of the sum secured by such policy or contract;

The expression “financial year” means each period of twelve months at the end of which the balance of the accounts of the assurance company is struck, or, if no such balance is struck, then the calendar year;

The expression “Court” means the High Court of Justice in England, except that in the case of an assurance company registered or having its head office in Ireland it means, in the provisions of this Act, the High Court of Justice in Ireland, and in the case of an assurance company registered or having its head office in Scotland it means, in the pro-

visions of this Act other than those relating to deposits, the Court of Session, in either division thereof;

The expression "Companies Acts" includes the Companies (Consolidation) Act, 1908, and any enactment repealed by that Act;

The expression "registrar" means the Registrar of Joint Stock Companies;

The expression "actuary" means an actuary possessing such qualifications as may be prescribed by rules made by the Board of Trade;

The expression "Gazette" means the London, Edinburgh, or Dublin Gazette, as the case may be.

Application to Special Classes of Business.

Application
to life assur-
ance com-
panies.

30.—Where a company carries on life assurance business, this Act shall apply with respect to that business, subject to the following modifications:—

- (a) "Policy on human life" shall mean any instrument by which the payment of money is assured on death (except death by accident only) or the happening of any contingency dependent on human life, or any instrument evidencing a contract which is subject to payment of premiums for a term dependent on human life;
- (b) Where the company grant annuities upon human life, "policy" shall include the instrument evidencing the contract to pay such an annuity, and "policy holder" includes annuitant;
- (c) The obligation to deposit and keep deposited the sum of twenty thousand pounds shall apply notwithstanding that the company has previously made and withdrawn its deposit, or been exempted from making any deposit under any enactment hereby repealed;
- (d) Where the company intends to amalgamate with or to transfer its life assurance business to another assurance company, the Court shall not sanction the amalgamation or transfer in

any case in which it appears to the Court that the life policy holders representing one-tenth or more of the total amount assured in the company dissent from the amalgamation or transfer;

- (e) Nothing in this Act providing that the life assurance fund shall not be liable for any contracts for which it would not have been liable had the business of the company been only that of life assurance shall affect the liability of that fund, in the case of a company established before the ninth day of August eighteen hundred and seventy, for contracts entered into by the company before that date;
- (f) In the case of a company carrying on life assurance business and established before the ninth day of August eighteen hundred and seventy, by the terms of whose deed of settlement the whole of the profits of all the business carried on by the company are paid exclusively to the life policy holders, and on the face of whose life policies the liability of the life assurance fund in respect of the other business distinctly appears, such of the provisions of this Act as require the separation of funds, and exempt the life assurance fund from liability for contracts to which it would not have been liable had the business of the company been only that of life assurance, shall not apply;
- (g) Any business carried on by an assurance company which under the provisions of any special Act relating to that company is to be treated as life assurance business shall continue to be so treated, and shall not be deemed to be other business or a separate class of assurance business within the meaning of this Act;
- (h) In the case of a mutual company whose profits are allocated to members wholly or mainly by annual abatements of premium, the abstract of the report of the actuary on the financial

condition of the company, prepared in accordance with the Fourth Schedule to this Act, may, notwithstanding anything in section five of this Act, be made and returned at intervals not exceeding five years, provided that, where such return is not made annually, it shall include particulars as to the rates of abatement of premiums applicable to different classes or series of assurances allowed in each year during the period which has elapsed since the previous return under the Fourth Schedule.

31.—Where a company carries on fire insurance business, this Act shall apply with respect to that business, subject to the following modifications :

Application
to fire insur-
ance com-
panies.

- (a) It shall not be necessary for the company to prepare any statement of its fire insurance business in accordance with the Fourth and Fifth Schedules to this Act :
- (b) Such of the provisions of this Act as relate to deposits to be made under this Act shall not apply with respect to the fire insurance business carried on by the company if the company has commenced to carry on that business within the United Kingdom before the passing of this Act :
- (c) Such of the provisions of this Act as relate to deposits to be made under this Act shall not apply where the company is an association of owners or occupiers of buildings or other property which satisfies the Board of Trade that it is carrying on, or is about to carry on, business wholly or mainly for the purpose of the mutual insurance of its members against damage by or incidental to fire caused to the houses or other property owned or occupied by them :
- (d) It shall not be necessary to make a deposit in respect of fire insurance business where the company has made a deposit in respect of any other class of assurance business, and, where a company, having made a deposit in respect of fire insurance business, commences to carry

on life assurance business or employers' liability insurance business, the company may transfer the deposit so made to the account of that other business, and after such transfer the deposit shall be treated as if it had been made in respect of such other business :

- (e) So much of this Act as requires an assurance company transacting other business besides assurance business, or more than one class of assurance business, to keep separate funds into which all receipts in respect of the assurance business or of each class of assurance business are to be paid shall not apply as respects fire insurance business :
- (f) The provisions of this Act with respect to the amalgamation of companies shall not apply where the only classes of assurance business carried on by both of the companies are fire insurance business, or fire insurance business and accident insurance business, and the provisions of this Act with respect to the transfer of assurance business from one company to another shall not apply to fire insurance business.

32.—Where a company carries on accident insurance business, this Act shall apply with respect to that business, subject to the following modifications :

- (a) In lieu of the provisions of sections five and six of this Act the following provisions shall be substituted :

“The company shall annually prepare a statement of its accident insurance business in the form set forth in the Fourth Schedule to this Act and applicable to accident insurance business, and the statement shall be printed, signed, and deposited at the Board of Trade in accordance with section seven of this Act” :

- (b) Such of the provisions of this Act as relate to deposits to be made under this Act shall not apply with respect to the accident insurance business carried on by the company if the

company has commenced to carry on that business in the United Kingdom before the passing of this Act :

- (c) It shall not be necessary to make or keep a deposit in respect of accident insurance business where the company has made a deposit in respect of any other class of assurance business, and, where a company, having made a deposit in respect of accident insurance business, commences to carry on life assurance business or employers' liability insurance business, the company may transfer the deposit so made to the account of that other business, and after such transfer the deposit shall be treated as if it had been made in respect of such other business :
- (d) So much of this Act as requires an assurance company transacting other business besides assurance business, or more than one class of assurance business, to keep separate funds into which all receipts in respect of the assurance business or of each class of assurance business are to be paid shall not apply as respects accident insurance business :
- (e) The provisions of this Act with respect to the amalgamation of companies shall not apply where the only classes of assurance business carried on by both of the companies are accident insurance business, or accident insurance business and fire insurance business, and the provisions of this Act with respect to the transfer of assurance business from one company to another shall not apply to accident insurance business :
- (f) The expression "policy" includes any policy under which there is for the time being an existing liability already accrued, or under which a liability may accrue :
- (g) Where a sum is due, or a weekly or other periodical payment is payable, under any policy, the expression "policy holder" includes the person to whom the sum is due or the weekly or other periodical payment payable.

Application
to employers
liability insur-
ance com-
panies.

33.—(1) Where a company carries on employers' liability insurance business, this Act shall apply with respect to that business, subject to the following modifications :

- (a) This Act shall not apply where the company is an association of employers which satisfies the Board of Trade that it is carrying on, or is about to carry on, business wholly or mainly for the purpose of the mutual insurance of its members against liability to pay compensation or damages to workmen employed by them, either alone or in conjunction with insurance against any other risk incident to their trade or industry :
- (b) This Act shall not apply where the company carries on the employers' liability insurance business as incidental only to the business of marine insurance by issuing marine policies, or policies in the form of marine policies, covering liability to pay compensation or damages to workmen as well as losses incident to marine adventure or adventure analogous thereto :
- (c) In lieu of the provisions of sections five and six of this Act the following provisions shall be substituted :

“The company shall annually prepare a statement of its employers' liability insurance business in the form set forth in the Fourth Schedule to this Act and applicable to employers' liability insurance business, and shall cause an investigation of its estimated liabilities to be made by an actuary so far as may be necessary to enable the provisions of that form to be complied with, and the statement shall be printed, signed, and deposited at the Board of Trade in accordance with section seven of this Act” :
- (d) Such of the provisions of this Act as relate to deposits to be made under this Act shall not apply with respect to the employers' liability

insurance business carried on by a company where the company had commenced to carry on that business within the United Kingdom before the twenty-eighth day of August nineteen hundred and seven:

- (e) As soon as the employers' liability fund set apart and secured for the satisfaction of the claims of policy holders of that class amounts to forty thousand pounds, the Paymaster-General shall, if the company has made a deposit in respect of any other class of assurance business, return to the company the money deposited in respect of its employers' liability insurance business, and it shall not thereafter be necessary for the company to keep any sum deposited in respect of that business, so long as the sum deposited in respect of any other class of assurance business is kept deposited :
- (f) Where money is paid into a county court under the provisions of the Eighth Schedule to this Act, the court shall (unless the court for special reason sees fit to direct otherwise) order the lump sum to be invested or applied in the purchase of an annuity or otherwise, in such manner that the duration of the benefit thereof may, as far as possible, correspond with the probable duration of the incapacity :
- (g) The expression "policy" includes any policy under which there is for the time being an existing liability already accrued, or under which any liability may accrue :
- (h) Where any sum is due, or a weekly payment is payable, under any policy, the expression "policy holder" includes the person to whom the sum is due or the weekly payment payable :
- (i) If the company carries on employers' liability insurance business outside the United Kingdom, that business shall not be treated as part of the employers' liability insurance business carried on by the company for the purposes of this Act.

Application
to bond
investment
companies.

(2) In the application of this section to Scotland the expression "county court" means sheriff court.

34.—Where a company carries on bond investment business, this Act shall apply with respect to that business, subject to the following modifications :

- (a) The expression "policy" includes any bond, certificate, receipt, or other instrument evidencing the contract with the company, and the expression "policy holder" means the person who for the time being is the legal holder of such instrument :
- (b) Such of the provisions of this Act as relate to deposits shall not apply with respect to the bond investment business carried on by the company, if the company has commenced to carry on that business in the United Kingdom before the passing of this Act :
- (c) As soon as the bond investment fund set apart and secured for the satisfaction of the claims of the policy holders of that class amounts to forty thousand pounds, the Paymaster-General shall, if the company has made a deposit in respect of any other class of assurance business, return to the company the money deposited in respect of its bond investment business, and it shall not thereafter be necessary for the company to keep any sum deposited in respect of that business, so long as the sum deposited in respect of any other class of business is kept deposited :
- (d) The first statement of the bond investment business of the company shall be deposited at the Board of Trade on or before the thirtieth day of June nineteen hundred and eleven :
- (e) The company shall not give the holder of any policy issued after the passing of this Act any advantage dependent on lot or chance, but this provision shall not be construed as in anywise prejudicing any question as to the application to any such transaction, whether in respect of a policy issued before or after

the passing of this Act, of the law relating to lotteries.

35.—The Board of Trade may, on the application of any unregistered trade union originally established more than twenty years before the commencement of this Act, extend to the trade union the exemption conferred by this Act on registered trade unions, and may on the application of an unregistered friendly society extend to the society the exemption conferred by this Act on registered friendly societies if it appears to the Board, after consulting the Chief Registrar of Friendly Societies, that the society is one to which it is inexpedient that the provisions of this Act should apply.

Power of
Board of
Trade to
exempt un-
registered
trade unions
and friendly
societies.

Provisions as to Collecting Societies and Industrial Assurance Companies.

36.—(1) Amongst the purposes for which collecting societies and industrial assurance companies may issue policies of assurance there shall be included insuring money to be paid for the funeral expenses of a parent, grandparent, grandchild, brother, or sister.

Provisions as
to collecting
societies and
industrial
assurance
companies.

(2) No policy effected before the passing of this Act with a collecting society or industrial assurance company shall be deemed to be void by reason only that the person effecting the policy had not, at the time the policy was effected, an insurable interest in the life of the person assured, or that the name of the person interested, or for whose benefit or on whose account the policy was effected, was not inserted in the policy, or that the insurance was not one authorised by the Acts relating to friendly societies, if the policy was effected by or on account of a person who had at the time a bona fide expectation that he would incur expenses in connection with the death or funeral of the assured, and if the sum assured is not unreasonable for the purpose of covering those expenses, and any such policy shall enure for the benefit of the person for whose benefit it was effected or his assigns.

(3) Any collecting society or industrial insurance company which, after the passing of this Act, issues

policies of insurance which are not within the legal powers of such society or company shall be held to have made default in complying with the requirements of this Act ; and the provisions of this Act with respect to such default shall apply to collecting societies, industrial insurance companies, and their officers, in like manner as they apply to assurance companies and their officers.

59 & 60 Vict.,
c. 25.

(4) Without prejudice to the powers conferred by section seventy-one of the Friendly Societies Act, 1896, the committee of management or other governing body of a collecting society having more than one hundred thousand members may petition the court to make an order for the conversion of the society into a mutual company under the Companies (Consolidation) Act, 1908, and the court may make such an order if, after hearing the committee of management, or other governing body, and other persons whom the court considers entitled to be heard on the petition, the court is satisfied, on a poll being taken, that fifty-five per cent. at least of the members of the society over sixteen years of age agree to the conversion ; and the court may give such directions as it thinks fit for settling a proper memorandum and articles of association of the company ; but, before any such petition is presented to the court, notice of intention to present the petition shall be published in the *Gazette*, and in such newspapers as the court may direct.

When a collecting society converts itself into a company in accordance with the provisions of this subsection, subsection (3) of section seventy-one of the Friendly Societies Act, 1896, shall apply in like manner as if the conversion were effected under that section.

(5) In this section the expressions "collecting society" and "industrial assurance company" have the same meanings as in the Collecting Societies and Industrial Assurance Companies Act, 1896.

59 & 60 Vict.,
c. 26.

Supplemental.

Repeal.

37.—The enactments mentioned in the Ninth Schedule to this Act are hereby repealed to the extent specified in the third column of that schedule :

Provided that nothing in this repeal shall affect any investigation made, or any statement, abstract, or other document deposited, under any enactment hereby repealed, but every such investigation shall be deemed to have been made and every such document prepared and deposited under this Act.

38.—(1) This Act may be cited as the Assurance Companies Act, 1909.

Short title
and com-
mencement.

(2) This Act shall come into operation on the first day of July nineteen hundred and ten, except that as respects section thirty-six it shall come into operation on the passing thereof.

FIRST SCHEDULE

N.B.—Where marine insurance business or sinking fund or capital redemption insurance business is carried on, the income and expenditure thereof to be stated in like manner in separate accounts. Any additional businesses (including employers' liability insurance business transacted out of the United Kingdom) to be shown in a separate inclusive general account.

(A).—Form applicable to Life Assurance Business.

Revenue Account of the for the Year ending in respect of Life Assurance Business.

	Business within the United Kingdom	Business out of the United Kingdom	Total		Business within the United Kingdom	Business out of the United Kingdom	Total
	£ s. d.	£ s. d.	£ s. d.		£ s. d.	£ s. d.	£ s. d.
Amount of life assurance fund at the beginning of the year	—	—	—		Claims under policies paid and outstanding:—		£
Premiums	•	•	•		By death	—	
Consideration for annuities granted	•	•	•		By maturity	—	
Interest, dividends, and rents	•	•	•		Surrenders, including surrenders of bonus	—	
Less income tax thereon	•	•	•		Annuities	—	
Other receipts (accounts to be specified)	•	•	•		Bonuses in cash	—	
					Bonuses in reduction of premiums	—	
					Commission	—	
					Expenses of management	—	
					Other payments (accounts to be specified)	—	
					Amount of life assurance fund at the end of the year, as per Third Schedule	—	

Note 1.—Companies having separate accounts for annuities to retain the particulars of their annuity business in a separate statement.

Note 2.—Companies having both Ordinary and Industrial branches to return the particulars of the business in each department separately.

Note 3.—Items in this Account to be net amounts after deduction of the amounts paid and received in respect of re-assurance of the Company's risks.

Note 4.—If any sum has been deducted from the Expenses of management account, and taken credit for in the Balance Sheet as an asset, the sum so deducted to be separately shown in the above Account.

Note 5.—Particulars of the new life assurances effected during the year of account to be appended to the above Account, showing separately, as respects business within and business out of the United Kingdom, the number of policies, the total sums assured, the amount received by way of single premiums, and the amount of the yearly renewal premium income, the items to be net amounts after deduction of the amounts paid and received in respect of re-assurances of the company's risks. The particulars as to yearly renewal premium need not be furnished in respect of industrial business.

Note 6.—The column headed "Business out of the United Kingdom," in the case of companies having their head office in the United Kingdom, apply

(B.)—Form applicable to Fire Insurance Business.

Revenue Account of the for the Year ending 19..... in respect of Fire Insurance Business.

Note I.— Items in this Account to be the net amounts after deduction of the amounts paid and received in respect of re-insurances of the Company's risks.

NOTE 2.--If any sum has been deducted from the Expenses of management account, and taken credit for in the Balance Sheet or the Company's risks.

(C.)—Form applicable to Accident Insurance Business.

Revenue Account of the.....for the Year ending.....19.... in respect of Accident Insurance Business.

	£ s. d.	£ s. d.
Amount of accident insurance fund at the beginning of the year:	£ s. d.	£ s. d.
Reserve for unexpired risks		
Total estimated liability in respect of outstanding claims		
Additional reserve (if any)		
Premiums	£ s. d.	£ s. d.
Interest, dividends, and rents		
Less income tax thereon		
Other receipts (accounts to be specified)		

£

Note 1.—Items in this Account to be the net amounts after deduction of the amounts paid and received in respect of re-insurances of the Company's risks.

Note 2.—If any sum has been deducted from the Expenses of management account, and taken credit for in the Balance Sheet as an asset, the sum so deducted to be separately shown in the above Account.

(1).—*Form applicable to Employers' Liability Insurance Business.*

Revenue Account of the.....for the Year ending.....in respect of Employers' Liability Insurance Business transacted within the United Kingdom.

£ s. d.	£ s. d.
Amount of employers' liability insurance fund at the beginning of the year :—	Payments under policies, including medical and legal expenses in connection therewith
Reserve for unexpired risks	Commission Expenses of management Other payments (accounts to be specified)
Total estimated liability in respect of outstanding claims	Amount of employers' liability insurance fund at the end of the year as per Third Schedule Reserved for unexpired risks, being per-cent of premium income for the year
Additional reserve (if any)	Total estimated liability in respect of outstanding claims, as per Fourth Schedule (D).
Premiums	Additional reserve (if any) .
Interest, dividends, and rents	£ —————
Less income tax thereon	£ —————
Other receipts (accounts to be specified)	£ —————

Note 1.—Items in this Account to be the net amounts after deduction of the amounts paid and received in respect of re-insurances of the Company's risks.

Note 2.—If any sum has been deducted from the Expenses of management account, and taken credit for in the Balance Sheet as an asset, the sum so deducted to be separately shown in the above Account.

(E.)—Form applicable to Bond Investment Business.

Revenue Account of the.....for the Year ending.....19....in respect of Bond Investment and Endowment Certificate Business.

	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Amount of Bond Investment and Endowment Certificate Fund at the beginning of the year.				Claims under bonds and certificates, paid and outstanding.
Additional reserve (if any)				Commission
Premiums				Expenses of management
Interest, dividends, and rents.				Other payments (accounts to be specified)
Less income tax thereon				Amount of Bond Investment and Endowment Certificate Fund at the end of the year, as per Third Schedule.
Other receipts (accounts to be specified)				Additional reserve (if any)
				£

NOTE 1.—Items in this Account to be the net amounts after deduction of the amounts paid and received in respect of re-insurances of the Company's risks.

NOTE 2.—If any sum has been deducted from the Expenses of management account, and taken credit for in the Balance Sheet as an asset, the sum so deducted to be separately shown in the above Account.

Section 4.

SECOND SCHEDULE.

PROFIT AND LOSS ACCOUNT OF THE

FOR THE YEAR ENDING

19

£ s. d.	£ s. d.
Balance of last year's account	£ s. d.
Interest and dividends not carried to other accounts	£ s. d.
Less income tax thereon	£ s. d.
Profit realised (accounts to be specified)	£ s. d.
Other receipts (accounts to be specified)	£ s. d.
Dividends and bonuses to shareholders	£ s. d.
Expenses not charged to other accounts	£ s. d.
Loss realised (accounts to be specified)	£ s. d.
Other payments (accounts to be specified)	£ s. d.
Balance, as per Third Schedule	£ s. d.

THIRD SCHEDULE.

Balance Sheet of the.....

19.....on the.....

	L	s.	d.	L	s.	d.
Shareholders' capital paid up (if any).	.	.	.	Assets,		
Life assurance funds—				Mortgages on property within the United Kingdom		
Ordinary branch				Do, out of the United Kingdom		
Industrial branch				Loans on mercantile and other public rates		
Annuity funds				Do, life interests		
Pure insurance fund				Do, Reversions		
Accident insurance fund				Do, Stocks and shares		
Employers' liability insurance fund				Do, Company's policies within their surrender values		
Bond, investment and endowment certificate fund				Investments:		
Machine insurance fund				British Government securities		
Sinking fund and capital redemption fund				Municipal and county securities, United Kingdom		
Profit and loss account				Do, Personal security		
Other funds (if any) to be specified				Do, Indian and Colonial Government securities		
Claims admitted or intimated but not paid				Do, Provincial securities		
Life assurance				Do, Municipal do.		
Fire insurance				Do, Provincial securities		
Bond investment				Do, Municipal do.		
Annuities due and unpaid				Do, Railway and other debentures and debenture stocks—Home		
Other sums owing by the company† (to be stated separately under				Do, ordinary stocks		
each class of business).				Do, Rent charges		
				Freehold ground rents		
				Leasehold do.		
				House property		
				Life interests		
				Reversions		
				Agents' balances		
				Outstanding premiums		
				Do, Interest accrued but not payable		
				Bills receivable		
				Cash:		
				On deposit		
				In hand and on current account		
				Other assets (to be specified)		

Life companies having separate annuity fund to show amount thereof separately.

† These items are or have been included in the corresponding items in the First Schedule.

Note 1.—When part of the assets of the company are specifically deposited, under local laws, in various places out of the United Kingdom, as security to holders of policies there issued, each such place and the amount comprising liability insurance business, it shall be sufficient to state the fact that a part of the assets has been so deposited.

Note 2.—A Balance Sheet in the above form must be rendered in respect of each separate fund for which separate investments are made.

Note 3.—The Balance Sheet must state how the values of the Stock Exchange securities are arrived at, and a certificate must be appended, signed by the same persons as sign the Balance Sheet, to the effect that, in their belief the assets set forth in the Balance Sheet are in the aggregate fully of the value stated therein, less any investment reserve fund taken into account. In the case of a company transacting life assurance business or bond investment business, this certificate is to be given on the occasions only when a statement respecting the amount and the Fourth Schedule is made.

Note 4.—In the case of a company required to keep separate funds under section 3 of this Act, a certificate must be appended, signed by the same persons as signed the Balance Sheet and by the auditor, to the effect that no part of any such fund has been applied, directly or indirectly, for any purpose other than the class of business to which it is applicable.

FOURTH SCHEDULE.

N.B.—Where sinking fund or capital redemption insurance business is carried on, a separate statement signed by the actuary must be furnished, showing the total number of policies valued, the total sums assured, and the total office yearly premiums, and also showing the total net liability in respect of such business and the basis on which such liability is calculated.

Sections 5, 30,
32, and 33.

(A.)—*Form applicable to Life Assurance Business.*

STATEMENT respecting the VALUATION of the LIABILITIES under LIFE POLICIES and ANNUITIES of the , to be made and signed by the ACTUARY.

(The answers should be numbered to accord with the numbers of the corresponding questions.)

1. The date up to which the valuation is made.
2. The general principles adopted in the valuation, and the method followed in the valuation of particular classes of assurances, including a statement of the method by which the net premiums have been arrived at, and whether these principles were determined by the instrument constituting the company, or by its regulations or byelaws, or how otherwise; together with a statement of the manner in which policies on under average lives are dealt with.
3. The table or tables of mortality used in the valuation. In cases where the tables employed are not published, specimen policy values are to be given, at the rate of interest employed in the valuation, in respect of whole-life assurance policies effected at the respective ages of 20, 30, 40, and 50, and having been respectively in force for five years, ten years, and upwards at intervals of five years respectively; with similar specimen policy values in respect of endowment assurance policies, according to age at entry, original term of policy, and duration.
4. The rate or rates of interest assumed in the calculations.
5. The actual proportion of the annual premium income, if any, reserved as a provision for future expenses and profits, separately specified in respect of assurances with immediate profits, with deferred profits, and without profits. (If none, state how this provision is made.)
6. The consolidated revenue account since the last valuation, or, in case of a company which has made no valuation, since the commencement of the business. (This return should be made in the form annexed. No return under this heading will be required where a statement under this schedule is deposited annually.)
7. The liabilities of the company under life policies and annuities at the date of the valuation, showing the number of policies, the amount assured, and the amount of premiums payable annually under each class of policies, both with and without participation in profits; and also the net liabilities and assets of the company, with the amount of surplus or deficiency. (These returns to be made in the forms annexed).

8. The principles upon which the distribution of profits among the shareholders and policy holders is made, and whether these principles were determined by the instrument constituting the company or by its regulations or byelaws or how otherwise, and the number of years' premiums to be paid before a bonus (*a*) is allotted, and (*b*) vests.

9. The results of the valuation, showing—

(1) The total amount of profit made by the company, allocated as follows:

(*a*) Among the policy holders with immediate participation, and the number and amount of the policies which participated;

(*b*) Among policy holders with deferred participation, and the number and amount of the policies which participated;

(*c*) Among the shareholders;

(*d*) To reserve funds, or other accounts;

(*e*) Carried forward unappropriated.

(2) Specimens of bonuses allotted to whole-life assurance policies for £100 effected at the respective ages of 20, 30, 40, and 50, and having been respectively in force for five years, ten years, and upwards at intervals of five years respectively, together with the amounts apportioned under the various modes in which the bonus might be received; with similar specimen bonuses and particulars in respect of endowment assurance policies, according to age at entry, original term of policy, and duration.

NOTE.—Separate statements to be furnished throughout in respect of Ordinary and Industrial business respectively, the basis of the division being stated.

(Form referred to under Heading No. 6 in Fourth Schedule (A).)

Comolidated Revenue Account of the....., for.....years
commencing....., and ending

Amount of life assurance fund at the beginning of the period	£ s. d.	Claims under policies paid and outstanding	£ s. d.
Premiums		By death	£ s. d.
Consideration for annuities granted		By maturity	
Interest, dividends, and rents	£ s. d.	Surrenders	
Less income tax thereon		Annuities	
Other receipts (accounts to be specified)		Bonuses in cash	
		" " reduction of premiums	
		Commission	
		Expenses of management	
		Other payments (accounts to be specified)	
		Amount of life assurance fund at the end of the period, as per Third Schedule	
		£ _____	

Note. If any sum has been deducted from the expenses of management account and taken credit for in the Balance Sheet as an asset, the sum so deducted to be separately shown in the above Statement.

(Form referred to under Heading No. 7 in Fourth Schedule (A).)

SUMMARY and VALUATION of the Policies of the.....as to.....19.....

Description of Transactions	Particulars of the Policies for Valuation			VALUATION		
	Number of Policies	Sums assured and Bonuses	Office Yearly Premiums	Value by the Table, Interest	Office Yearly Premiums	Net Yearly Premiums
ASSURANCES.						
I. <i>With immediate participation in profits.</i>						
For whole term of life						
Other classes (to be specified)						
Extra premiums payable						
II. <i>With deferred participation in profits.</i>						
For whole term of life						
Other classes (to be specified)						
Extra premiums payable						
Total assurances with profits.						
III. <i>Without participation in profits.</i>						
For whole term of life						
Other classes (to be specified)						
Extra premiums payable						
Total assurances without profits.						
Total assurances						
Product re-assurances (to be specified according to class in a separate statement).						
Other classes (to be specified)						
Extra premiums payable						
Adjustments, if any (to be separately specified)						
ANNUITIES ON LIVES.						
Immediate						
Other classes (to be specified)						
Total of the results						

Note 1.—The term "extra premium" in this Act shall be taken to mean the charge for any risk not provided for in the minimum contract premium. If policies are issued in or for any country at rates of premium reduced from tables other than the European mortality tables adopted by the company, separate schedules similar in form to the above must be furnished.

Note 2.—Separate returns and valuation on results must be furnished in respect of classes of policies valued by different tables of mortality, or at different rates of interest, also for business at other than European rates.

Note 3.—In cases also where separate valuations of any portion of the business are required under local laws in places outside the United Kingdom, a summary statement must be furnished in respect of the business so valued in each such place showing the total number of policies, the total sums assured and bonuses, the total office yearly premiums, and the total net liability on the bases as to mortality and interest adopted in each such place, with a statement as to such bases respectively.

(Form referred to under Heading No. 7 in Fourth Schedule (A).)

VALUATION BALANCE SHEET of

as at 19

Dr.

Cr.

C

To net liability under Life Assurance and
Annuity transactions (as per summary
statement provided in Fourth Schedule (A))

To surpluses, if any

By Life Assurance and Annuity Funds (as
per Balance sheet under Schedule 3)

By deficiency, if any

C

(C.)—Form applicable to Accident Insurance Business.

STATEMENT of the ESTIMATED LIABILITY in respect of OUTSTANDING CLAIMS arising in the year of Account, and in the preceding year or years; computed as at the end of the year in which the claims arose, and as at the end of the year of Account; with particulars as to the number and amount of the claims actually paid in the intervening period.

I.—Claims arising during the year of account ending 19 .

(a) Particulars as to Claims arising, and settled, during the year of Account :—

Class of Claim (1)	No. of Claims (2)	Total amount paid	
		By sums insured (3)	By Weekly Allowance (4)
(i) Fatal claims			
(ii) Non-fatal claims			
Totals			

(b) Particulars as to Claims arising during and outstanding at the end of the year of Account :—

Class of Claim (1)	No. of Claims (2)	Amount paid during Year of Account (3)	Estimated Liability (4)
(i) Fatal claims			
(ii) Non-fatal claims, involving payment of sums insured			
(iii) Non-fatal claims, involving payment of temporary weekly allowances :—			
With maximum duration, not exceeding 26 weeks			
With maximum duration exceeding 26 weeks, but not exceeding 52 weeks			
And so on, at intervals of 26 weeks, up to the longest period over which temporary weekly allowances are granted.			
(iv) Non-fatal claims, involving payment of yearly allowances during permanent total disablement			
Totals			

**H.—Outstanding Claims which arose during the *first year* preceding
the year of account ending 19 .**

Particulars of Claims	Estimated Liability in respect of Claims Outstanding as at the above Date	Claims paid during the Period of One Year between the above Date and the End of Year of Account		Estimated Liability in respect of Claims Outstanding as at the End of Year of Account	Totals of Columns (3), (4), and (5)
		Terminated within such Period	Not terminated within such Period		
(1)	(2)	(3)	(4)	(5)	(6)
	No. Amount. No. Amount.	No. Amount. No. Amount.	No. Amount. No. Amount.		
(i) Fatal claims					
(ii) Non-fatal claims involving payment of sums insured					
(iii) Non-fatal claims involving payment of temporary weekly allowances :—					
With maximum duration not exceeding 25 weeks					
With maximum duration exceeding 25 weeks, but not exceeding 52 weeks					
And so on, at intervals of 26 weeks, up to the longest period over which temporary weekly allowances are granted					
(iv) Non-fatal claims, involving payment of yearly allowances during permanent total disablement					
Totals					

NOTE.—If temporary weekly allowances are granted by the Company for periods exceeding 52 weeks, particulars are to be furnished, in a form or forms similar to H, as above, showing, in respect of claims involving such extended allowances, the estimated liability as at the end of the year in which such claims arose, and as at the end of the year of account; and the number and amount of such actual claims paid during the intervening period of two (or more) years; distinguishing claims terminated, and not terminated, within such period.

III.—Summary of estimated liability, in respect of claims outstanding as at the end of the year of account—

As per column (4) of Statement I. (b) . . . £

" " (5) " " II. . . .

" " (5) of further schedules in the form of Statement II. (if required) . . .

In respect of yearly allowances during permanent total disablement, outstanding at the end of the year of account, but not included in the above Statements . . .

Total estimated liability, in respect of outstanding claims as at the end of the year of account, as per First Schedule (C.) . . . £

(D).—Form applicable to Employers' Liability Insurance Business.

STATEMENT of the **ESTIMATED LIABILITY** in respect of **OUTSTANDING CLAIMS** arising during each of the five years preceding the year of Account, and in such year; computed as at the end of the year in which the claims arose, and as at the end of the year of Account; with particulars as to the number and amount of the claims actually paid in the intervening period.

I.—Claims arising during the year of account ending 19 .

(a) Particulars as to claims arising and settled during the year of account :

Class of Claim (1)	Number (2)	Amount paid £ (3)
Fatal claims		
Non-fatal claims		
Total		

(b) Particulars as to the claims arising during, and outstanding at end of, the year of account :

Class of Claim (1)	Number (2)	Amount paid during year of account (3)	Estimated Liability (4)
Fatal Claims			£
Non-fatal claims			
Total			

II.—Outstanding claims which arose during the first year preceding the year of account, ending 19 .

Particulars of Claims (1)	Estimated Liability in respect of Claims outstanding as at the above date (2)	Claims paid during the period of 1 year between the above date and the end of the year of Account (3)	Estimated Liability in respect of Claims outstanding as at the end of the year of Account (4)	Total of Columns (3) and (4) (5)
	Number	Amount	Number	Amount
Fatal claims		£	£	£
Non-fatal claims				
Terminated			—	—
Not terminated			—	—
Total				

III.—Outstanding claims which arose during the second year preceding the year of account, ending the 19 .

Particulars of Claims	Estimated Liability in respect of Claims out- standing as at the above date	Claims paid during the period of 2 years		Estimated Liability in respect of Claims outstanding as at the end of the year of Account	Total of Columns (3) and (4)
		(3)	(4)		
		Number	Amount	Number	Amount
			£		£
Fatal claims					
Non-fatal claims—					
Terminated					
Non-terminated					
Total					

IV.—Outstanding claims which arose during the third year preceding the year of account, ending the 19 .

Particulars of Claims	Estimated Liability in respect of Claims out- standing as at the above date	Claims paid during the period of 3 years		Estimated Liability in respect of Claims outstanding as at the end of the year of Account	Total of Columns (3) and (4)
		(3)	(4)		
		Number	Amount	Number	Amount
			£		£
Fatal claims					
Non-fatal claims—					
Terminated					
Non-terminated					
Total					

V.—Outstanding claims which arose during the fourth year preceding the year of account, ending the 19 .

Particulars of Claims	Estimated Liability in respect of Claims out- standing as at the above date	Claims paid during the period of 4 years		Estimated Liability in respect of Claims outstanding as at the end of the year of Account	Total of Columns (3) and (4)
		(3)	(4)		
		Number	Amount	Number	Amount
			£		£
Fatal claims					
Non-fatal claims—					
Terminated					
Non-terminated					
Total					

VI.—Outstanding claims which arose during the fifth year preceding the year of account, ending the 19 .

Particulars of Claims	Estimated Liability in respect of Claims out- standing as at the above date	Claims paid during the period of 5 years between the above date and the end of the year of Account	Estimated Liability (Included in Statement VII. and valued by the method there specified) in respect of Claims out- standing as at the end of the year of Account		Total of Columns (3) and (4)
			(1)	(2)	
Number Amount Number Amount Number Amount					
		£	£	£	£
Fatal claims . . .					
Non-fatal claims—					
Terminated . . .				—	—
Non-terminated . . .				—	—
Total . . .					

NOTE.—In cases where the date at which the estimated liability required under column (2) in Forms IV., V., and VI. above would fall in any year prior to 1908, such estimated liability is to be returned as at the end of the year of account terminated in 1908, and the claims paid, required under column (3) of such forms, are to be in respect of the period between the end of the year of account terminated in 1908 and the end of the year of account rendered.

VII.—Statement respecting claims of five years' duration and upwards outstanding as at the end of the year of account. (To be made and signed by an actuary.)

(1) The number of claims incumbent and having durations of five years and upwards as at the end of the year of account, including those separately returned under Form VI. above; and the amount of the weekly payment, and of the annual payment, due in respect of such claims; separately stated in respect of each year of life of the workmen, from the youngest to the oldest. (These particulars to be returned under columns (1) to (4) of the tabular statement given below.)

(2) The estimated liability in respect of the claims specified above, computed, as at the end of the year of account, on the basis of the amount which would be required to purchase from the National Debt Commissioners, through the Post Office Savings Bank, an immediate life annuity for the workmen equal to 75 per cent. of the value of the weekly payment, according to the sex and true age of the workers. (These particulars to be returned under column (5) of the tabular statement given below, in respect of each year of life of the workmen, from the youngest to the oldest.)

(3) If the estimated liability, as reserved under the First Schedule in respect of the claims specified above, is computed on any basis other than that specified under Heading No. (2) above, the whole of the particulars required under Headings (1) and (2) above are to be returned in columns (1) to (5) of the tabular statement given below, together with the following additional particulars:—

(i) If the estimated liability is determined on the basis of the value of an immediate life annuity:—

(a) The table of mortality upon which such life annuity values are based;

(b) The rate of interest at which such life annuity values are computed;

(c) Whether such life annuity values are discriminated according to the sex of the workers;

(d) The proportion of such life annuity values representing the estimated liability;

(e) The modifications (if any) made in the true ages of the workmen, in deducing the estimated liability;

(f) The amount of the estimated liability. (To be returned, in respect of each year of life, in column (6) of the tabular statement given below.)

(ii) If the estimated liability is not determined on the basis of the value of an immediate life annuity, full particulars are to be specified as to the precise method adopted in deducing such estimated liability, and the total amount of estimated liability is to be returned under column (6) of the tabular statement given below.)

Number of Claims	Ages of the Workmen as at the end of the Year of Account	Amount of Weekly Payment	Amount of Annual Payment	Estimated Liability	
				Basis of 5 percent of Value of Life Annuity purchased through the Post Office	Estimated Liability if computed on Basis other than that specified in Column 5
(1)	(2)	(3)	(4)	(5)	(6)

NOTE.—Separate particulars to be furnished in respect of male and female workers.

Summary of estimated liability in respect of outstanding claims as at the end of the year of account—

As per column (4) of Statement I (b)
" "	(4)	"	II	"	"
" "	I)	"	III	"	"
" "	I)	"	IV	"	"
" "	I	"	V	"	"
" "	(5, or (6))	"	VII	"	"

Total estimated liability in respect of outstanding claims as at the end of the year of account as per First Schedule (D). £

(E.)—Form applicable to Bond Investment Business.

STATEMENT respecting the VALUATION of the LIABILITY under BONDS and ENDOWMENT CERTIFICATES of the to be made and signed by the ACTUARY.

(The answers should be numbered to accord with the numbers of the corresponding questions.)

1. The date up to which the valuation is made.

2. The principles adopted in the valuation of the liabilities under bond investment policies and endowment certificates, and whether these principles were determined by the instrument constituting the company, or by its regulations or byelaws, or how otherwise.

3. The rate or rates of interest assumed in the calculations.

4. The actual proportion of the annual income from contributions, if any, reserved as a provision for future expenses and profits. (If none, state how this provision is made.)

5. The consolidated revenue account since the last valuation, or, in the case of a company which has made no valuation, since the commencement of the business. (This return should be made in the form annexed. No return under this heading will be required where the valuation is made annually.)

6. The liabilities of the company under bond investment policies and endowment certificates at the date of the valuation, showing the number of policies or certificates, the amounts assured, the amount of contribution payable annually, and the provision for future expenses and profits; also the net liabilities and assets of the company, with the amount of surplus or deficiency. (These returns should be made in the forms annexed.)

7. The principles upon which the distribution of profits among the bond and certificate holders and shareholders is made, and whether those principles are determined by the instrument constituting the company, or by its regulations or byelaws, or how otherwise, and the time during which a bond investment policy or endowment certificate must be in force to entitle it to share in the profits.

8. The results of the valuation, showing—

(1) The total amount of profit made by the company, allocated as follows :—

(a) among participating bond or certificate holders, with the number so participating and the total amount of their bonds or certificates;

(b) among the shareholders;

(c) to reserve funds, or other accounts;

(d) carried forward unappropriated.

(2) Specimens of profit allotted to policies or certificates for £100, effected for different periods, and having been in force for different durations.

(Form referred to under Heading No. 5 in Fourth Schedule (E).)

**CONSOLIDATED REVENUE ACCOUNT of the.....for.....Years Commencing.....19.....
and ending.....19.....**

£ s. d.	£ s. d.	£ s. d.
Amount of Bond Investment and Endowment Certificate Fund at the beginning of the period		Claims under Bonds and Certificates
Additional reserve, if any		Commission
Premiums		Expenses of management
Interest, dividends, and rents		Other payments (accounts to be specified)
Less Income Tax thereon		Amount of Bond Investment and Endowment Certificate Fund at the end of the period, as per Third Schedule
Other receipts (accounts to be specified)		

Note.—If any sum has been deducted for the Expenses of management account and taken credit for in the Balance Sheet as an asset, the sum so deducted to be separately shown in the above Statement.

(Forms referred to under Heading No. 6 in Fourth Schedule (E).)

**SUMMARY and VALUATION of THE BOND INVESTMENT POLICIES or ENDOWMENT CERTIFICATES of
the
as at**

19

Particulars of the Policies or Certificates for Valuation

Description of Transactions	Particulars of the Policies or Certificates for Valuation		Valuation Interest at — per cent.)				
	No. of Policies	Sums Assured and Bonuses (if any)	Full Yearly Premiums	Value of Sums Assured and Bonuses (if any)	Value of Full Yearly Premiums	Provisions for future Expenses and Profits	Net Liability
With participation in profits,							
Without participation in profits,							
Totals							
Deduct re-assurances (to be specified according to class)							
Net Totals							
Adjustments (if any)							
Total of the results							

(Form referred to under Heading No. 6 in Fourth Schedule (E).)

VALUATION BALANCE SHEET of the

as at
49

Dr.	Cr.	£
To net liability under Bond Investment and Endowment Certificate transactions (as per summary statement provided in Fourth Schedule (E))	By Bond Investment and Endowment Certificate Fund (as per balance sheet under Schedule 3)	
To surplus (if any)	By deficiency (if any)	

Section 6

FIFTH SCHEDULE.

N.B.—Where sinking fund or capital redemption business is carried on, a separate statement, signed by the actuary, must be furnished showing the total sums assured maturing in each calendar year and the corresponding office premiums.

(A.)—*Form applicable to Life Assurance Business.*

STATEMENT of the LIFE ASSURANCE and ANNUITY BUSINESS
of the _____ on the 19 ,
to be signed by the Actuary.

(The answers should be numbered to accord with the numbers of the corresponding questions. Statements of reassurances corresponding to the statements in respect of assurances are to be given throughout.) Separate statements are to be furnished in the replies to all the headings under this schedule for business at other than European rates. Separate statements are to be also furnished throughout in respect of ordinary and industrial business respectively.

1. The published table or tables of premiums for assurances for the whole term of life and for endowment assurances which are in use at the date above mentioned.

2. The total amount assured on lives for the whole term of life which are in existence at the date above mentioned, distinguishing the portions assured with immediate profits, with deferred profits, and without profits, stating separately the total reversionary bonuses and specifying the sums assured for each year of life from the youngest to the oldest ages, the basis of division as to immediate and deferred profits being stated.

3. The amount of premiums receivable annually for each year of life, after deducting the abatements made by the application of bonuses, in respect of the respective assurances mentioned under Heading No. 2, distinguishing ordinary from extra premiums. A separate statement is to be given of premiums payable for a limited number of years, classified according to the number of years' payments remaining to be made.

4. The total amount assured under endowment assurances, specifying sums assured and office premiums separately in respect of each year in which such assurances will mature for payment. The reversionary bonuses must also be separately specified, and the sums assured with immediate profits, with deferred profits, and without profits, separately returned.

5. The total amount assured under classes of assurance business, other than assurances dealt with under Questions 2 and 4, distinguishing the sums assured under each class, and stating separately the amount assured with immediate profits, with deferred profits, and without profits, and the total amount of reversionary bonuses.

6. The amount of premiums receivable annually in respect of each such special class of assurances mentioned under Heading No. 5, distinguishing ordinary from extra premiums.

7. The total amount of premiums which has been received from the commencement upon pure endowment policies which are in force at the date above mentioned.

8. The total amount of immediate annuities on lives, distinguishing the amounts for each year of life, and distinguishing male and female lives.

9. The amount of all annuities on lives other than those specified under Heading No. 8, distinguishing the amount of annuities payable under each class, and the amount of premiums annually receivable.

10. The average rate of interest yielded by the assets, whether invested or uninvested, constituting the life assurance fund of the company, calculated upon the mean fund of each year during the period since the last investigation, without deduction of income tax.

It must be stated whether or not the mean fund upon which the average rate of interest is calculated includes reversionary investments.

11. A table of minimum values, if any, allowed for the surrender of policies for the whole term of life and for endowments and endowment assurances, or a statement of the method pursued in calculating such surrender values, with instances of the application of such method to policies of different standing and taken out at various interval ages from the youngest to the oldest. In the case of industrial policies, where free or paid up policies are granted in lieu of surrender values, the conditions under which such policies are granted must be stated, with specimens as prescribed for surrender values.

(E.)—*Form applicable to Bond Investment Business.*

STATEMENT of the BOND INVESTMENT BUSINESS of the
on the 19 . (To be signed by the Actuary.)

(The answers should be numbered to accord with the numbers of the corresponding questions. Statements of re-insurances, corresponding to the statements in respect of insurances, are throughout to be given.)

1. The published table or tables of rates of contribution for bond investment policies and endowment certificates which are in use at the date above mentioned; with full particulars as to the terms and conditions on which advances are made under such policies or certificates, whether on security of house property or land, or otherwise.

2. The total amounts assured under policies or certificates which are in existence at the date above mentioned, distinguishing the portions insured with and without profits, stating separately the total additions by way of bonus, and specifying such sums insured and bonuses respectively according to the number of complete years unexpired at such date.

3. The amount of premiums receivable annually, in respect of the respective insurances mentioned under Heading No. 2, separately specified according to the number of complete years unexpired at the date above mentioned.

4. The total amount of premiums which have been received from the commencement upon all policies or certificates mentioned under Headings Nos. 2 and 3, separately specified according to the number of complete years unexpired at the date above mentioned.

5. The average rate of interest realised by the assets whether invested or uninvested, constituting the bond investment and endowment certificate fund of the company, calculated upon the mean fund of each year during the period since the last investigation, without deduction of income tax.

6. Full particulars as to the terms and conditions upon which surrenders of policies and certificates are granted, with specimens of the values allowed in respect of different durations, and different unexpired terms at the date of surrender.

7. Full particulars as to the terms and conditions upon which allowances are made on the death of a policy or certificate holder, with specimen values as required under Heading No. 6.

8. Full particulars as to the terms and conditions upon which transfers of the interest in a policy or certificate are granted, whether on the death of the policy or certificate holder, or during his lifetime.

9. Full particulars as to the terms and conditions upon which redemption of advances is granted, with specimens of redemption values in respect of bonds or certificates of different durations, and having different unexpired terms, at the date of redemption.

10. A tabular statement in respect of policies or certificates lapsed during the period since the last investigation, showing the number, the amount insured, the yearly premiums, and the total premiums received from the commencement; classified according to the year in which such policies or certificates were effected, and lapsed, respectively; with a similar tabular statement in respect of policies or certificates surrendered during the period: Provided that policies or certificates which have lapsed and been revived shall not be entered as lapses.

11. A statement of the total number of advances made under policies or certificates to the holders thereof, whether on the security of house property or land or otherwise, and the total amount of such advances outstanding at the date above mentioned, distinguishing the advances on first mortgage and those on second or subsequent mortgage.

Section 17.

SIXTH SCHEDULE.

RULES FOR VALUING POLICIES AND LIABILITIES.

(A)—*As respects Life Policies and Annuities.*

Rule for valuing an Annuity.

An annuity shall be valued according to the tables used by company which granted such annuity at the time of granting the same, and, where such tables cannot be ascertained or adopted to the satisfaction of the court, then according to such rate of interest and table of mortality as the court may direct.

Rule for valuing a Policy.

The value of the policy is to be the difference between the present value of the reversion in the sum assured according to

the contingency upon which it is payable, including any bonus or addition thereto made before the commencement of the winding-up, and the present value of the future annual premiums.

In calculating such present values interest is to be assumed at such rate, and the rate of mortality according to such tables, as the court may direct.

The premium to be calculated is to be such premium as according to the said rate of interest and rate of mortality is sufficient to provide for the risk incurred by the office in issuing the policy, exclusive of any addition thereto for office expenses and other charges.

(B)—*As respects Fire Policies.*

Rule for valuing a Policy.

The value of a current policy shall be such portion of the last premium paid as is proportionate to the unexpired portion of the period in respect of which the premium was paid.

(C)—*As respects Accident Policies.*

Rule for valuing a periodical Payment.

The present value of a periodical payment shall, in the case of total permanent incapacity, be such an amount as would, if invested in the purchase of a life annuity from the National Debt Commissioners through the Post Office Savings Bank, purchase an annuity equal to seventy-five per centum of the annual value of the periodical payment, and, in any other case, shall be such proportion of such amount as may, under the circumstances of the case, be proper.

Rule for valuing a Policy.

The value of a current policy shall be such portion of the last premium paid as is proportionate to the unexpired portion of the period in respect of which the premium was paid.

(D)—*As respects Employers' Liability Policies.*

Rule for valuing a Weekly Payment.

The present value of a weekly payment shall, if the incapacity of the workman in respect of which it is payable is total permanent incapacity, be such an amount as would, if invested in the purchase of an immediate life annuity from the National Debt Commissioners through the Post Office Savings Bank, purchase an annuity for the workman equal to seventy-five per cent. of the annual value of the weekly payment, and in any other case shall be such proportion of such amount as may, under the circumstances of the case, be proper.

Rule for valuing a Policy.

The value of a current policy shall be such portion of the last premium paid as is proportionate to the unexpired portion of the period in respect of which the premium was paid, together with, in the case of a policy under which any weekly payment is payable, the present value of that weekly payment.

(E)—*As respects Bonds or Certificates.*

Rule for valuing a Policy or Certificate.

The value of a policy or certificate is to be the difference between the present value of the sum assured according to the

date at which it is payable, including any bonus or addition thereto made before the commencement of the winding up, and the present value of the future annual premiums.

In calculating such present values, interest is to be assumed at such rate as the court may direct.

The premium to be calculated is to be such premium as, according to the said rate of interest, is sufficient to provide for the sum assured by the policy or certificate, exclusive of any addition thereto for office expenses and other charges.

Section 17.

SEVENTH SCHEDULE.

Where an assurance company is being wound up by the court or subject to the supervision of the court, the liquidator, in the case of all persons appearing by the books of the company to be entitled to or interested in policies granted by such company, is to ascertain the value of the liability of the company to each such person, and give notice of such value to such persons in such manner as the court may direct, and any person to whom notice is so given shall be bound by the value so ascertained unless he gives notice of his intention to dispute such value in manner and within a time to be prescribed by a rule or order of the court.

Sections 28
and 33.

EIGHTH SCHEDULE.

REQUIREMENTS TO BE COMPLIED WITH BY UNDERWRITERS
BEING MEMBERS OF LLOYD'S OR OF ANY OTHER
ASSOCIATION OF UNDERWRITERS APPROVED
BY THE BOARD OF TRADE.

(A)—*As respects Life Assurance Business.*

1. Every underwriter shall deposit and keep deposited in such manner as the Board of Trade may direct a sum of two thousand pounds. The Board of Trade may make rules as to the payment, repayment, investment of, and dealing with, a deposit, the payment of interest and dividends from any such investment, and for any other matters in respect of which they may make rules under section 2 (6) of this Act in relation to deposits made by assurance companies. The sum so deposited shall, so long as any liability under any policy issued by the underwriter remains unsatisfied, be available solely to meet claims under such policies.

2. The underwriter shall furnish every year to the Board of Trade a statement in such form as may be prescribed by the Board showing the extent and character of the life assurance business effected by him.

(B) and (C)—*As respects Fire and Accident Insurance Business.*

1. Except as hereinafter provided, every underwriter shall comply with the following requirements:—

(a) He shall deposit and keep deposited in such manner as the Board of Trade may direct a sum of two thousand pounds in respect of each class of business. The Board

of Trade may make rules as to the payment, repayment, investment of, and dealing with, a deposit, the payment of interest and dividends from any such investment, and for any other matters in respect of which they may make rules under section 2 (6) of this Act in relation to deposits made by assurance companies. The sum so deposited shall, so long as any liability under any policy issued by the underwriter remains unsatisfied, be available solely to meet claims under such policies.

- (b) He shall furnish every year to the Board of Trade a statement, in such form as may be prescribed by the Board, showing the extent and character of the fire or accident insurance business effected by him.

2. An underwriter who carries on fire insurance or accident insurance business may, in lieu of complying with the above requirements, elect to comply with the under-mentioned conditions :

- (a) All premiums received by or on behalf of the underwriter in respect of fire and accident insurance or re-insurance business carried on by him, either alone or in conjunction with any other insurance business for which special requirements are not laid down in this schedule, shall without any apportionment be placed in a trust fund in accordance with the provisions of a trust deed approved by the Board of Trade;
- (b) He shall also furnish security to the satisfaction of the Board of Trade or, if the Board so direct, to the satisfaction of the committee of the association, which shall be available solely to meet claims under policies issued by him in connexion with fire and accident business and any other non-marine business carried on by him for which special requirements are not laid down in this schedule.

The security may be furnished in the form of either a deposit or a guarantee, or partly in the one form and partly in the other.

The amount of the security to be furnished shall never be less than the aggregate of the premiums received or receivable by the underwriter in the last preceding year in connexion with such fire and accident and other non-marine business :

- (c) The accounts of every underwriter shall be audited annually by an accountant approved by the committee of the association, who shall furnish a certificate to the committee of the association and to the Board of Trade in a form prescribed by the Board of Trade;
- (d) For the purpose of these requirements "non-marine insurance business" means the business of issuing policies upon subject-matters of insurance other than the following, namely :

Vessels of any description, including barges and dredgers, cargoes, freights, and other interests which may be legally insured by, in, or in relation to vessels, cargoes, and freights, goods, wares, merchandise, and property of whatever description insured for any transit by land or water, or both, and whether or not including warehouse risks or similar risks in addition or as incidental to such transit.

(D)—*As respects Employers' Liability Insurance Business.*

1. Every underwriter shall deposit and keep deposited in such manner as the Board of Trade may direct a sum of two thousand pounds. The Board of Trade may make rules as to the payment, repayment, investment of, and dealing with, a deposit, the payment of interest and dividends from any such investment, and for any other matters in respect of which they may make rules under this Act in relation to deposits made by assurance companies. The sum so deposited shall, so long as any liability under any policy issued by the underwriter remains unsatisfied, be available solely to meet claims under such policies.

2. Where the person insured by any policy issued by an underwriter is liable to make a weekly payment to any workman during the incapacity of the workman, and the weekly payment has continued for more than six months, the liability therefor shall before the expiration of twelve months from the commencement of the incapacity be redeemed by the payment of a lump sum in accordance with paragraph (17) of the First Schedule to the Workmen's Compensation Act, 1906, and the underwriter shall pay the lump sum into the county court, and shall inform the court that the redemption has been effected in pursuance of the provisions of this schedule.

3. The underwriter shall furnish every year to the Board of Trade a statement in such form as may be prescribed by the Board showing the extent and character of the employers' liability business effected by him.

4. For the purposes of this schedule "policy" means a policy insuring any employer against liability to pay compensation or damages to workmen in his employment.

(E)—*As respects Bond Investment Business.*

1. Every underwriter shall deposit and keep deposited in such manner as the Board of Trade may direct a sum of two thousand pounds. The Board of Trade may make rules as to, the payment, repayment, investment of, and dealing with a deposit, the payment of interest and dividends from any such investment, and for any other matters in respect of which they make rules under section 2 (6) of this Act in relation to deposits made by assurance companies. The sum so deposited shall, so long as any liability under any policy issued by the underwriter remains unsatisfied, be available solely to meet claims under such policies.

2. The underwriter shall furnish every year to the Board of Trade a statement in such form as may be prescribed by the Board showing the extent and character of the bond investment business effected by him.

NINTH SCHEDULE.

Section 37.

ENACTMENTS REPEALED.

Session and Chapter	Short Title	Extent of Repeal
33 & 34 Vict. c. 61.	The Life Assurance Companies Act, 1870 .	The whole Act.
34 & 35 Vict. c. 58.	The Life Assurance Companies Act, 1871 .	The whole Act.
35 & 36 Vict. c. 41.	The Life Assurance Companies Act, 1872 .	The whole Act.
39 & 40 Vict. c. 22.	The Trade Union Act Amendment Act, 1876	Section seven.
7 Edw. 7. c. 46.	The Employers' Liability Insurance Companies Act, 1907.	The whole Act.

American Railway Securities as Investments for Insurance Companies. By HUBERT ANSELL, Fellow of the Institute of Actuaries, Manager of the Anglo-American Debenture Corporation, Ltd., and a Director of the London Scottish American Trust, Ltd.

[Read before the Institute, 29 November 1909.]

IN writing upon the subject of this paper, I met at the outset with a difficulty which, I am afraid, will tend to make the paper less interesting to some of you than I would like it to be. The difficulty I refer to is this—that a considerable proportion of such information as I have upon the subject, and that which occurs most readily to me, is what I may perhaps call “particular” information, that is information relating to particular investments; whereas the information which appears to me to be most suitable to give here is that of a more general character, relating, primarily, to the principles which should guide us in selecting investments.

The first point to consider in connection with any railway the securities of which are under consideration for investment, is the value of the undertaking itself in relation to the amount of the issue of bonds or other security charged upon it. There are

various ways of considering the value, the principal of which are :

1. The cost of the undertaking.
2. The probable cost of building another similar railway.
3. The actual present value as evidenced by the revenue derived from it.

As regards the first of these, it may be noted that this criterion of value is the least reliable of the three, but it is interesting not to overlook it, as it is in this respect that we find the greatest contrast between English and American railways. Whilst English railways were built at very great expense, large sums being paid for land both in the country and in towns, and the actual work of construction being done upon the most expensive scale, American railways were, in early days, built very cheaply, the land being obtained without payment, and the railway construction being on an economical scale. The Capital was also raised in a totally different manner, the English railway capital being for the most part raised in the proportion of one part Debenture capital and three parts share capital, whilst the original method of building American railways was to raise all the capital required for the prime cost of construction by the issue of bonds, the share capital for the most part being issued to promoters and others for considerations of no real value. Even in recent times many railways have increased their mileage by issuing new bonds as a first charge upon extensions built with their proceeds, no increase being made in the share capital. It is obvious that if these principles had been followed continuously by all the American railways, the position would be thoroughly unsound, and there would be no occasion for anyone connected with this Institute to read a paper on the subject of American Railway Securities as investments for Insurance Companies, for the reason that there would probably be few or no securities answering to this description. This, however, is not the case.

Two very important methods of American railway finance have operated to modify the original system. One is that many railways which were, in the first instance, built very roughly have been improved from year to year by the Companies using a substantial part of their revenue to improve the railway in various ways, such as making the road bed more substantial, widening curves, reducing grades, laying heavy steel rails in place of light iron ones, and increasing and improving the rolling stock. The other important method has been that of obtaining cash

subscriptions upon share capital. Cash subscriptions have been obtained in more than one way. The earlier instances occurred in this way—the Companies got into financial difficulties and had to be reorganized, and in the case of most reorganizations the plan provided that shareholders should lose their shares unless they paid assessments upon them. This was so common that it became a saying in America that railway shares were divided into three classes: those which paid dividends; those which did not; and those which, besides not paying dividends, called for assessments at fairly regular intervals from their shareholders.

This state of affairs ceased, or at any rate became much less common, after some very drastic reorganizations which took place in the years 1893, 1894 and 1895. Partly as a result of these reorganizations, and partly in consequence of the natural development of the country, the railways from this time became very much more prosperous, so much so that many of them have been able to make large issue of shares for cash. Others adopted the system of issuing for cash what are called Convertible Bonds, that is, bonds which are entitled to a fixed rate of interest, but which generally have no mortgage upon the railway and rank after the ordinary mortgage bonds. By way of compensation for what they lacked in security they were given a speculative value by being made convertible into shares. In many cases the right to convert has been availed of because the shares now pay substantial dividends and have become more valuable than the bonds. Whether the bonds have been converted or not, the use of the proceeds of their issue has gone to improve the value of the railway, without increasing the mortgage debt.

I believe no statistics have been compiled which show, in reference to all the railways of the United States, what proportion of the share capital represents cash invested in the undertaking. As regards any given railway, it would be possible to make an approximate estimate from general information and by searching records. A general impression can be formed, however, in an indirect way. For this and other purposes I extract the following figures from *Poor's Manual*.

Year	Mileage	Share Capital		Bonded Debt Millions of Dollars
		Millions of Dollars	Millions of Dollars	
1888	154,222	4,392	4,555	
1893	175,412	5,021	5,510	
1898	181,894	5,581	5,635	
1903	206,886	6,355	6,722	
1908	230,085	7,642	8,788	

It will be seen that the amount of share capital of all the railways was 5,021 million dollars in 1893, since which time comparatively few issues have been made other than for cash, and it may also be taken that some appreciable proportion—probably not less than a third of the capital issued before that date was issued for cash. The total amount of all railway share capital being now about 7,800 million dollars, it is probably safe to assume that less than half of this was issued without payment of cash. I am not losing sight of the fact that there has been considerable duplication of capital in comparatively recent years by the issue of Collateral Trusts Bonds, that is bonds which, instead of being directly charged upon a railway or any portion of it, are secured by deposit of shares of some other railway which have been acquired. This has increased the total bond issues of the railways, but not the total stock issues, so that the operation is not in any sense a watering of stock.

The Table given above gives some approximate idea of the proportion of total bonded debt to share capital, and if we assume that for our purpose we are only considering mortgage bonds, to the exclusion of collateral trust bonds, and convertible bonds (which in most instances are appreciably inferior in security to the mortgage bonds of the same railways), it is evident that the proportion of direct mortgage bonds to other capital invested is somewhat better than would be inferred from the table on page 57. I give figures for three railways, showing the proportion of the bonded debt which is a mortgage upon the line.

	Mortgage Bonds	Convertible Bonds
	\$	\$
Union Pacific RR.	152,900,000	46,119,000
Atchison Topeka and Santa Fe	190,653,500	114,851,000
Norfolk and Western	108,971,000	25,376,000

The next gauge of value to consider is the cost of replacement, although this is, for reasons which I will shortly give, less often a consideration of importance than it was formerly.

Not many years ago the railway systems of the United States were considerably more numerous than at present, and little traffic went through from the point of origin to its final destination upon the lines of one railway company. A railway might have plenty of traffic originating on its own lines, but be dependent upon other railways to carry it through to its

destination, and the rate which the railway obtained for carrying goods depended naturally, as it still does to a great extent, upon the terms which it was able to make for interchange of traffic between the local and the trunk line. The rate which a local line was able to charge and keep for itself depended very much upon what it would cost the trunk line to build a branch line into the same locality. Then again, the rate which a trunk line could charge between two important points was, and still is to some extent, influenced by the question of what it would cost to build a new line between the same points. Most of the small local lines which existed as separate undertakings 15 or 20 years ago have, however, now been absorbed in the larger systems, as will be seen by reference to a list of bonds of some of the important companies. It will be seen on looking at the list that there are a number of bonds bearing the names of smaller railways which have been absorbed. The bonds of the absorbed railways have in many cases been assumed or guaranteed by the larger company now in control of the property, but in many more cases the controlling company maintains the line and pays interest, and will no doubt in due course pay the principal—not because it has guaranteed the bonds or assumed liability by any legal process, but because it is less expensive to meet these obligations than to build a new line. Before buying a bond secured upon the line of an absorbed company, it is therefore still worth while to ascertain whether the bond has been guaranteed or assumed, and, if not, the question of the cost of duplicating the line is a practical one to consider. Before the process of absorption took place, an investor in bonds, as well as in shares, was running the constant danger of some other company building a new line regardless of the fact that there was insufficient traffic to justify it. In addition to the improvement in the situation which has taken place in consequence of small railways having been absorbed and, in many cases, the securities guaranteed, another protective feature has developed gradually—namely the enhanced cost of new construction. Towns have grown up on the line of route of most of such railways, in which case another railway entering the towns would have to pay heavily for purchase of land in and near the towns, besides also having to pay fairly high prices for ordinary agricultural lands through which the railway might pass. Before leaving this branch of the subject, I would like to refer to what no doubt many of you regard as ancient history.

Up to about 20 years ago it was not uncommon for a comparatively powerful company to actually build a duplicate line under the circumstances referred to above, with the inevitable result that one or both of the companies concerned suffered serious loss. In many cases the companies continued the competition until one was starved out of existence. This of course was the crudest form of railway warfare. Then followed a period during which these occurrences were less frequent, as it was generally found sufficient for the more powerful company to threaten the process to enable it to make satisfactory terms for the absorption of the smaller company. In many instances the terms were also quite fair to the latter. This process had the effect of bringing most of the railways of the country into systems of 2,000 miles and upwards. During the last decade, however, a more extensive application of the same process has been in vogue. The largest railways have availed themselves of the aid of powerful financial groups, to obtain control of other companies by the purchase of a sufficient number of shares, partly in the open market and partly by negotiations with large holders, with the result that there are several railways which, according to official records, appear to be separate and independent, but which are really parts of a larger group or system. One might name groups extending from the Pacific Coast to the Atlantic practically under one control. One such group consists of the Union Pacific, the Southern Pacific, the Illinois Central and the Baltimore and Ohio Railways. It is obvious that a system of that size cannot be duplicated with the aid of any sum of money at all likely to be found for the purpose. It is still possible for a railway company belonging to one group to build a new line through territory at present served by another, but the process has not the same attraction for the aggressor now, for the reason that a railway already in the field would be supported by the full financial strength of the group and so could not be starved out at any reasonable cost. This condition of affairs has, on the whole, had an advantageous effect for investors, and during the last decade there have been comparatively few instances of absolute financial disaster to any American railway due to excessive competition.

The most useful measure of value is the regular revenue of the railway; in fact, this is so important that, perhaps, if I had not mentioned the other considerations first, there would have been some danger of their being overlooked altogether. The revenue accounts of important railways are, or were recently,

summarized in about the following form, and the summaries giving the principal items over a series of years can be found in books of reference which are published annually:

Miles of road operated.	
Passenger receipts.	
Freight ,,,	
Miscellaneous.	
	Total earnings.
Maintenance expenses.	
Transportation ,,,	
General ,,,	
	Total expenses.
Net earnings from operation of railroad.	
Other income.	
	Total net income.
Fixed charges.	
Taxes.	
Interest.	
Miscellaneous.	
	Total.
Surplus over fixed charges.	
Dividends paid.	
Improvements, betterments, &c.	
	Surplus.

Most of the above items have been standardized by custom for many years, but until recently there has been very little uniformity in the system of charging different kinds of expenses to the various subdivisions into which the main items are divided in the companies' accounts. These accounts are now regulated by instructions given from time to time by the Interstate Commerce Commission with a view to instituting a uniform system. The instructions, besides prescribing certain somewhat numerous subdivisions of revenue and expense, aim at enforcing a uniform principle in such matters as determining what should be charged to maintenance as an operating expense, and what should be charged as improvements or betterments, or paid out of capital. "Improvements and betterments" appear as an item in many railway company revenue accounts, and the effect is the same as if the amount so charged had been carried to reserve and the items paid for out of capital. Some companies have been under suspicion of charging in this way some of the expenses which should have been charged under the heading

of maintenance, the object apparently being to make it appear that the company realized a large net profit and paid only a moderate proportion of it in dividend, while perhaps a more correct view would be that the company made a smaller profit, most of which it distributed. The effect of a central supervision of accounts will make it a little easier for experts to form a correct judgment of any company's position.

In studying the revenue account, the first thing to look at is the proportion of gross and net revenue from the operation of the railway. Some people commence with the net revenue, but that is not going quite far enough back. There is an important difference in the case of various railways in the percentage of their net to their gross earnings. Taking the figures for the year ended 30 June 1908, most of the railways had net earnings varying from 27 per-cent to 35 per-cent, but several were as low as 20 per-cent, and several over 40 per-cent. If in normal times two companies were receiving net earnings, say, equal to twice the interest and rentals paid by the company, and in one case the net earnings were 40 per-cent of the gross earnings, and in the other case only 25 per-cent, there might be a considerable difference in the safety of the two securities. In the possible case of any long-continued depression, it might be expected that the stress would be felt most by those railways which were working on the narrowest margins. I give in the following table a few figures bearing on the years 1906, 1907 and 1908:

Name of Company	RATIO OF NET EARNINGS TO GROSS EARNINGS AND FIXED CHARGES					
	1906		1907		1908	
	Net to Gross	Net to Fixed Charges	Net to Gross	Net to Fixed Charges	Net to Gross	Net to Fixed Charges
	per-cent	per-cent	per-cent	per-cent	per-cent	per-cent
Great Northern . . .	49	360	41	305	33	244
Northern Pacific . . .	51	321	45	317	42	283
Southern Pacific . . .	36	178	37	216	33	178
Union Pacific . . .	48	301	47	311	45	234
Illinois Central . . .	33	177	31	189	27	156
Baltimore and Ohio . . .	36	210	33	213	26	148
Norfolk and Western . . .	40	276	37	245	36	191
Southern	28	131	24	101	23	89

The above figures do not in every case confirm the supposition that companies whose net revenue is a larger

percentage of the gross suffer less than others in bad years. Notwithstanding the figures, I believe, however, that this is the case, and that the explanation of its not always being apparent is that there have been changes in methods of bookkeeping. Some important items not always dealt with in the same way are the payments made in connection with the hire purchase of equipment. These have sometimes been charged as an operating expense, but are now generally included in the fixed charges. These and some other changes have no doubt been made with a view to making the accounts more correct, but they nevertheless disturb the basis of comparison.

Another lesson to be learnt from the above table is that, on the whole, companies have shown remarkable power of reducing expenses to partially meet reductions in gross revenue. It might be supposed that the very large decrease in gross earnings which were experienced in many cases in the year 1908 would have had an almost disastrous effect in reducing the percentage of profit, but it will be seen from the table on page 62 that in most cases the reduction in the rate of profit was moderate.

Bearing upon the question of expenses, which of course regulates the proportion of net to gross revenue, an important item to look at is maintenance. It is very easy in any year for a company which has kept its railway in good condition to make a substantial economy in the amount paid for maintenance. It appears, however, that this was not done to a very great extent in 1908. For instance, the Great Northern Railway maintenance charges per mile were \$1,908 in the year 1906, \$2,281 in the year 1907 and \$2,728 in 1908, showing an actual increase instead of decrease which might perhaps have been expected. This is an instance of a company which could very well afford a reduction in earnings, its fixed charges being covered two and a-half times over by net earnings even in the year 1908. Taking, however, another Railway, the Southern, which in the best of times only earns a moderate surplus, the maintenance charges for the respective years were \$2,131, \$2,284 and \$2,193 per mile. This is a case in which the condition of the railway was greatly inferior to the other one quoted, so that it was less practicable to make reductions in this item.

An item appearing now in many Revenue accounts is "Joint Facilities Revenue", and there is a corresponding item among the deductions from revenue. It appears that the former of these items is a proportion of the gross revenue collected from the

public by other Companies and paid over to the Company, whilst the latter is a proportion of gross revenue received by the Company and paid over to other Companies. As we wish to arrange the Summary of the Revenue Account so that the percentages and ratios may have an intelligible meaning, it is best to add and deduct these items from the gross earnings. In the Companies' own statements this is sometimes done, but in other cases the receipts from this source appear as other revenue alongside dividends from investments and the payments among the fixed charges—which appear to be wrong ways of treating the items.

Outside operations appear in some companies' reports in both sides of the revenue account, whilst some companies simply add or deduct the net profit. It depends a good deal upon what the outside operations consist of, whether the former or latter method is more correct. If the operations referred to are mostly such as maintaining dining car services and refreshment rooms at stations, *i.e.*, things intimately connected with the working of the railway—generally resulting in a net loss—I suppose the theoretically correct way is to add these receipts and payments to the general receipts and general expenses of the railway operations. But when many of the outside operations are less intimately connected with the actually railway business, it is probably just as correct to bring in the net result as one separate item and work out all statistical figures relating to operations of the railway with this item omitted. A reason in favour of the latter method is that in the monthly Railway Returns published with authority of the Interstate Commerce Commission, the monthly results are given with the net result of outside operations shown separately, but, on the other hand the various annual publications merge these outside operations with those of the Railway proper.

Hire of Equipment, which used to be treated as one of the expense items, is now brought in as a fixed charge.

Rentals are also a fixed charge, not a working expense.

TAXES.—These are now included as an item in expenses account, but they used to be shown amongst the fixed charges, and the various statistical books showing total expenses per mile of road or per-cent of gross earnings do not include taxes as expenses. It is therefore best at present to continue this practice for convenience in comparing recent results with past years.

The next item in the account to consider is that generally referred to as "Other Income" or "Income from Investments." These are sometimes comparatively insignificant, but of late years have become considerable in the case of railways which have acquired large holdings of shares in other railways in the process of establishing community of interest. The item is now of very great importance. Last year the Union Pacific paid dividends of 10 per-cent, but of this 6 per-cent was taken from the Company's own earnings and 4 per-cent from the interest and dividends on its investments. Speaking without special reference to this instance, the figure we have to consider in estimating the margin of security behind the Company's own Bonds is the net earnings from its own operations, not the total income, including interest and dividends upon its investments. It must be remembered that the receipt of dividends upon the shares it owns depends upon the revenue of the Company from which the dividends are received, and this revenue is subject to the fixed charges of that Company. Any circumstance which might decrease the revenue of the railway holding the securities would in all probability affect the revenue of the Companies whose securities it held. It is therefore clear that the method which should be adopted is either to calculate the margin of earnings of the principal Company alone, without regard to its income from investments, or, in cases where the Company owns a large proportion of the shares of another Company, as in the case of the New York Central, owning shares in the Lake Shore, Michigan & Southern, &c., the combined earnings of the several companies and their combined interest charges should be taken into consideration. In the above table I have given a column showing percentage of net earnings to fixed charges. I think this figure shows more correctly than any other percentage or statement of margin the relative safety of bonds of different Companies, so far as can be measured by figures derived from Revenue Account. To illustrate the different impression given by taking the total income including dividends, I have calculated the percentage of total net income to fixed charges of the Union Pacific system. The percentage is 345 as compared with 241, if we take its own net earnings alone, whilst the corresponding figures for the Southern Pacific, Illinois Central and Baltimore & Ohio, in which it holds shares were respectively 178, 156 and 148 a year ago (probably about 20 points better now, but at the moment of writing the reports are not issued).

The above point appears somewhat laboured, but a very important statistical book which I have before gives the margin of safety for a number of railroads, and the percentages, which are all calculated from the total income, are materially different to what they would be if calculated upon net earnings from operation alone, so that it seems worth while emphasising the point. The figure given as the "Margin of safety", is stated to be "the proportion of total net income remaining over after payment of all current fixed obligations", so that if, for instance, the total net income were \$1,000,000 and the fixed charges \$600,000, the margin of safety would be entered at 40 per-cent.

This appears perfectly clear, but it is a little difficult to state the result in this way when there is no margin, but a deficit. Therefore I prefer tabulating the percentage of net earnings to fixed charges, which, in the case mentioned above, would be 167, if net earnings were the same as total net income.

Although for the purpose of an Insurance Company we may only have to take account of bonds which are not likely to be seriously affected, even by a considerable set back in the prosperity of a railway, it is worth while considering the nature of its traffic. To investors who consider junior securities, questions of traffic and prospects of increase and decrease are even more important. The important point is to look up the Company's record of what is called Density of Traffic, *i.e.*, the total passenger and freight receipts per mile, and see whether they are increasing or decreasing. In this connection it should be remembered when there is an increase in mileage we cannot expect the same traffic per mile of new line as on the old, and also it should be noted that the usual mileage denominator for such statistics is the number of miles of first track. It is not usual to take into consideration the extra mileage due to some portion of the route having two or more tracks.

The next point is the class of traffic. In many annual reports we find a classification, such as :

Agricultural products	.	.	.	10 per-cent.
Animal	do.	.	.	3 per-cent.
Mining	do.	.	.	60 per-cent.
Forest	do.	.	.	3 per-cent.
Manufacturers	do.	.	.	22 per-cent.
Other	do.	.	.	2 per-cent.
				<u>100 per-cent.</u>

In other cases we can only go by knowledge acquired in less precise manner, but sufficiently near. In most cases there is one staple industry or group of industries upon which the railway mainly depends. For instance, the principal crop carrying systems are the Atchison, Union Pacific, St. Paul, Burlington, Great Northern, Northern Pacific, &c., whilst the Reading, Lehigh Valley and Lackawana are amongst the railways whose principal business is coal carrying. Apart from any opinion we may hold as to the comparative permanence of different classes of business it is useful to have this information, as one should divide investments between railways serving different industries and avoid getting too much into one group.

By way of summarizing the points referred to above, I recommend an abstract of the Revenue account in the following form, in which I have filled in figures applicable to the Union Pacific Railroad Company:

Year ending 30 June	1909	1908	1907
Number of miles—			
First Track	6,062	5,781	5,645
Additional Track	415	352	288
Sidings	1,882	1,802	1,743
Gross Traffic Earnings—	\$	\$	\$
Passenger, Mail, and Express	22,027,074
Per mile	3,634	3,564	3,374
Freight and Sundry (including outside operations)	56,723,387
Per mile	9,357	9,589	10,028
Total	78,750,461
Total per mile	12,991	13,153	13,402
Expenses—			
Operating, General, &c	22,828,340
Per mile	3,766	4,289	3,939
Maintenance	15,116,745
Per mile	2,493	2,923	3,175
Total	37,945,085
Total per mile	6,259	7,212	7,114
Gross Earnings from operation	40,805,376
Per mile	6,732	5,941	6,288
Per-cent of gross	51·81%	45·17%	46·60%
Per-cent of fixed charges	241	234	311
Excluded charges (including Taxes)—			
Total	16,943,368
Per mile	2,795	2,533	2,016
Required for Bonds and others ranking in priority
Per mile
Other Income	17,736,393	16,019,691	12,064,408

Principal sources of "Other Income"—

Dividends on Shares in the Southern Pacific Railroad, Illinois Central Railroad, and Baltimore and Ohio Railroad.

Principal Freight of Railroad—

Crops—but Traffic not classified in Annual Report.

APPROXIMATE STATEMENT OF PRESENT ANNUAL FIXED
CHARGES OF WHOLE SYSTEM.

Taxes		\$2,600,000
Hire of Equipment		1,400,000
Union Pacific First Mortgage .	\$100,000,000	4% 4,000,000
Union Pacific First Lien and Refunded	52,900,000	„ 2,116,000
Union Pacific Convertible Bonds	46,419,000	„ 1,856,760
Oregon Railroad and Navigation Co. Consolidated Mortgage .	23,380,000	„ 935,200
Oregon Short Line First Mort- gage	14,931,000	6% 895,860
Utah and Northern First Mort- gage	4,991,000	4% 199,640
Utah and Northern Consoli- dated Mortgage	1,802,000	5% 90,100
Oregon Consolidated First Mortgage	12,328,000	„ 616,400
Oregon Refunding	45,000,000	4% 1,800,000
Oregon Income Bonds, Series “A”	330,000	5% 16,500
Oregon Income Bonds, Series “B”	35,000	4% 1,400
		<hr/>
		816,527,860

So far I have spoken of the fixed charges, including interest on all the Company's Mortgages, as if the latter all ranked *pari passu*, but of course, in practice, we are generally considering the safety of a particular bond, and it is necessary to go deeper than the Company's statement of its total fixed charges. It will be necessary to take one of the leading books on the subject, such as *Poor's Manual*, and read carefully what issues of bonds are outstanding, and what portions of the line they are charged upon. It is the exception rather than the rule to find a bond which is an absolute first mortgage upon the whole Railroad. In many cases, as in that of the Union Pacific Railroad, for which figures are given above, you cannot place the securities in any order and say that is the order of priority. Many of the securities have special liens on various sections, and some of these overlap. For instance, the Union Pacific 1st Mortgage Bonds have a first charge upon 2,090 miles and only a general charge on the

remainder, whilst the Union Pacific First Lien Refunding Mortgage is a first charge upon another 1,178 miles, and a second charge upon the 2,090 miles. The various issues of the Oregon Short Line are specially charged upon other portions of the system. If we were considering the Union Pacific First Mortgage or First Lien and Refunding Mortgage, it would be hard to justify the omission of any of the charges, except the Convertible Bonds and Income Bonds, in calculating the ratio of safety.

The nearest approach to a First Mortgage is usually a Consolidated or General Mortgage upon the whole line subject to a few prior charges on several portions. These prior charges are generally called underlying bonds. In American finance a Consolidated Mortgage does not mean a mortgage which has been consolidated, but a mortgage which is intended to be consolidated at some future date more or less distant. It is best in such a case to add the amount of all underlying bonds and the consolidated mortgage bonds together and see what the total charge is. In the case of many of the railways it will be found that the whole of the interest and other fixed charges of the Company is covered, say, twice over, or, in other words, that the ratio of safety is 200 per-cent, but that, if we bring in only those charges which are equal or superior to the one under consideration, there is a ratio of safety of 300 per-cent, or over. You may have under consideration a bond which is undoubtedly a first mortgage upon some small portion of a big railway system, and, if you are invited to buy it, will probably be informed that it is charged upon one of the most essential and valuable parts of the system. That information may be perfectly true, but, on the other hand, it is a little difficult to confirm. Also instances have happened in the reorganization of railways in which it has been found by experience that concessions have had to be made by bondholders, notwithstanding that the particular section of line securing the bonds was worth the full amount of the bonds issued against it. I am therefore a strong advocate of paying comparatively little attention to the supposed value of sections of a line, and looking to the margin afforded by the total earnings over the total charges, excluding only those bonds which are clearly a junior charge on the whole railway or have no mortgage at all.

Amongst such bonds I might name convertible bonds, which, so far as security is concerned, are equivalent to notes. Another

kind of bond which may or may not be so considered is a Collateral Trust bond. These are generally secured by shares in other railways which have been bought up. In considering these, it is impossible to lay down any general rule. The collateral security referred to may be simply common stock in another company, but if that company supplies an important portion of the traffic, or affords valuable facilities, the loss of the shares, and with them, the control, might have a very disastrous effect.

An analysis of the company's accounts, something on the lines indicated above, goes a long way to enable us to estimate the safety of any particular issue of bonds; and the next point is to consider the ratio of safety in reference to the rate of interest which an investment would yield. A remarkable thing in this connection is the great disparity in point of security between bonds yielding 4 per-cent and those yielding 5 per-cent or a little under 5 per-cent. One would expect some difference, but not so great a difference as actually exists. The following are some bonds which, at current prices, yield about 4 per-cent; and I have filled in approximately the ratio of safety as indicated by the revenue accounts for the year ending 30 June 1909:

	Yield.	Ratio of Safety %.
Atchison, Topeka 4 per-cent General Mortgage Bonds	£3 18 6	312
Baltimore and Ohio 4 per-cent First Mortgage Bonds	3 19 3	163
Denver and Rio Grande First 4 per- cent. Consolidated Mortgage Bonds	4 1 3	155
Lake Shore and Michigan Southern 4 per-cent Debentures . . .	4 2 9	163
Louisville and Nashville 4 per-cent Unified Bonds	4 0 0	181
Northern Pacific 4 per-cent Prior Lien Bonds	3 17 6	367
Northern Pacific 3 per-cent General Lien Bonds	4 0 0	319
Union Pacific First Mortgage 4 per-cent Bonds	3 14 6	280

It will be seen that in most of the above cases there is a very high ratio of safety, and in other cases there are special reasons

why the securities are ranked high; but if we look for a 5 per-cent yield, we have to take such bonds as St. Louis and San Francisco 4 per-cent Refunding Bonds or Southern Railway Development and General 4 per-cent Bonds, in both of which cases the margin of net earnings over amount required for interest is only nominal. It used to be a practice with some general investors to buy bonds yielding 5 per-cent, with the hope that an improvement in the security, following the natural development of the country through which the railway passed and the consequent increase in the railway company's earnings, would give an enhanced value to the bonds. I have known this hope realized in many instances, but the opportunities do not appear so great in the case of American railways now as ten or fifteen years ago. I am inclined to think that the present is not a good time for buying third-class bonds, quite apart from the fact that securities of such a class are not suitable as investments for an insurance company.

A feature which appears to me to be an argument in favour of buying the best bonds at the present time is that if we compare the margin, or ratio of safety, with the course of prices in each of the last ten years, it will be noticed that, whilst the trend of surplus earnings above fixed charges during that period has been generally upward, the trend of prices during the same years has been rather down than up.

In preparing these notes as a basis for discussion, I have endeavoured rather to point out a way in which the security of an investment may be gauged rather than to supply statistics or give an opinion upon particular investments. For statistics bearing upon the subject, other than the full reports of the companies themselves, I would refer to the weekly, monthly, and quarterly issues of the *Commercial and Financial Chronicle* (of New York), *Poor's Manual of Railroads* (an annual publication giving considerable details and historical information regarding each railroad), and *Moody's Analyses of Railroad Investments*, which endeavours to bring the principal features of each railway together in a uniform shape. Valuable statements are also issued from time to time by the Interstate Commerce Commission.

ABSTRACT OF THE DISCUSSION.

MR. G. E. MAY, in opening the discussion, congratulated the author sincerely on the paper he had produced under severe limita-

tions as to time. In the commencement of the paper the author stated that the majority of the information at one's disposal was what he called "particular" information—information gained in particular companies—but what was required chiefly for a paper of such a nature was general information, and with that he thought all the members would agree. In discussing the question of American Railway Securities, Mr. Ansell had dealt first with the cost of the undertaking, and then with the probable cost of building another railway, and finally with the actual present value as evidenced by the revenue derived from it. Mr. Ansell did not appear to attach very much value to the first two methods, but relied mainly, and rightly so, on the third one. He, however wished to say one or two things with regard to the first two methods. The author stated that the total amount of all railway share capital in the United States of America was now about 7,800 million dollars, and that it was probably safe to assume that less than half of that was issued without payment of cash. Elsewhere Mr. Ansell stated that the Railways of the United States had considerably increased in value, owing to the various causes which he enumerated, and he wished to add, and thought he was safe in saying it, that the total mileage of the United States could hardly at the present time be replaced for the total net capitalization. When he said "total net capitalization", he meant the total amount of the bonds, added to the share capital, and the market value of the securities held deducted; if this deduction were not made a duplication of capital would be introduced.

Further on the author said: "Before buying a bond secured upon the line of an absorbed company, it is therefore still worth while to ascertain whether the bond has been guaranteed or assumed, and, if not, the question of the cost of duplicating the line is a practical one to consider." He quite agreed, but he was afraid it was a job very few people would like to undertake; and if he himself did get out any figures with regard to it he was afraid he should not like to rely upon them in forming an estimate of the security of the bonds. He wished to refer to a particular instance, in regard to the cost of a railway being of importance in considering the security of bonds. The Seaboard Air Line Railway Company some time ago built a small line called the Atlanta and Birmingham Seaboard Air Line. The total amount of money spent on the railway was about twice the amount of the first mortgage bonds, which were guaranteed both as to principal and interest by the Seaboard Air Line. The parent company recently got into difficulties, and a Receiver was appointed. The Receiver, however, continued to pay the interest on the bonds, and in the reconstruction which had recently taken place those bonds were not adversely affected. On the contrary, he thought the security was improved, as one of the conditions of the reconstruction was that fresh money should be brought in.

Further on in his paper, Mr. Ansell commenced his analysis of the net earnings, and the remainder of the paper was practically

devoted to that subject. The analysis was described as a most useful measure of value, and of this there could be no question. He was pleased to see that the author had not overloaded his definitions and conclusions with a mass of detail, but had simply pointed out the salient features. On the whole his methods of examination and conclusions were admirable. On page 62 there was a table showing the factor of safety in a novel way; this factor might be described as one showing the number of times the fixed charges were covered by net earnings. It was quite different from the way in which it was generally tabulated; though, perhaps, Mr. Ansell's method enabled one to realize more readily the margin of security than did the old method. Yet as he (Mr. May) had always used the old method, which was also adopted in all his reference books, he did not feel prepared to change.

Later on, the author dealt with the income from investments, and gave a table with a column showing percentage of net earnings to fixed charges, and then quoted the figures for the Union Pacific in the two ways, 345, taking surplus income into account, and 241, leaving it out of consideration altogether. In the case of railways where the income from investments was small, it did not seem to him of much importance whether it was included or not, but at the same time, if there was reason to think it was not likely to be a varying quantity, or would not vary in much greater proportion than that of the parent company, he thought it should be included, because, after all, income from investments was an additional security for bonds. In the case of large companies, or rather companies where the surplus income was large, it was a mistake to neglect it. Mr. Ansell suggested a way of getting over it, and the instance he took was the New York Central; he said that where a company owned a large proportion of the capital stock of another company, one might take into consideration the whole of the fixed charges and the whole of the net income of that company. If that were done, in the first place, in the company quoted, a very large income from other investments was excluded, namely, those where the holdings were not a large proportion of the stocks of the subsidiary company; and in the second place, he did not think the difficulty was got over in the instance taken. The author said he would include the Lake Shore, Michigan and Southern, which was the great holding company of the New York Central, and held a large block of shares of the Reading Company, while the Reading Company owned a large block of shares of the Central of New Jersey. If the total earnings were included, the profit from the investments of these companies would also be included, and so the author would not be attaining his object. It was possible that one might be able to solve the difficulty by including a proportion of the fixed charges, and the net earnings, according to the holding in the subsidiary company, but he was afraid that would be extremely complicated, and not worth entering into. He thought, however, Mr. Ansell had given the solution of the difficulty when he said: "To illustrate the different

impression given by taking the total income, including dividends, I have calculated the percentage of total net income to fixed charges of the Union Pacific system. The percentage is 345 as compared with 241, if we take its own net earnings alone." Looking at those figures it would appear that the true margin of safety must be some intermediate figure, and in that way a very good idea could be obtained of the security offered. He did not think the author would really neglect surplus earnings altogether. In the present case he thought Mr. Ansell would probably have worked out his proportion as 241, but would have cast an occasional glance with at least one eye on the average surplus earnings.

On page 67, Mr. Ansell set out in schedule form the results of his investigation, and subject to the remarks made he (Mr. May) agreed with him, but he thought it was an advantage to take the investigation further. When the net results with regard to a particular company had been obtained, it was of very great service to compare them with those of similar companies running through similar territories, as by that means a comparison was obtained which could not be made from the figures themselves. Taking, as an instance, the subject of maintenance, and comparing the maintenance per mile of the railway under consideration with the maintenance of similar railways running through similar territory, one obtained an idea as to whether a sufficient amount had been spent on maintenance, or whether the line was being starved. In the latter case, the margin of security did not give a true idea of the real security. A further very useful comparison was one of the net capitalization per mile and of the net earnings on the total net capitalization. A reasonable way of arriving at the total net capitalization was to add the total bonds to the total stocks, and to those add the rentals capitalized on a 4 per-cent basis, and from the total obtained deduct the market value of the securities held; the resulting figure would be the total net capitalization, and for the purpose for which he was going to use it this was near enough. Then, knowing the total mileage operated by the railway and also the total net earnings, it was easy to obtain the total net capitalization per mile, and the rate per-cent of the net earnings on net capitalization. Comparing these figures with those of similar railways running through similar territories, one could obtain an idea of the prosperity of the line, and by comparing the capitalization per mile a very good idea was obtained as to whether the railway was over-capitalized or not, and that, he thought, was a very valuable consideration.

He wished, also, to make a few remarks on the more practical side of the question that had to be dealt with when bonds were offered for sale. In the ordinary run of business it would be absolutely impossible to go through an exhaustive investigation of the nature laid down in the paper. The system adopted in his own office was, when there was any spare time, to make an investigation into all the principal American railways, and in that way a pretty good idea could be formed as to the bonds of any company, taken

as a whole. When a bond of any particular company was offered, one would at once have a fair idea as to the security; but, if not, it was easy to refresh one's memory by referring to the notes. There was a book entitled, "American Railways as Investments", by Carl Snyder, which was extremely useful, in spite of being somewhat out of date, but a more valuable one was that to which the author had referred, "Moody's Analyses of Railroad Investments." Another book, which was a very useful one, but unfortunately extremely expensive, was "White and Kemble's Atlas and Digest of Railway Mortgages."

With regard to the time to buy bonds, he did not suppose there was a similar class of security that varied so much in price as American railway bonds. There was a time to buy, and a time not to buy. By buying at a cheap time one could profit very much. In times of crises like 1907, bonds could be picked up very cheaply. A bond that would ordinarily stand at 90 might then have been purchased at 70, though the security had not varied one iota. Anyone having funds at his disposal at that time would then do well to throw the whole of them into American railway bonds, as he might be sure of reaping a golden harvest.

MR. H. W. ANDRAS, after welcoming the Author, not only as a Fellow of the Institute, but as a prominent man in the financial world, said he had recently been in the habit of consulting a book of reference entitled "Moody's Analysis of Railroad Investments" (first annual number, 1909), referred to by the previous speaker. This book included in the "margin of safety" for each railroad the income from outside investments, which additional revenue must, in his opinion, influence the distribution of the total net income, and therefore the dividends declared. For this reason, he preferred Mr. Moody's "margin of safety" factor to the "rate of safety" factor recommended by Mr. Ansell. At the same time, he was fully aware that it was important for many other purposes to separate the operating figures, and that course was insisted upon in the annual returns required from the railways by the Inter-State Commerce Commission. He found, from Mr. Moody's book, that the annual average for the decade ending 1908, in the case of the Pennsylvania Railroad, was 78 per-cent of the total net income from operation, and 22 per-cent from surplus earnings. The corresponding figures for the Union Pacific were 78 per-cent and 22 per-cent, and for the New York Central 83 per-cent and 17 per-cent: on the other hand, the similar annual average for the Atchison was only 3 per-cent from other sources, and 97 per-cent from operation. The tendency in many lines during recent years had been to increase the "outside income" sources, and a comparison of the last few years' results would show much higher percentages for such lines as the Pennsylvania, New York Central and Union Pacific. In many cases the outside revenue was, therefore, a substantial proportion of the revenue from other sources, and should, presumably, be taken into consideration in any estimate of any "margin of safety." A large mass of statistics had

been calculated by Mr. Moody on that basis, and, although they could, with some labour, revise them in accordance with Mr. Ansell's method, the end would not, in his opinion, justify the means. Of course, although the "margin of safety" factors introduced into Mr. Moody's calculations were the main consideration, they were not the only factors given by him as a means of classifying and forming an opinion of any particular bond or stock.

Life assurance companies had for many years invested in first-class American railway bonds, and were well acquainted with the leading securities. Their yield, however, of recent years had become less than was originally looked for. He had taken out the variations in market price, at the end of each year from 1905 to 1908, of a group of sixteen first-class bonds. In 1905 their average price was £106 per £100, and yield £3. 15s. 5d. per-cent per annum : in 1906 the average price was £103, and yield £3. 18s. 6d. At the end of 1907, from the effect of the financial depression then prevalent, the average price dropped to £96. 18s. 6d., with a yield of £4. 6s. 6d. ; and at the end of 1908, the average price recovered to £101, with a yield of £3. 19s. 6d. Insurance companies, therefore, expecting a yield of at least 4 per-cent on foreign investments, would probably in future be disposed to invest in a somewhat lower, but nevertheless well-secured, class of American railway bonds, and the statistics in Mr. Moody's book were of great assistance in forming a fair estimate of their security. At the same time, when an estimate had been formed by this means, the confirmatory opinion of an expert was desirable, and he himself had found that such a further opinion had, in the few cases of which he had experience, only tended to increase his confidence in Mr. Moody's principles of analysis and classification.

A very useful article was published in *The Times* on 5 November 1909, on the "Terminology of American Securities", both as to their serious titles, and, what might be called, their serio-comic titles : this was a useful supplement to the information given on that head by Mr. J. Burn in his valuable book on investments, just published, and to that given by Mr. Ansell in this paper. A paper on "American Methods of Railway Accounting" by S. Chapman, published in the *Journal of the Royal Statistical Society* for December 1908 (p. 619), was also a useful contribution to the literature of the subject.

After some remarks by Mr. Wilkinson,

MR. J. DOUGLAS WATSON, said that he found in practice that not only was it difficult to investigate an investment offered by means of the various books of reference, and reconcile the information contained therein with that obtained from various brokers and others, but that, having succeeded in buying an investment, there was a good deal of difficulty in keeping one's eye upon it and watching it. He had often wondered how trust companies systematically kept in view the investments they had made, and watched the various market influences affecting their value from time to time, producing those results which during the last ten years or so had

been seen with so much gratification. During last year, he had elaborated a little system for assisting the work of the heads of offices in collecting information about investments. The offices were constantly receiving offers of investments, and some system was necessary for selecting and collating information and putting it in a definite form. The system he had adopted was to have certain blank forms printed, and in adopting them he had had the advantage of consulting other members of the profession, who had found similar methods of great practical use. The investments could be put through the mill, so to speak, in the office, and the information be always ready at hand when required.

MR. W. P. PULLEY asked the author, when replying to criticisms, kindly to give some explanation of the final table set out in his paper, because there seemed a want of relation between the yield of the bonds and the ratio of safety. It appeared to him that there should be some better agreement between the two. For instance, where the ratio of safety was 312 the yield was £3. 18s. 6d., and in the next security, where the ratio of safety was only 163, the rate of interest was only £3. 19s. 3d. Again, a little lower down in the table, where the ratio of safety of the Lake Shore and Michigan Southern was also 163, the yield of interest was £4. 2s. 9d., the highest yield that appeared in the list. It would naturally have been thought that the reduction in the rate of interest yielded would have closely followed the superior security as represented by the ratio of safety. His meaning would be more clearly seen by re-arranging the eight securities given in the list according to the magnitude of the ratio of safety per-cent and the corresponding yield, as follows :

Ratio of safety per-cent						Yield per-cent
367	£3 17 6
319	4 0 0
312	3 18 6
280	3 14 6
181	4 0 0
163	3 19 3
163	4 2 9
155	4 1 3

MR. W. O. NASH drew attention to the author's remark that the present was not a good time for buying third-class bonds, quite apart from the fact that securities of such a class were not suitable as investments for an insurance company. It was not obvious why that statement was made. If insurance companies were going to invest in American railway bonds, they should discriminate between the different classes of bonds, and try to grasp, not only the particular features of the several railways, but also the social and ethical conditions that governed the progress of the country, and affected the masses of American securities. Personally, he believed that America was progressing rapidly along the road of civilization. It seemed to him that it was very doubtful

whether the present was a time to buy first-class American bonds, since one could obtain as good a return from English securities. But if it were possible to select the right kind of second and third class bonds, such bonds, with the progress of civilization in the United States, the increase of commercial morality, and so on, would be bound to approach to the first class, and there would be a considerable rise in their value.

With reference to the problem whether it was a good thing for British capital to go out of the country, there was an increasing tendency for capital to go where it would find its greatest return and greatest security, and it seemed vital to a study of the question to consider whether the United States was one of those countries where the security of capital was improving. Another factor which seemed likely to lead to an improvement in the value of second and third class bonds was the tendency for securities to be what might be termed "standardized", i.e., a great many different classes of bonds and securities were amalgamated, and so become much more easily understood, more marketable, and of increasing value. An instance of this was our own Local Loans Stock, where the securities of many corporations were amalgamated and the whole stock was guaranteed by the Government. As railways in America continued to amalgamate, and to come more and more under the influence of the American Government, there would be a sort of standardization of the security, which would increase the value of the lower class bonds more than it did that of the higher class. The same sort of thing would happen in Australia with the amalgamation of the State debts under the Commonwealth. He should be very much obliged to the author if he would state more specifically why he considered the present was a bad time to buy second and third class bonds.

MR. A. T. WINTER said that it might be of some interest to institute a comparison between the ratio of safety of some of the American bonds mentioned by Mr. Ansell, and similar securities in other countries. Taking the Argentine as an example, the debenture stock of one of the chief companies—the Buenos Aires and Great Southern—showed a ratio of safety of 410, based on a similar analysis to that given by the author; while the Buenos Ayres Pacific, even after allowing for the amount that had been passed from reserve into revenue, showed a ratio of safety in the first debenture stock of 870. The interest yields were slightly lower than the average of the American bonds given by Mr. Ansell. In one case it was £3. 16s. 9d., and in the other case £3. 17s. As an example of Canadian Railways, he took the Canadian Pacific. The Canadian Pacific Debenture Stock yielded a much lower rate of interest than United States Railway Bonds, but the 4 per-cent preference, which yielded £3. 17s., would serve as a basis of comparison, and this showed a ratio of safety of 220. Coming to home railway debenture stocks, there was, of course, a very much smaller yield, and it was necessary again to go to the preference for comparison. The

Midland 2½ per-cent preference yielded £3. 15s. 6d., and showed a ratio of safety of 165. There were many other considerations, besides the margin of revenue charges, which affected the stability and interest yield of American and other railway bonds. One of the most important was the possibility of the expansion of traffics. Many of the lines were running through districts which were to a certain extent undeveloped, and much capital had been placed in them which was not at present producing full revenue. The future increases of revenue likely to result in such lines would improve the balance of income ranking behind the debenture issues. Other considerations were the class of traffic, and the probability of its permanence. A general traffic was a much more favourable feature than a specific traffic, depending on trades liable to great fluctuations.

Mr. May had hinted at the difficulties sometimes met with in arriving at the exact security underlying American bonds; no doubt most members had experienced that difficulty. Immediately one entered the domain of American railway finance, things were not often what they appeared to be. Mr. Ansell had mentioned the case of the consolidated mortgage, which was not at present a consolidated mortgage, but would be at some future date; and often one came across a first mortgage, which was not a first mortgage, or only so to a very limited extent. Taking, for instance, one of the securities referred to, the Baltimore and Ohio 4 per-cent first mortgage bond, when looked into it was found there were 75 million dollars prior lien bonds charged on the main line and the important branches, and a sum was reserved to replace those on maturity in 1925, but until that date arrived it was not a first mortgage on the main system, but only a second mortgage, with a very large charge in front of it. In consulting books of reference, there were a number of considerations to be borne in mind, which had an important bearing on the facts deduced from such books, and the author had earned the gratitude of the members by emphasizing some of the more important of these points.

MR. R. TODHUNTER, in summing up the discussion, said that, although the paper dealt with some general features of American railway history and finance, its main idea was to systematize and reduce to something in the nature of a rule the process to be followed in examining the accounts and statistics of an American railway company, with a view to considering the eligibility of some one of its bonds, or the whole of its bonded debt. It pre-supposed the intention to invest in American securities, and proceeded at once to the practical question—the problem of selection. The fact, however, could not be entirely overlooked that, before arriving at the point from which the paper started, there were certain general considerations that presented themselves, bearing on the eligibility of American railway securities as a class. There was for example the contingency of war, and of the consequent suspension of interest payments. Again, there was the possibility of the taxation of foreign capital, and of an income tax in the States. It must be

remembered that the idea of an income tax had been mooted in responsible quarters in the United States of late years, notwithstanding the fact that such a tax had been declared by a small majority of the Supreme Court to be unconstitutional. Then again, there was the element of violent fluctuation in American securities, as to which, however, there was a reassuring article in a recent number of the *Economist*, comparing the comparatively mild fluctuation that followed the recent crisis with that which followed the crisis of 1893. In the year following 1893, the railways of the United States showed a decline of something like 11½ per-cent in gross traffic, and recovered very slowly. In 1908 the decline was only about 2½ per-cent, and already one-half of it had been recovered. Then there was the final question of the general future of railway enterprise, having regard particularly to the relations of labour and capital. Such general considerations as those formed the staple of the references to the subject of American railways in the earlier investment literature of the Institute. He thought that they still had some bearing upon the question whether investments in American railway bonds should be strictly limited to the best class of bonds, and that they would probably so far weigh with most members of the Institute as to incline them to agree most thoroughly with Mr. Ansell in his opinion as to the inadvisability of investing in third rate bonds.

The recent history of British railways was certainly not encouraging to any element of speculation in railway securities extending over a long term. But, with that reservation, it might be admitted that the question of the eligibility of American railway bonds had been settled in their favour by the common action, if not the common consent, of the great majority of large insurance companies. It was impossible to say what amount of British insurance funds was at present invested in American railroad bonds. The Act of 1870 did not require—and the new Act would not require—investments in American railroad bonds to be separately stated. But whether one judged from the balance sheets of the few companies that had always made a practice of stating this asset separately, or whether one drew an inference from the success of recent American railway bond issues in this country, a degree of success which would hardly have been obtained without the assistance of insurance capital, one arrived at the conclusion that the amount of British life assurance funds invested in American railroad bonds must have greatly increased since the reconstruction period, *i.e.*, since the early nineties, and that it had particularly increased during the last year or two. That, no doubt, might be attributed to the belief that the age of reconstruction was practically over, that the principal American railways had attained stability or practical stability, and that the security for the bonded debt generally was much better than it used to be. That belief was probably well founded, but it should be noticed that the present tendency was to increase bonded debt, relatively both to mileage and to capital stock. The 1909 figures were not ready when the

paper was written, and if the author had been able to make use of the last figures in "Poor's Manual" he would have found, as no doubt he was aware, that the bonded debt, instead of being about one-sixth greater than the share capital, was now one-fourth greater. During the first half of 1909, out of 103 millions sterling of new capital issued by American companies, no less than 89 millions took the form of bonded debt; and although that was partly due to replacements of earlier issues of bonds, still it had the effect of increasing the proportion of bonded debt to share capital to the extent he had mentioned.

For the purpose of shortly representing the margin of safety, Mr. Ansell recommended the ratio of net income to fixed charges, and there had been some discussion as to whether that was a reliable measure. The criterion was a familiar one, whether in connection with mortgages or Stock Exchange securities, and the explanation the author gave of the way in which the various items in the accounts of American railways should be dealt with in arriving at the ratio formed a most useful feature of his paper. There was, however, one main objection to such a criterion as Mr. Ansell recommended, namely, that one was apt to depend too much upon it, owing to its very attractive simplicity. It was quite obvious that a charge covered ten times over by a net revenue, which was itself only 10 per-cent of the gross income from which it was derived, would be very poorly secured indeed, although its margin of safety according to the rule under discussion would be 1,000. The author, of course, recognized this in his statement that to begin with the net revenue was not going back quite far enough; but the question was whether it was sufficient to recognize it, and still use the ratio for ordinary purposes. Would it not be better to adopt some measure of the margin of safety which involved the factor of operating expense? Such a measure might be the ratio of gross income to the sum of net income and expenses, to be used not as an alternative to, but in conjunction with, the more usual ratio. This ratio would bring out, for example, in connection with the figures of such companies as the Illinois Central and the Baltimore and Ohio—companies in which the ratio of net income to gross income was exceptionally low—and it should be borne in mind that the ratio of net income to gross income was generally much lower in the States than it was in this country—that a comparatively small percentage increase of operating expenses would cause the margin of safety entirely to disappear; although not, of course, so far as the first mortgage bonds were concerned. A point brought out by Mr. Ansell's analysis of the Union Pacific, and a characteristic feature of all American railways, was the large proportion of the income derived from goods traffic, a proportion, of course, very much higher than the proportion in the United Kingdom, or in densely-populated countries. That was probably an advantageous feature; but it suggested the necessity of considering the possible effect which the opening of the Panama Canal would have on the goods traffic of companies doing a trans-

Continental business, or carrying goods from the Southern States to the Western coast. The competition of the Canal might not cause a reduction of traffic, but it might very well cause a reduction of rates, and a consequent reduction of income. There was only one other point he wished to allude to, upon which Mr. Ansell might perhaps be willing to say something, namely the question of the relative eligibility in general of bonds dealt with in London, and bonds not dealt with in London. Roughly, the total of bonded issues of the various American railway companies was something like 2,000 millions sterling, and considerably less than one-half of that was quoted in the *Stock Exchange Daily List*. Might one expect to find amongst the securities quoted only in the States many of equal eligibility with those known in London, and at the same time, owing to the fact of their not being known in London, yielding a somewhat higher rate of interest?

On the motion of the PRESIDENT, a hearty vote of thanks was accorded to Mr. Ansell for his paper.

MR. HUBERT ANSELL, in reply, thanked the meeting for the very kind reception that had been given to his paper by the Council and the members. Mr. May took some exception to the figure adopted in the paper as a measurement of safety. Of course, he meant only a measure of safety so far as could be judged from the revenue account, without regard to other features; but still, judging it in that way, both Mr. May and Mr. Andras, and probably other members, disagreed with him as to that being the most advantageous figure to take. He had endeavoured to state as clearly as possible the reasons which rather influenced him in leaving out "Other income" in calculating the ratio of safety, but he entirely agreed with the speaker that he was accustomed to look at it with one eye at least. Mr. May also referred to the undoubted advantage of comparing the statistics of the particular company under consideration at the time with those relating to other companies operating in the same portion of country. That was a thing one habitually looked at, and it had been brought recently into focus in a very valuable book that had been referred to several times—"Moody's Analyses of Railroad Investments." He thought perhaps the question of the net capital per mile, and the net earnings, if taken into consideration in conjunction with other things known with regard to the railway, was undoubtedly of value. For instance, if there were two railways which were almost entirely similar, such as some of the big western roads, and it was found that one was capitalized at a very much greater charge per mile than the other, one would judge from that that the latter company was considerably sounder and safer than the former.

He might refer to an illustration which was of considerable prominence and importance many years ago. The Northern Pacific Railroad and the Great Northern Railroad practically competed with each other all the way across the Continent from St. Paul to the Pacific coast, and they were carrying on very

similar business. In one case, the capitalization was almost double what it was in the other case, and that was a feature of very great use and service to ordinary simple people who had very little except figures to go by, and it enabled a good many people in this country to avoid losses which they might otherwise have made. He was familiar with the book that Mr. May had referred to, giving a map of the routes; it was a book which was used by all the bond-dealing houses in New York. Undoubtedly the information would be extremely useful in any case, and much more so ten or fifteen years ago, when railways were in more danger of going into liquidation than they were now. In case of default, one would have to look very particularly to the special piece of line which was mortgaged to secure one's bond, and one had to fight pretty hard, although the line might be worth the amount of money it was mortgaged for.

With reference to the advisability of investing in first-class bonds, or bonds of a lower class, he had assumed in writing the paper that insurance companies, and more particularly English insurance companies, had for some years practically confined themselves to first class bonds rather than lower class bonds. But, of course, if such companies felt inclined to invest in lower class bonds—not rubbish—but bonds which might be called second or third class, there was no reason why they should not be as successful in investing in them as some other people were. It would be very inadvisable, however, to do that, unless the company meant to make a closer study than most directors and officers of insurance companies appeared to give to the subject. He was sorry he missed the article upon "American Terminology" in the *Times* of 5 November, to which Mr. Andras referred, because he thought it might be very interesting. The figures which Mr. Andras quoted for 1905-6-7-8 were fairly representative for the average price of first-class bonds and the yield in those years: they bore out to some extent the remarks he had made in the paper.

Mr. Watson had asked some practical questions, as to methods of looking after investments when they had been made. He did not know that he should like to claim any particular talent or ability to do that: but some people did attempt to watch things, very much in the way that Mr. Watson had described. For instance, they obtained reports of companies in which they were interested, and of course it was usual to analyze them and bring the principal points to the observation of the principal officer of the company, who in most cases took the trouble to read them through himself, and he believed that was really more valuable than looking at figures that had been prepared by somebody else. With regard to the values of quotations, that he thought should not be left to casual observations. In all companies he believed it was made the duty of some clerk to record them every week and report progress regularly. With reference to the question of the ratio of safety to the yield, one speaker had referred to the table, and had expressed surprise that the yield ratios came out in the manner they did. The one specially remarked was

the Atchison Topeka, where there was a ratio of safety of 312 given in the paper, and the yield was £3. 18s. 6d., and another was the Baltimore and Ohio with a ratio of 163, and yield a little more—£3. 19s. 3d. He had that and one or two others in view when he wrote in the paper, "It will be seen in most of the above cases there is a very high ratio of safety, and in other cases there are special reasons why the securities are ranked high." The first one which occurred was the Baltimore and the Ohio, a railroad bond well known in New York and London for a much longer period than the Atchison had been favourably known. Circumstances of that description did undoubtedly influence the prices of the bonds; but he did not know that it should necessarily influence to such a great extent any well-informed person's appreciation of them. The Lake Shore and Michigan South 4 per-cent Debenture was another with apparently not a very large margin, but it had a great reputation, being connected with the New York Central.

He was extremely interested in the comparative ratios brought out in the case of some other securities, including some Argentine railways. After getting beyond about 300 per-cent ratio of safety he did not think it made a very great deal of difference to the price of securities: beyond that, the consideration that weighed mostly with people was the general repute of the business. Reference was made to the question of the improvement of a security, in consequence of large amalgamations, which were supposed to make them in some way similar to a Government security. He did not think that was so. The big amalgamation of railways that had been going on in the States was rather contrary to Government views than otherwise.

Perhaps the most interesting remarks were those made by previous speakers as to the general question of investing in United States railways. It was perfectly true, as Mr. Todhunter said, that in the paper it was taken for granted that these were a suitable form of investment for insurance companies. Some of the risks which were referred to he thought were not serious, but some were. He thought that risks due to the possibility of war were not very great. Nearly all the American railway bonds were bearer securities, and if anyone was in the habit of keeping such bonds in this country he would simply have to send the coupons to a French or German banking house, and there would be very little difficulty in getting them cashed. If the securities were kept on the other side of the water, things might not be quite so easy, because he imagined that, however friendly one's banker might be, and although he might not wish to allow a political, or patriotic, question to interfere with business, he would be precluded from remitting the interest. He did not think taxes on income could be escaped, if the Americans adopted an income tax. Up to quite recently, the British Government used to allow foreigners to have rebates on their taxes if they received dividends from British companies: but now they did not do that, and presumably there would be a similar law in the United States.

He did not fear fluctuations in prices, because it occurred to him that it mattered less, from the point of view of an insurance company, than it did from the point of view of other investors, because the insurance company did not as a rule have to realize. Most of the members were much more intimately acquainted with business as conducted in life assurance offices than he was: but, so far as he had observed, the funds were generally increasing—it was rather exceptional for them to decrease.

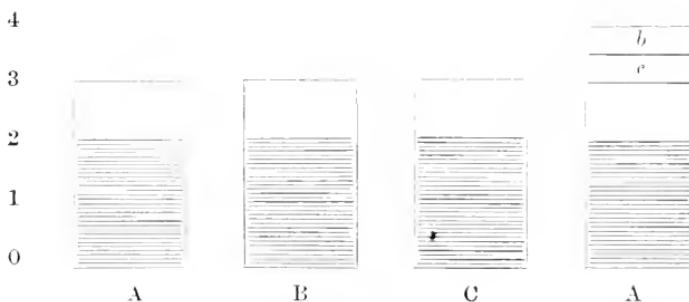
The recent increase in the ratio of bonded debt to share capital was a fair criticism, but any objection to this feature was a little modified by the circumstance that a good deal of the old share capital of American railways consisted of "water", whilst most of the share capital coming out now was paid for in cash at par, in some cases over par, and so provided a better margin than any amount of "water." He was entirely in agreement with the remarks made, with regard to not depending entirely upon the ratio of safety, that it would be dangerous to follow simplicity by just getting someone to work out the figures and neglect other points. With regard to the ratio of the net earnings to gross earnings, he had tried to bring that out. The form he used in his office he had practically copied out in the paper. He brought this percentage close up to the percentage of fixed charges. He put down the amount per mile of the percentage of gross earnings and percentage of fixed charges, and he imagined those were the figures that Mr. Todhunter had thought should be brought together.

In reference to the Western traffic, that might be affected by the opening of the Panama Canal, but he did not believe it would make such a difference as could affect a bond the interest on which was covered two or three times over. Nearly all the traffic had its origin somewhere inland, and instead of having a long haul to the Atlantic coast a good deal of it would have a short haul to the Pacific coast. He did not believe any great volume of freight was taken in at the Pacific coast and shipped across to the Atlantic or *vice versa*. A good deal, of course, would have a shorter railway haul by going through the canal. With regard to keeping securities in London or New York, and the marketability of them according to where they were kept, he had not found that it made a great deal of difference. In the case of trust companies, they had their agents on the other side, and dealt quite indifferently with one side or the other. But as a rule, one could deal a little closer by doing the business on the other side, because the securities belonged to that side, and therefore there was rather a freer and more extensive market. It was not always so, however, and in many cases equally advantageous quotations could be obtained on both sides.

Note by the Author as to Net Earnings and other Income.

SUPPOSE there are three connecting railways A, B, and C, which have net earnings and fixed charges as indicated by the first three

columns in the diagram below, where the total height of the column represents the net earnings, the shaded part the fixed charges, and the plain part the margin available for dividends. It will be seen that the fixed charges are covered one and a-half times, or the ratio of safety is 150 per-cent. The position in each case is that if one-third of the net earnings were swept away there would still be sufficient to pay the bond interest, but if one-half of the net earnings disappeared there would be a deficit.



Now, suppose A purchase half the share capital of B and C (raising the money for the purpose by an issue of its own shares, and so not increasing its own fixed charges), then its total income will be as shown in the fourth column in the diagram, where the plain parts *b* and *c* represent the dividends received by A from B and C. This total income is now 200 per-cent of A's fixed charges, which remain as before, and if we treat this as the measure of safety the inference is that if the railways lost half their net earnings the interest on A's bonds would still be covered. This, however, is not correct, because if all the railways lost half their net earnings there would be no dividend from B and C, and A would be in no better position than it was before the financial transaction referred to.

If, instead of buying shares in B and C, A had bought B's or C's bonds, or even shares in some undertaking that had not any loan capital, the above argument would not apply, and the income from the investments might form as good a security as A's own net earnings.

The conclusion appears to be that each case must be decided upon its merits, and that while it may not be fair to say that income from investments should always be excluded, it is not safe to include it without considering what the investments consist of. It will be observed that in the form recommended for general use

a space is reserved in which to state what are the principal sources of other income.

It will, of course, not be overlooked that the community of interest established by a purchase of shares is often a valuable safeguard against diminution of net earnings, but that is, in my opinion, a separate consideration which cannot be measured in figures.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., Barrister-at-Law.

Policy on life of
bankrupt
concealed from
trustee. Conflict
of claims to
policy moneys.

THE case of *Tapster v. Ward* [1909] 101 L.T. 25, deals with an interesting point which occasionally arises in office practice, as to the claim to a policy on the life of a bankrupt, when the existence of the policy has been concealed from the trustee in bankruptcy and the assured has continued to pay premiums on it for many years after his discharge. The case in question had its origin in the form of a special case stated by consent for the opinion of the Court. The facts were as follows: One John Tapster effected a policy of assurance on his life in 1879 for £250. On 30 December 1879 he presented a petition for liquidation under the Bankruptcy Act, 1869, and, in January 1880, a resolution was passed for the liquidation by arrangement of the debtor's affairs. The existence of the policy was not disclosed. In October 1880 the debtor received his discharge. At the date of his discharge only one premium had been paid on the policy, and it had no surrender-value. The policy was retained by the debtor, and all subsequent premiums were paid by him down to the date of his death, on 19 April 1907. The assured died intestate, and his administrator applied to the assurance company for the policy moneys; but the company declined to pay except on the joint receipt of the administrator and the official receiver in bankruptcy. The latter, in whom the policy was vested under sections 160 and 161 of the Bankruptcy Act, 1883, declined to concur without an order of the Court; but, upon his suggestion, payment of the policy moneys was obtained from the company to himself and the solicitor of the administrator on their joint receipt, and the amount was paid into the bank on their joint account, pending the decision of the Court. The

question then submitted for the opinion of the Court was :
“ Whether, under the circumstances above stated, the plaintiff,
“ as the legal personal representative of John Tapster, deceased,
“ is entitled to the whole or any and what part of the policy money
“ as against the defendant as official receiver and trustee for
“ the creditors of John Tapster.”

The case came before Eve, J., who held that the trustee was entitled to the policy money, and that the administrator of the assured was not entitled to any part of it. In the course of his judgment, he said : “ These facts must be borne in mind : “ first of all, that on the evidence that is now available the “ existence of this policy was unknown either to the trustee in “ the liquidation by arrangement with the creditors of John “ Tapster or his successor, the official receiver in bankruptcy; and, “ secondly, I think there is no evidence that the liquidating debtor “ paid the subsequent premiums in the belief that the policy “ moneys would not be his. I think that is an inference “ I may draw from the fact that he has paid them during the “ twenty-six years and more which elapsed between his discharge “ and death, and, for the purposes of this judgment, I can “ assume that the liquidating debtor did pay the premiums “ believing that his estate would be entitled to the policy “ moneys, and in ignorance of the fact that the trustee in his “ liquidation had a claim upon the moneys. Even assuming “ that such is the fact, is there any authority that as “ between the liquidating debtor on the one hand, and his “ creditors on the other, he is entitled to follow the whole of the “ policy moneys, or, as I thought at first, the amount of “ premiums paid, with interest from the time of payment ? “ The case which is nearest to this is *Re Tyler*, “ *Ex parte Official Receiver* ([1907] 1 K.B. 865, *J.I.A.*, xli, 411), but when one comes to examine the facts of that case and “ the lines upon which the Court there proceeded, and compares “ those grounds with those upon which the Court of Appeal “ founded its decision in the case, decided by it on the same day, “ of *Re Hall, Ex parte Official Receiver* ([1907] 1 K.B. 875, “ *J.I.A.*, xli, 413), it is quite clear that what the Court decided “ there is that the Court was bound to assume that the trustee “ in liquidation, knowing that the widow was paying the “ premiums—knowing that the premiums were being paid under “ a contract made with the bankrupt previously to the “ bankruptcy—could not allow the official receiver to hold the

" policy moneys without discharging the obligation which a " person would feel himself under who had stood by and " suffered another to pay the premiums on a policy to which " he was not entitled, until that policy had fructified into the " subject matter in dispute. Would such considerations operate " as between a debtor and his creditors, who are, no doubt, " prior in equity, or enable the debtor to say, 'my money " brought into existence this property, and I am entitled to it " against him to whom, *prima facie*, it belonged'? I do not " think so. I think I have appreciated the true *ratio decidendi* " in *Re Tyler . . .* and that that decision ought not to be " extended to a case where the debtor is making the payments " in respect of property which is vested in a trustee in " bankruptcy and, *prima facie*, belongs to his creditors. . . . " I therefore answer the question in the special case by declaring " that the plaintiff is not entitled to any part of the policy " moneys."

The administrator appealed against this decision, but on the matter coming before the Court of Appeal on 14 October 1909, the previous decision was affirmed.

**Effect of valuation
of securities by
secured creditor
in bankruptcy.** Some interesting points relating to the position of a secured creditor in bankruptcy came before the Court in the recent case of *In re Pearce* [1909] 2 Ch. 492. In this case one Henry Pearce, on 24 December 1898, assigned by way of mortgage certain property, consisting of two policies of life assurance, some book debts, and freeholds in Devonshire, to Messrs. Bullard, King & Co., to secure a guarantee which they had given to Pearce's bankers. The deed contained the common provision, excluding section 17 of the Conveyancing Act, 1881, and so permitted consolidation. The mortgagees also claimed to have a lien for unpaid purchase money on certain shares in steamships. In August 1899 the mortgagor was adjudicated bankrupt, and on the following May 10 the mortgagees lumped all their securities together, valued them at £3,806, and proved for the balance of the debt due to them. The total amount of the debt was in the first instance fixed at £8,206, but it was subsequently settled at £6,497. On receiving the valuation, the trustee did not object to all the securities being lumped together, but he wrote for particulars of the values placed on the various portions of the mortgaged property. A statement was furnished on May 22nd

in reply to this request, showing that one policy was valued at £5 and the other at £200: that the shares in steamships were valued at £1,325, and the Devonshire property, which was subject to a prior mortgage, and the book debts, at £2,276. This valuation was accepted by the trustee, and in July 1900 he admitted the proof to rank for £2,691, being £6,497 less £3,806, the assessed value of the securities. The mortgaged book debts subsequently realized much more than was expected, and out of the amount received from them the trustee paid to the mortgagees the sum of £2,276, the amount at which those debts, together with the Devonshire property, were valued. He also paid the mortgagees a dividend of 12*s.* in the pound on £2,691, the uncovered balance of their debt. In 1901 the bankruptcy was annulled, and an order made re-vesting his property in Pearce. In 1904 Pearce created certain further charges on the Devonshire property, and again became bankrupt. In 1908 the Devonshire property was sold by the prior mortgagees, and the balance of the proceeds of sale paid into Court under the Trustee Act, 1893. The subsequent mortgagee then claimed an account of what was due to Messrs. Bullard, King & Co., on the mortgage of 24 December 1898, and redemption. On the case coming before Warrington, J., he held that in the circumstances Bullard, King & Co. were entitled to hold all the remaining property on which they claimed to have a charge, namely, policies, steamship shares, and Devonshire freeholds, as security for the sum of £3,806, the aggregate amount at which they had valued their securities in the mortgagor's bankruptcy.

Against this decision the subsequent mortgagor appealed. On the case coming before the Court of Appeal, the mortgagees abandoned their claim to consolidate their lien on the steamship shares with the securities comprised in the deed of 24 December 1898, but contended that the effect of the transaction between Bullard, King & Co. and the trustee in bankruptcy was to give the former a charge for the whole of their debt over the whole of the securities, a charge to which, it was admitted, they would not be entitled but for the proceedings in bankruptcy. The Court of Appeal, however, held that the property comprised in the mortgage of 24 December 1898 could be redeemed for £2,481, the amount at which it was valued in the bankruptcy, subject to an allowance for what had already been received by the mortgagees on this security. It was, however, also held that the mortgagees were entitled to be repaid all premiums paid by

them on the life policies from the date of the receiving order, with interest from that date. Cozens-Hardy, M.R., in the course of his judgment, said: "I certainly shall not be the first to "sanction the view that it is competent for a trustee in bank- "ruptcy to give away from the unsecured creditors, and to "confer upon a secured creditor, part of the bankrupt's property "to which the secured creditor is not entitled. I do not think "the trustee ever contemplated doing that, and I therefore think "that the claim which has been given effect to by the order "under appeal must be wrong. That order declares that by "virtue of the proceedings in bankruptcy, and the composition "and the subsequent bankruptcy, the respondents became "entitled to hold the shares in the steamships, and the policies, "debts, and hereditaments in the mortgage of 24 December "1898, by way of security for the principal sum of £3,806, the "amount at which the securities in the aggregate were valued in "their proof. That, it seems to me, must be wrong, and must "be set right."

He went on, however, to express his dissent from the view that because the Devonshire property and the book debts had been valued at £2,276, and this amount had been paid, the mortgagees had no further claim of any sort on that property. On this point he said: "There is no power under the Bank- "ruptcy Act, as I read it, when a security is given upon "Blackacre and Whiteacre in one deed, for the trustee to compel "the mortgagee to put a separate value on Blackacre and a "separate value on Whiteacre; it is all one security, although "it contains several parcels, and it seems to me that the true "effect of this letter of 10 May 1900 was this: 'We value the "property comprised in the mortgage of 24 December 1898 at "the aggregate sum of £2,481, that being, with the £205 for "the policies, the aggregate value put upon all the items "comprised in that security.' That being so, we must have "regard to this fact also, that it was a mortgage debt, carrying "interest, of course; it was a mortgage debt comprising, amongst "other things, policies of assurance in respect of which premiums "were payable, and in respect of which there would be, or "might be, a lien upon them. . . . I think therefore that what "we ought to do is to take an account of what is due under the "mortgage of 24 December 1898, leaving out of consideration "any question of consolidation and simply taking an account of "what is due under that mortgage. There will be credited on

" the one side the interest, of course, on the total debt of £2,481.
 " There will be credited on the date when the £2,276 was paid
 " that amount—that would be credited as a receipt. When
 " premiums were paid on the policies they would be put on one
 " side of the account, and the money received from the sale of
 " the policies will, of course, be put on the other side of the
 " account, and the ultimate result, including any mortgagee's
 " costs properly incurred in reference to the securities, will show
 " what, if anything . . . is due to Bullard, King & Co. in
 " respect of their security of 24 December 1898, and that
 " amount must be paid out of the funds in Court, and the
 " balance paid to . . . the incumbrances immediately behind."

In a subsequent discussion as to the terms of the order, it was held that Bullard, King & Co. were entitled to be repaid, not only the premiums paid by them since the proof of their debt and the valuation of their security, but also the premiums paid by them between the date of the receiving order, 19 April 1899, and the date of their proof, 31 July 1900, on the ground that the proceedings now arose, not in bankruptcy, but in the Chancery Division, in an action between prior and subsequent mortgagees as to their respective rights. It was pointed out that Pearce himself could not have redeemed the property without repaying the premiums not payable in bankruptcy, the payment of which by the mortgagees was necessary to preserve the property, and that the subsequent mortgagees were in no better position, although if the trustee in bankruptcy had been seeking to redeem, the result might have been different.

**Period of risk
in coupon
assurances**

Coupon assurances have become common in recent times, and the case of *General Accident Fire and Life Assurance Corporation v. Robertson (or Hunter)*

[1909], A.C. 404, which deals with the determination of the period covered by the assurance in such cases, will therefore, no doubt, be of interest to the readers of these Notes. In this case one Hunter purchased, on or before 25 December 1905, a diary for 1906. It contained a coupon for assurance, under which the sum of £1,000 was made payable if the purchaser should meet with a railway accident, either immediately fatal or causing death within three months of the accident, while travelling in a passenger train. The assurancce was made conditional, *inter alia*, upon the following proviso, contained in

the coupon : "Provided that at the time of such accident the person so killed or injured was the owner of the publication in which this insurance coupon is inserted, that such person had duly caused his or her name to be registered at the head office of the corporation in Perth and had paid the fee for registration and cost of acknowledgment, and that notice of claim is sent to the registered office of the corporation at Perth within fourteen days of the occurrence of the accident, and that such claim be made within twelve months of the registration of the holder's title." Mr. Hunter duly completed the necessary form for registration and posted it, together with the required fee of 6d., on 25 December 1905, and it was admittedly received at the corporation's offices in Perth on 26 December 1905. As the office was closed for holidays on 25 and 26 December, the letter was not opened until the 27th, and on that day it was impressed with the stamp "6d., 27 December 1905." An acknowledgment, dated 29 December 1905, but not posted until 3 January 1906, was sent to Mr. Hunter, stating that the corporation had registered his name as being insured in terms of the coupon. Hunter was injured in a railway accident on 28 December 1906 and died on the following day ; and notice of his death was given to the corporation on 2 January 1907. The claim was disputed on the ground that registration of the name of the assured within the meaning of the policy was duly completed on 27 December 1905, when the assured's application form was stamped, placed in a bundle with others of the same class, and entered, and, therefore, that the year covered by the assurance had expired when the accident had occurred on 28 December 1906, and, moreover, the claim was not made within twelve months of registration, as required by the condition referred to above.

On the case coming before the House of Lords, they affirmed the decision of the First Division of the Court of Session, Scotland, in favour of the legal personal representative of the assured. In doing so, Lord Loreburn, L.C., after referring to the facts as given, said : "On 27 December 1905, at latest, the risk attached ; and it attached whether or not registration was effected on that day, because if it were not effected the only persons to blame were the defenders. The late Mr. Hunter had fully done his part, and they could not take advantage of their own default.

"That being so, how long was the insurance to continue ?

" On this the documents are silent, except for the provision . . .
" that the claim must be 'made within twelve months of the
" registration of the holder's name.' This involves that when
" once the defenders have registered the name, no liability can
" arise more than twelve months after the date of registration.
" On the other hand it is quite possible that the accident may
" have occurred within the twelve months and yet no claim been
" made within the twelve months, which gives to the defenders
" the advantage of the time elapsing between accident and
" claim. If however there is no registration, then the time
" during which the liability continues is protracted, and
" protracted, as it seems to me, without limit; or if registration is
" delayed, then the cesser of liability is deferred accordingly.

" In considering, therefore, whether this contract of insurance,
" which commenced at latest on 27 December 1905, was still in
" force on 28 December 1906, the date of the accident, and
" whether the plaintiff can succeed, two things are to be
" ascertained: first, what was the date of the registration of
" Hunter's name; secondly, on what date was the claim made?

" Registration of the name is not whatever defenders choose
" to call it, but must mean something in the nature of a record
" which may be available for use if disputes arise. It was not
" enough that a date stamp was impressed on Hunter's coupon
" slip on 27 December 1905, nor that this slip was temporarily
" placed in a bundle with others on which the same fee had been
" paid, nor that the daily total of remittances, including Hunter's
" 6*ds.*, was entered in a book. . . . Nor do I think the acknowledgement
" with a reference number, dated 29 December 1905,
" but sent to Hunter on 3 January 1906, was a registration
" of his name. It was merely a receipt for the form or coupon
" slip he had sent. But I do think that when Hunter's coupon
" slip was taken from the bundle in which it had been placed
" temporarily, and was placed in another bundle alphabetically,
" and that bundle filed to be 'kept until the liability thereon
" 'expired', then the name of Hunter was registered. . . . It does
" not appear in the evidence when that was done, but as the
" bundle in question contained other coupon slips dated as late
" as 31 December 1905, and 1 January 1906, it cannot have
" been done before 1 January 1906. Very likely it was not
" done till later—almost certainly after 1 January, for the
" bundle includes a slip of that date. Now the claim was made
" on 2 January 1907, and the defenders have not proved that

" registration took place before 2 January 1906, or, indeed, at " what time it did take place. . . .

" Accordingly I am of opinion that the registration must be " taken to have been within twelve months of the claim, not " merely because the defenders have failed to prove the contrary, " but also because, upon a balance of probabilities, I infer that " was the fact for the reason stated. . . . It follows that the " accident occurred within the period of insurance, and the claim " was made within twelve months of the registration."

Lord Shaw said : " I think the insurers under such a policy " are liable to a person insured for twelve months after they have " accomplished and acknowledged the registration in fact. On " 3 January 1906 the appellant's general manager wrote to the " late Mr. Hunter, ' Enclosed please find official acknowledgment " of the registration of your coupon.' It reached Mr. Hunter " on 4 January. . . . It appears . . . that the date of the " 'official acknowledgment', which was enclosed with the letter of " 3 January 1906 was 29 December 1905, and that the insurance " company plead that they are thus in the position of having " officially acknowledged the registration of the insurance more " than twelve months prior to the notice of death. After much " consideration, my Lords, I do not think this argument is " sound. I think an acknowledgment is not something private " in the books of the company, but is an acknowledgment to the " insurer ; that an 'official acknowledgment to Adam Turnbull " Hunter' is of no effect so long as it is lying in the archives " of the company or until it is in the course of transmission to " the addressee ; and, finally, until this acknowledgment is not " only made but in the course of transmission, there has been no " datum arrived at upon which both parties to the contract could " rely as the definite and specific time to which, namely, until " twelve months thereafter, the insurance should run."

Stamping of
substituted
security in
settlement.

The case of *Inland Revenue v. Oliver* [1909] A.C.

427 is of some interest, throwing light as it does on the methods of the Inland Revenue in stamping matters, and eliciting some suitable remarks thereon from Lord Macnaghten. The case came before the House of Lords by way of appeal from a decision of the First Division of the Court of Session, sitting as the Court of Exchequer, Scotland. The facts are as follows: In 1875 one David Turnbull, by his marriage

settlement, executed prior to his marriage, covenanted for the payment to his wife, if she should survive him, of an annuity of £400, to be charged upon certain specified lands. This settlement was impressed with a stamp of £25. 17s. 6d., and was adjudicated duly stamped. From time to time certain portions of the land in question were sold, with the concurrence of the wife, enough remaining to form ample security for the annuity. In 1898 the settlor wished to sell the remainder of the land, and the wife agreed, while retaining the personal covenant for the payment of the annuity, to release the land from the charge on condition that other sufficient security was provided. This was done, and by a deed executed in 1898, which recited the facts as to the proposed arrangement, David Turnbull conveyed to trustees certain property consisting of houses and rights of property, four policies of assurance on his own life amounting to £4,950 with bonus additions of £490, and a sum of £2,582, part of the proceeds of the sale of the original land, being the sum required to pay up in full the premiums on the four policies, the whole being in substitution for the land released. This trust deed was impressed with a 10s. stamp, but when, in February 1908, it was submitted to the Inland Revenue authorities for adjudication, they held that the deed was a "settlement", and therefore liable to settlement duty in respect of £8,022, this representing the value of the policies and bonuses and the residue of the price of the land. They accordingly assessed the total duty at £20. 15s., and claimed that amount, less the 10s. already paid, together with interest and penalty amounting to £19. 18s. 1d. On appeal, this decision of the Commissioners of Inland Revenue was reversed, the Court of Session holding that the deed in question was not a settlement. The Inland Revenue then appealed to the House of Lords, but the appeal was dismissed and the decision of the Court of Session affirmed.

The judgment of the House of Lords was delivered by Lord Maenaghten who said: "I am of opinion that the judgment under appeal is right, and that there is no foundation whatever for the claim put forward by the Commissioners of Inland Revenue There is no definition of the term 'settlement' in the body of the Stamp Act. In the schedule, 'settlement' is explained as 'any instrument . . . whereby 'any definite and certain principal sum of money . . . or any 'definite and certain amount of stock or any security is settled 'or agreed to be settled in any manner whatsoever.' And so.

"the Commissioners, finding definite and certain sums of money mentioned in the deed of substituted security, boldly claimed settlement stamp duty upon them all. The simple answer is, "that a mortgage or a deed of security by which the destination of the equity of redemption is not altered, is not a settlement in the ordinary and proper acceptance of the term, even though the mortgage or deed of security be couched in the form of a trust, and the trust be developed or unfolded in a series of provisions which have the semblance of successive steps or stages. . . . The claim, whether it is to be regarded as an ingenious experiment or an unhappy blunder, fails altogether, and the appeal must, I think, be dismissed with costs."

An important case, dealing with the liability to income tax of interest collected abroad and reinvested there, recently came before the Scottish Courts.

Income tax
on interest
collected abroad.

The case is that of *Scottish Widows' Fund Life Assurance Society v. Inland Revenue* [1909] 46 S.L.R. 993, and was in the form of an appeal to the First Division of the Court of Session, as the Court of Exchequer in Scotland, from a decision of the Commissioners of Inland Revenue in respect of an assessment to income tax in the following circumstances: The assurance society carries on the business of mutual life assurance and granting of annuities within the United Kingdom, and has no agencies for assurance or annuity business and no deposits to make or claims to meet, outside the United Kingdom. A portion of its funds is invested in American railroad and other bearer bonds and in mortgages on real estate in America and British Colonies. The interest from such investments is included in the revenue of the society, and is treated as an asset in the general statement of its affairs. The revenue, apart from such interest, was sufficient to meet all the claims, expenses and liabilities of the society in the particular year in respect of which the claim for tax was made. All surplus revenue not required to meet current charges is invested and added to the funds of the society. During the year 1905 a total sum of £101,607 was received in respect of certain of the foreign investments referred to, this amount consisting of £76,477 in respect of coupons of the bearer bonds and £25,130 in respect of mortgage interest. The majority of the coupons on the bearer bonds are payable in New York or elsewhere in

America, but some of them are payable alternatively in America or London. The coupons amounting to £76,477 were all cashed abroad and consisted, as to £3,534, of coupons payable alternatively in America or in this country, and, as to £72,943, of coupons payable in America. The bearer bonds, with their coupons, are kept at the head office of the society in Edinburgh, but it is the practice to send such of the coupons as are not cashed in this country to America, to arrive there before they fall due. The mortgages are also kept at the head office and are usually accompanied by interest notes, on presentation of which in America on their due dates interest is paid. Some of these notes are kept by the society's agents in America and some at the head office, the latter being sent to America to arrive there before they fall due. The interest on both bonds and mortgages is invested in America in bonds and mortgages as directed by the head office, and the bonds with their coupons, and the mortgages with their interest notes, representing the investment of interest, are sent to the head office at Edinburgh.

On these facts the Society contended that no part of the sum of £101,607 was subject to tax, the whole of it having been received abroad and reinvested there. The Surveyor of Taxes having claimed tax on the whole of this amount, the society appealed to the Commissioners of Inland Revenue at Edinburgh, who in part sustained and in part refused the appeal, holding the society liable for tax in respect of £84,414, whereupon eases were stated, at the instance of both parties, for the opinion of the Court of Session as the Court of Exchequer for Scotland.

On the case coming before the Court in June last, the contention of the assurance society was upheld on all points, the interest in question was held not liable to income tax in this country, and the amount of tax already paid was ordered to be returned, with interest at 4 per-cent from the date of payment until it was repaid. In delivering judgment to this effect, the Lord President said: "How can this money be said to have "been received in this country? As far as the bond itself is "concerned, it is, of course, merely a piece of paper, but it "represents a debt. The debt is not presently payable, and "taking the bond we have here as an illustration, is not payable "until the year 1935, and then is not payable in this country, "but in New York. In the same way the interest is not payable "here; it is only payable—again taking the specimen coupon

"as an example—on the first day of October 1907, at the agency in the city of New York. That is to say, there is still the debt of the principal until 1953 ; and if one were speaking of a period before the first of October 1907, the interest is not payable until October 1907 comes, when it is paid. What I have been unable to understand is the answer to the question I put—and put in vain so far as any answer was given—how money could be in two places at once. According to the argument of the Crown, the money was received in this country the moment the bond came into the company's safe in London or in Edinburgh. Equally it was in America, because the day of payment had not yet come, and therefore it was, so to speak, in the pocket of the debtor. How it can be at the one time both in America and in this country is, I think, a difficulty which surpasses even the powers of legal fiction. . . . Although the bearer bonds are marketable securities, that is surely neither here nor there, because in one sense everything is a marketable security at a price. The fact that a bearer bond is a marketable security, and easily marketable, and therefore a negotiable instrument, does not seem to me to touch the question whether it is an ordinary form of remittance. . . . On the whole matter I am clearly of opinion that this money has not been received in the United Kingdom. It is perfectly easy for the Legislature, if they so wish, to make money in this condition fall within the net of the tax gatherer. At present I do not think they have done so, and accordingly I think the determination of the Commissioners ought to be reversed. In the case of the mortgage interest, I do not need to say anything separately, because the considerations are precisely the same."

The Court accordingly reversed the decision of the Commissioners of Inland Revenue in respect of their assessment on £84,414, and ordered repayment of the income tax paid on that amount, with interest thereon at the rate of 4 per-cent per annum from the dates of payment thereof until repaid.

REVIEW.

Stock Exchange Investments in Theory and Practice.

By JOSEPH BURN.

[Published for the Institute of Actuaries by C. & E. LAYTON, London.]

In his presidential address, delivered at the Annual General Meeting of the Institute in 1908, Mr. Wyatt referred to the revised syllabus of the Institute examinations, which was then about to be published, and mentioned particularly the fact that some knowledge of finance would in future be required from a candidate presenting himself for the examination in Part II. When the revised regulations were published, the syllabus for Part II was found to contain the following sections :

- (4) The Constitution and Operations of the Bank of England : the National and Local Debts of the United Kingdom.
- (5) The principal classes of Stock Exchange Securities, and practical questions arising in connection with their purchase and sale.

It is obvious that the scope allowed to the Examiners for setting questions on the above sections is somewhat wide, and though it is understood that no questions of a very intricate nature, involving the theory of banking or currency, will be set, it would have been a difficult matter for a student to prepare for these sections without the aid of an additional text-book specially designed to cover the necessary ground. To remedy this difficulty, the Council, in pursuance of the policy they have adopted of assisting students as far as possible in their preparation for what may be called the special subjects of the examinations, arranged with Mr. Burn for the delivery of a series of twelve lectures on financial subjects during the session 1908-9. Of Mr. Burn's peculiar qualifications for undertaking a task of this nature, it is not necessary to say anything, as his experience in the matter of investments in general, and of Life Assurance Companies' investments in particular, is possibly unique, and it may confidently be asserted that the publication of these lectures has placed the members generally, and the students in particular, of the Institute under a heavy debt of gratitude.

Though, as Mr. Burn says in the preface, the lectures were intended primarily for those who were studying for Part II of the examination syllabus, there is no doubt—and the many favourable reviews which have appeared in the general press confirm the view—that the lectures in their book form will be a very useful addition to the library of any member of the Institute, or of any person interested in financial matters, more especially as they are eminently practical. It is, however, desirable that in the pages of the *Journal*, the original purpose of the author should be borne in mind, and it is therefore intended in this review to deal with the book more particularly as an examination text-book.

The first three lectures explain the Constitution and Operations of the Bank of England, starting with an interesting account of its origin and the difficulties which encompassed it at various periods of its history. The main provisions of that important statute, the Bank Act of 1844, are discussed in detail, and the author rightly draws attention to the importance of a full understanding of this Act as a precedent to following the intricacies of the banking and financial system of the country.

Full explanations are given regarding the various items included in the Weekly Returns, and the significance of the periodical variations in these items, while the conditions which culminated in the Bank crises of 1866, 1890, and 1907 are admirably described. There is one point which may perhaps be noted, and that is in regard to the expedient of selling Consols for cash, resorted to by the Bank in order to make the Bank rate effective. Just recently the Bank found itself in difficulty, owing to the drain of gold and the consequent reduction in its reserve, and it is possible that this method may have again been used, though Mr. Burn states that he was informed, upon enquiry, presumably just before the delivery of the lectures, that this course had not been taken for several years.

In the next three lectures, which are devoted to the various securities constituting the National Debt, an account is given of the gradual change in the methods of providing for national expenditure, starting with the early methods of borrowing by means of "Privy Seals" and "Tallies," and ending up with a description of the National Debt in its present-day form. In connection with the grant of life annuities by the State, on the basis of the Northampton Table, the author mentions that Mr. John Finlaison estimated that the ultimate loss to the nation by the grant of these annuities on an incorrect basis would be £24,000,000, while in one of the Government Reports of 1891 another estimate is given, which places the loss at £891,000, and it would be interesting if some further information on the point could be obtained, which would allow some discrimination to be made between two so widely differing estimates.

It is on such points as the management of the National Debt that Mr. Burn has conferred a great boon on students: information of this kind is particularly difficult to obtain, as it is very widely spread, and much valuable time is lost in referring to various books before the exact information required can be extracted. There have been several questions in recent examination papers which are understood to have been, very generally, left unanswered, but which would have presented no difficulties had this book been in existence.

After discussing fully the growth of the National Debt, the author gives a full description of the methods which have been adopted for its reduction, dealing successively with the old and new sinking funds, and with terminable annuities. The various conversion schemes which have been propounded and passed, and the causes of their failure or success, are thoroughly analyzed.

Of the remaining six lectures, the next five are devoted to consideration of Stock Exchange Securities, and after a description of the constitution of the Stock Exchange, the various documents which should come before an investor in any particular Stock Exchange transaction are examined in detail. Passing on to the securities themselves, the author takes the various classes in the order in which they appear in the official list, and clearly and concisely explains the differences existing in the various forms of stocks and shares which are dealt in on the market, considerably adding to the value of his book by the inclusion of a glossary of the many peculiar terms which are made use of on 'Change.

The troublesome question as to the exact security offered by that peculiar kind of bonds, known as "Lloyd's Bonds", may now be considered definitely settled, as the author quotes statutory authority for the statement that, so far as railway companies are concerned, there is no doubt as to the prior right of debenture stockholders.

Returning now to American Railway Bonds — a favourite investment for Life Assurance Companies at the present time—a full account is given of the three types of such securities, namely, gold, sterling and currency bonds, and thus another "preparation" difficulty is abolished. Several examples are given of arbitrage transactions, and the differences which exist in various markets as to the exact meanings which are to be attached to the prices quoted in the respective official lists are clearly laid down. Towards the end of the eleventh lecture the author describes the method of dealing in practice with terminable Stock Exchange securities, and also makes certain references to trustee investments.

The last, and perhaps the most remarkable, lecture in the book is devoted to Local Indebtedness, and the way in which the author has arranged the information he gives on the subject is admirable. After pointing out the difference in character between Local and National Indebtedness, Mr. Burn goes on to give full particulars regarding the methods followed by local bodies in raising loans, the statutory powers by which such loans are authorized, and the various rates which can be levied to provide for municipal and other expenditure. Loans to municipal bodies were, and are still to a certain extent, favoured as investments by insurance companies, and the method adopted by the author of dealing with these investments under the various heads of a proposal form which the borrowing body would have to complete should be very useful in practice. This lecture should, without doubt, cover the ground for the Part II Examination, and should also be very helpful for the Final, as it goes more fully into the question of the selection of investments than do the previous lectures. In fact, it is noteworthy how much information the author has managed to introduce into this lecture, as we have always found that, owing to the scattered form of information available, this subject is a very difficult one to deal with. It might, perhaps, be remarked in passing that the total amount of

municipal indebtedness is now about £500,000,000, which represents an increase of 100 per-cent on the total of ten years ago, and, further, that the municipal debt at the present time is equal in amount to approximately four-fifths of the funded debt of the United Kingdom. These figures show how important the question of municipal finance is becoming.

The book is very well produced, and has a copious index, whilst the author has stated his authorities very fully. There is one point, however, on which we may be allowed to sound a note of regret, and that is, that Mr. Burn seems to have studiously avoided giving his own opinion on debateable points. For instance, it would have been interesting, had he seen his way to do so, for us to have had his own views on such a burning question as the adequacy of the Bank Reserve.

In conclusion, Mr. Burn has given us a most valuable book, and one which will well repay careful study. It has already been received in a very appreciative manner by the Press, and should have a far wider sphere of educational usefulness than amongst the immediate circle of students for whom it was originally intended.

R. G. M.

SIXTH INTERNATIONAL CONGRESS OF ACTUARIES.

THE INSTITUTE OF ACTUARIES.

AT the first Ordinary Meeting of the Session, held on 29 November 1909, the PRESIDENT (Mr. G. F. HARDY) said that since the Institute last met, the Congress at Vienna, to which many of the members were then looking forward, had been held, and he thought it might be said that it proved an eminently successful one. It was only fitting that he should take advantage of what was practically the first public opportunity that had occurred to offer, on behalf of the Institute, their congratulations to the organizers of the Congress upon the very satisfactory results of their efforts, and at the same time express to them the appreciation of the members of the Institute of the very hospitable welcome extended, not only to them, but also to all the British actuaries who were present in Vienna. (Cheers.) All who were at the Congress, and experienced that welcome, and also saw with admiration the energy and enthusiasm with which the Austrian actuaries threw themselves into the business of the gathering, would be glad that the Council had been able to recognize the services thus rendered to actuarial science.

through the Congress, and the flattering welcome extended to the members of the Institute, by recommending that three of the eminent Austrian actuaries, Dr. Blaschke, Dr. Graf and Dr. Klang, should be elected as Corresponding Members of the Institute. Time that evening did not permit him entering into any detailed reference as to the proceedings of the Congress. The printed "Transactions" were now published, and he had no doubt that many of the members would be studying them during the ensuing winter; and he was quite sure they would derive very considerable benefit from the great number of essays that were read. It was a matter of congratulation that so many of the papers were contributed by members of the Institute, many of them of very great interest and value, and that in that way, and also by some very useful speeches contributed to the discussions, the Institute had some important share in the success of the meetings.

AT the Ordinary Sessional Meeting, held on 29 December, the three gentlemen referred to by the President in the above remarks, and recommended by the Council, were unanimously elected as Corresponding Members of the Institute, namely :

PROFESSOR ERNEST BLASCHKE, Actuarial Adviser (Regierungsrat) to the Ministry of the Interior, Vienna.

DR. JULIUS GRAF, Actuary and Secretary (Technischer Sekretär) of the "Assicurazioni Generali" Company, Triest.

DR. JAMES KLÄNG, Managing Director of the "Österreichischer Phoenix" Assurance Company, Vienna.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Mortality of Female Assured Lives, with graduated tables deduced from the British Offices' Experience, 1863-1893. By CHARLES WILLIAM KENCHINGTON, F.I.A., of the Prudential Assurance Company.

[Read before the Institute, 20 December 1909.]

1. THE Mortality of Female Assured Lives has not been, so far as I am aware, the main subject under discussion at a sessional meeting of this Institute since February 1875,* when the late Mr. Cornelius Walford read extracts from his exhaustive article on "Female Life", which was subsequently published in the Insurance Cyclopedia. It is hoped, therefore, that in bringing forward this subject as the main theme for consideration, a useful purpose may be served by eliciting the current views of the profession in regard to assurances on Female Lives, and that the tables now submitted may prove of value in practice.

2. A secondary object of the paper is to bring under notice a new summation formula of graduation, and it is hoped that some part of the discussion may centre round the method adopted in graduating the tables.

3. Although the subject of Female Mortality has not received exclusive consideration at a sessional meeting of the Institute for nearly 35 years, it has incidentally been mentioned in papers read in this Hall, and has been the topic of valuable contributions to the *Journal*.

The paper by Mr. G. F. Hardy on the mortality of assured lives and annuitants in France (*J.I.A.*, vol. xxxiii, p. 485), and Mr. Ackland's paper on "The British Offices' Life Tables, 1893" (*J.I.A.*, vol. xxxvii, p. 113), both deal at some length with the

* The proceedings at this meeting are reported *J.I.A.*, vol. xix, p. 174.

subject. Mr. G. H. Ryan's editorial remarks on the practice of Life Offices in regard to Female Assurances (*J.I.A.*, vol. xxix, p. 75), with tables showing the experience of four French and twenty-three German Companies, and Dr. T. B. Sprague's observations on the Mortality of Healthy Female Lives insured with the Scottish Equitable Life Assurance Society (*J.I.A.*, vol. xxxi, p. 226) are also valuable contributions.

The practice of British companies in regard to female risks was more recently tabulated by Mr. H. E. W. Lutt (*J.I.A.*, vol. xli, p. 472). Outside the transactions of the Institute, the matter has received the attention of Mr. J. Chatham in two papers read before the Faculty of Actuaries in Scotland (*T.F.A.*, vol. i, pp. 109 and 145) wherein he gives, *inter alia*, the premiums and reserves respectively, at $3\frac{1}{2}$ per-cent interest, for Female Assured Lives, deduced from the ungraduated British Offices' Experience. A particular aspect of the subject was also dealt with by Mr. H. Moir (*T.F.A.*, vol. i, p. 17) in his paper on First Year's Risk.

From a medical officer's point of view the subject was discussed by Dr. Hingston Fox (*Transactions of the Life Assurance Medical Officers' Association*, 1898 and 1899, p. 49), and more recently in an exhaustive essay by Dr. A. L. Galabin (*T.L.A.M.O.A.*, 1906 and 1907, p. 21).

Insurance on the lives of women was one of the topics for discussion at the Fifth International Congress of Actuaries held in Berlin in 1906, and foremost amongst the contributions was that of Mr. H. J. Baker (*Transactions*, Fifth Congress, vol. i, p. 591).

This list is not intended as a complete record of the writings on the value of female assured life, but it may be sufficient for the purpose of reference, so far as the present paper is concerned.

4. It is generally considered that the number of assurances effected on female lives is increasing, and if this be the case the matter is one which should receive careful attention.

No precise information is available on this point, because the Board of Trade Returns do not furnish particulars as to assurances on male and female lives separately, and it is not known that any Office analyses its business in this manner, except in the case of industrial business, which it is no object of this paper to investigate.

5. The facilities afforded by the Married Women's Property Acts, whereby a married woman can effect a policy on her own life for her separate use, or for the benefit of her husband, or

of her husband and children, were by some thought to be inducements which would result in an increase in the number of assurances effected on female lives. How far this expectation has been realized is open to question.

6. The other chief reason given for the supposed increase in female assurances is that, as women are now taking a greater share than formerly in business pursuits and professional activities, they are the more likely to require the protection afforded by a policy either as a provision for their dependents, or for their own declining years.

7. We may perhaps draw an inference as to the relative number of assurances effected on male and female lives respectively from the following table :

TABLE A.

British Offices' Experience. *New Entrants.* *Full Aggregate Data.*

Class of Assurance	Males	Females	Total	Percentage of Females to Total
Whole-Life, With Profits	402,272	27,640	429,912	6·43
Whole-Life, Without Profits	41,541	7,869	49,410	15·93
Endowment Assurances, With Profits	106,361	4,871	111,232	4·38
Endowment Assurances, Without Profits	20,507	1,297	21,804	5·95
Whole-Life, Limited Premiums, With Profits	30,706	530	31,236	1·70
Whole-Life, Limited Premiums, Without Profits	1,622	80	1,702	4·70
Whole-Life, Ascending Scale, With Profits	17,548	583	18,131	3·22
Whole-Life, Ascending Scale, Without Profits	3,573	269	3,842	7·00
Joint-Life Assurances, With Profits	5,049	3,835	8,884	48·16
Joint-Life Assurances, Without Profits	1,567	1,048	2,615	40·07
Contingent Survivorship Assurances	2,996	1,236	4,232	29·21
Temporary Assurances	10,639	1,713	12,352	13·87
Totals	644,381	50,971	695,352	7·33

8. Bearing in mind that the 20 Offices' Experience was constructed, so far as the elimination of duplicates was concerned, on a similar basis to that adopted in the compilation of the aggregate tables in the 60 Offices' Experience, and that in the former case the observations were made irrespective of the class of policy, the following comparison between the new entrants according to the total aggregate data of the

20 Offices' and 60 Offices' Experience is thought to be interesting :

TABLE B.
New Entrants. Full Aggregate Data.

Experience	Males	Females	Total	Percentage of Females to Total
20 Offices . . .	130,243	16,604	146,847	11·31
60 Offices . . .	644,381	50,971	695,352	7·33

9. In the whole-life assurance class of the 60 Offices' Experience, we can compare the number of new entrants on the basis of select data, and the figures are as follows :

TABLE C.
*60 British Offices' Experience. New Entrants. Select Data.
Whole-Life Assurances.*

Class	Males	Females	Total	Percentage of Females to Total
With Profits . . .	485,431	30,094	515,525	5·84
Without Profits . . .	45,197	8,726	53,923	16·18

10. It would appear, therefore, that although the number of assurances on female lives has largely increased, the relative number as compared with the total assurances effected has diminished; but it must be borne in mind that it is now 16 years since the observations, which formed the basis of the British Offices' Experience, terminated, and that during this period much of the progress in the business employment of women has taken place.

11. Comparing the percentages of new female entrants, as deduced from the full aggregate data, with the corresponding percentages from the select data, we have :

TABLE D.
*60 British Offices' Experience. Whole-Life Assurances.
Female New Entrants. Percentage to total New Entrants.*

Class	Full Aggregate Data	Select Data
With Profits . . .	6·43	5·84
Without Profits . . .	15·93	16·18

We may conclude, therefore, that the number of policies effected on the same life is greater amongst males than amongst females in the case of with profit whole-life assurances, while the opposite result is shown in the case of without profit policies.

12. It will also be observed from Table A that the percentages in the case of the without profit policies are greater than those for the corresponding with profit policies of the same class of assurance, with the exception of joint-life assurances.

13. In the paper alluded to in paragraph 3, Dr. Hingston Fox gave the figures relating to the Office for which he was acting, showing that out of 3,230 policies issued in the period from October 1896 to December 1897, 200 were on the lives of women, 111 being spinsters, 70 wives, and 19 widows. The average ages at entry were 27·5, 38·1, and 46·1, respectively.

14. In order to throw some additional light on the number of assurances effected on female lives, I recently investigated a representative section of business effected in the Ordinary Branch of my own Office during 1908. The result of the analysis is given in Table E.

TABLE E.
Analysis of a number of New Entrants, 1908.

Class of Assurance	Males	Females			Percentage to Total		
		Spinster	Wives and Widows	Total Males and Females	Spinster	Wives and Widows	Females
Whole-Life Assurances, with Profits	475	14	92	581	2·41	15·83	18·24
Endowment Assurances, with Profits	4,186	486	215	4,887	9·94	4·40	14·34
Double Endowments, without Profits, and not subject to medical examination	718	454	245	1,417	32·04	17·29	49·33

The average ages at entry were, in the case of spinsters, 33·9, 27·4 and 29·7 in the three classes whole-life, endowment assurances, and double endowment assurances respectively, and in the case of married women 50·0, 38·2 and 39·7 respectively.

It will be seen that the ratio of female cases to the total issued is much greater than that indicated by the British

Offices' Experience, and no doubt the relatively small average Sum Assured under the policies is the chief reason for the difference.

Taking the whole-life with profit policies it will be observed that the percentage of policies issued on the lives of spinsters is very small, being only 2·41 per-cent of the total issue in that class. On the other hand, policies issued on the lives of married women, including widows, are 15·83 per-cent of the total, but it should be pointed out that no fewer than 65 out of the 92 cases in this section were on lives aged 48 years next birthday and upwards, and were for the most part policies effected upon the maturity of previously existing endowment assurances.

In the endowment assurance class, the number of policies effected on the lives of spinsters is $2\frac{1}{4}$ times the number effected by married women.

The combination of the two elements—investment, and absence of medical examination—in the case of Double Endowment Assurances makes this class of business exceedingly attractive to female lives.

It would be of interest to know if the experience of other Offices tends to confirm these statistics.

15. All the authors above referred to agree that the rate of mortality of female assured lives, from age 15 to about age 45, is higher than that of males of corresponding age, and that after that age the conditions are reversed, and most of them are concerned to find reasons for this divergence in the mortality curves.

The reasons adduced may be summarized as follows :

1. The relatively large proportion of married women amongst assured females, as compared with the general population. This has the effect of giving increased weight to the special risks attendant upon childbirth.
2. The medical examination prior to assurance is much less searching in the case of female lives than in that of males.
3. Women, by nature, are more prone than men to conceal material facts.
4. Assurances effected on female lives for financial purposes bear a larger proportion to the total assurances on lives of their own sex than is the case with male lives.

16. Before proceeding to the consideration of these arguments in detail it is desirable to investigate more closely the results brought out by an analysis of the rates of mortality of female assured lives in the several classes of assurance furnished by the British Offices' Experience.

BRITISH OFFICES' EXPERIENCE, 1863-1893.

AGGREGATE TABLES.

17. I give in Appendix II, Table 1 (pp. 144-5), graduated values of q_x , deduced from the full aggregate data in the following classes :

1. Whole-life participating assurances, O^F .
2. Whole-life non-participating assurances, O^{NF} .
3. Joint-life assurances, O^{JF} .
4. Endowment assurances, O^{EF} .

18. The graduation has, in each case, been effected by a new summation formula, which may be expressed in Mr. G. F. Hardy's notation as

$$\begin{aligned} u'_0 &= \frac{1}{385} [5][7][11] \{ [3] + [5] - [7] \} u_0 \\ &= \frac{1}{385} [5][7][11] \{ -u_{-3} + u_{-1} + u_0 + u_1 - u_3 \} \end{aligned}$$

The formula is developed at length in Appendix I (pp. 138-143), but it may here be remarked that the chief advantage claimed for the formula is its high smoothing power, combined with facility of application. The large theoretical error, namely $-44.8 \Delta^4 u_0$, is thought to be of little consequence throughout the main section of any life table, as the values of the fourth differences are practically negligible. This is shown by the very close agreement in every case between the expected and actual deaths up to about age 75. Above this period of life the error is of more consequence, and to overcome this difficulty it is advisable to use a formula of more limited range and with a smaller theoretical error. Should the juncture between the curves require adjustment, this can be completed arbitrarily by blending the values derived from the two series by the "Curve of Sines", or by some other convenient process.

The function operated upon in the case of the aggregate tables was the unadjusted value of q_x .

19. In the case of the whole-life participating assurance section in Table 1, the values of q_x up to age 79 were derived directly from the use of the formula. From age 84 to the end of life, the values are the same as those given in the ultimate table of the select experience (*see* paragraph 32), and the intervening values for ages 80 to 83 were interpolated by assuming constant third differences.

In the other classes of assurance, no attempt has been made to complete the tables to the end of life, as it was not thought that any useful purpose would be served by so doing.

The values of q_x below age 25 in the Joint-life table were inserted by a graphic process, and the extremities of the Endowment assurance table were dealt with in a similar arbitrary manner.

20. In Tables 2 to 5 (pp. 146-9) the numbers exposed to risk, expected and actual deaths, the deviation of the latter from the former, the accumulated deviation, and the expected deviation from the average are given in quinquennial groups of ages. It may be mentioned that the expected deaths were computed by multiplying the exposed to risk at each age by the corresponding value of q_x , and the totals were summarized in quinquennial groups for the sake of convenience. Any other quinquennial groupings would produce similar results. The accumulated deviations show the sum of the deviations, from the age group against which they are tabulated to the end of the table.

21. The values inserted in the columns headed "Expected deviation" were computed approximately as follows:

The expected deviation is given by the expression

$$\pm \cdot8 \sqrt{E_x(1-q_x)q_x}$$

where q_x is the true, or graduated rate of mortality. It is to be noted that the product under the radical can be expressed as $\theta'_x \times p_x$, where θ'_x is the number of expected deaths. A close approximation to the expected deviation in a quinquennial age group will therefore be furnished by the expression

$$\pm \cdot8 \sqrt{p_x \sum_{t=2}^{t=2} \theta'_{x+t}},$$

and accordingly this method was adopted. Thus, for example the number of expected deaths for the group 70-74 in the whole-life participating assurance table is 2061·7, which, being

multiplied by $p_{72} = .93390$, gives an expected deviation of ± 35.10 , as shown in Table 2 (p. 146). The correct value, as found by the formula

$$\pm 8 \sqrt{\theta'_{70} \times p_{70} + \theta'_{71} \times p_{71} + \dots + \theta'_{74} \times p_{74}} \text{ is } \pm 35.09.$$

22. By the inclusion of the numbers exposed to risk in these tables, it is thought that a clearer idea of the bearing of the expected and actual deaths to each other is obtained than would otherwise be the case. The numbers exposed to risk exhibited in this way also serve the purpose of furnishing information as to the extent of the data.

The agreement between the expected and actual deaths is close throughout, and with few exceptions the deviations are well within those expected.

The smoothness of the curves is shown in the last column of each of the tables 2 to 5, wherein is tabulated the sum of $\Delta^3 q_x \times 10^5$, irrespective of sign.

The rates of mortality are plotted graphically in Diagram 3, with the exception of the O^{SF} curve, which has been omitted in order that the curves given may not be confused.

I return, in paragraph 35, to the discussion of the special features exhibited by the curves.

SELECT TABLES.

23. In the whole-life participating assurance class, the data are sufficient to provide a basis for the construction of Select Tables, and graduated select rates of mortality are submitted in Table 6 (p. 150). The new formula was used as the basis of the graduation, but certain modifications were adopted, which I proceed to describe.

24. Before the new formula had been devised, I had constructed by means of Mr. Spencer's 21-term formula tables of the ultimate rates of mortality, excluding the first 5 and the first 10 years of assurance, respectively. A comparison of the tables showed that the benefit of selection was, for all practical purposes, eliminated by the exclusion of the shorter term.

25. The numbers exposed to risk, and the corresponding deaths were extracted, therefore, from the experience volume for each of the first five years of assurance, and for the experience excluding the first five years. The ultimate table is consequently

based upon select data. It will be remembered that Mr. G. F. Hardy found it necessary to employ select data in the construction of the ultimate table of the O^[M] experience.*

The numbers exposed to risk, and the deaths for each of the first five years of assurance, and for the ultimate table excluding the first five years, were graduated separately by the new formula.

26. Special advantages, peculiar to this experience, were obtained by the graduation of E_x and θ_x separately, quite apart from the general advantages to which attention is drawn in Appendix 1. Graduated values of $q_{[x]}$, $q_{[x]+1}$, $q_{[x]+2}$, $q_{[x]+3}$, $q_{[x]+4}$, and q_x resulted at once from the division of the final summation column of the deaths by the final summation column of the exposed to risk. The values so obtained, while exhibiting a smooth progression for each year of assurance by itself, showed some irregularity in regard to the progression from year to year according to duration.

27. In order to understand more clearly the bearing of the figures to each other, the curves were plotted graphically. It was then observed that the rates of mortality in year of assurance 4 did not differ widely from those of the ultimate table, excluding the first five years of assurance. Accordingly, the values of $385 E_{[x-4]+4}$ and $385 E_x^{(5)}$ were combined, as were also the corresponding deaths, and from these totals the values of $q_x^{(4)}$, excluding the first four years of assurance, were obtained.

The largest difference between $q_x^{(5)}$ and $q_x^{(4)}$ was -00023 at age 24, and at 17 ages out of the 42 for which they were computed the differences were confined to the fifth place of decimals, whilst at two ages the rates were identical.

28. In order to test more closely the applicability of the table as a representation of the ultimate rate of mortality after the exclusion of the benefit of selection, the expected deaths were computed for the experience excluding the first 10 years of assurance, using select data, and a comparison between them and the actual deaths in quinquennial groups of age is given in the following table:

* See also T.F.A., vol. iii, pp. 286-296, as to the employment of the select data in the construction of the "ultimate" table based on the experience of Life Annuitants, 1863-93.

TABLE F.

*British Offices' Experience—Females—Whole-Life Participating Assurances.**Select Data, excluding first 10 years of Assurance.**Expected Deaths, computed by Ultimate Table excluding first 4 years of Assurance, compared with Actual Deaths.*

Age Group	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation	Age Group
-19	...	3	-19
20-24	6·9	7	- ·1	15·9	20-24
25-29	23·1	16	7·1	16·0	25-29
30-34	62·1	47	15·1	8·9	30-34
35-39	147·8	159	-11·2	- 6·2	35-39
40-44	262·9	257	5·9	5·0	40-44
45-49	415·7	423	- 7·3	- ·9	45-49
50-54	641·3	634	7·3	6·4	50-54
55-59	945·2	941	4·2	- ·9	55-59
60-64	1,347·2	1,332	15·2	- 5·1	60-64
65-69	1,838·8	1,867	-28·2	-20·3	65-69
70-74	2,162·4	2,138	24·4	7·9	70-74
75-79	2,102·2	2,099	3·2	-16·5	75-79
80-84	1,608·0	1,640	-32·0	-19·7	80-84
85-89	754·1	754	- ·1	12·3	85-89
90-94	204·0	190	14·0	12·2	90-94
95-99	39·2	41	- 1·8	- 1·8	95-99
Total	12,560·9	12,545	15·9

The agreement is so close that I did not hesitate to adopt the ultimate table, excluding four years of assurance only, more especially as the above comparison shows that if any difference exists it is in the direction of producing a lower ultimate rate of mortality by excluding the first 10 years of assurance than by the exclusion of four years only.

29. The values of $q_{[x]+6}$ were throughout lower than those of any other year, as might have been expected, and were adopted without modification.

It only remained to complete the table for years of assurance one, two and three, and an inspection of the plotted curves showed that, by combining the values so as to obtain $q_{[x]+1}$ and $q_{[x]+2}$, the rates of mortality would run smoothly not only for each year of assurance, taken by itself, but also for any given entry age according to duration of assurance.

The values of $385E_{x+1}$ and $385E_{x+2}$ were combined, therefore, as were also the corresponding deaths, giving by division the values of $q_{[x]+1}$. Values of $q_{[x]+2}$ were obtained by a similar process.

30. To interpolate for q_{x+1} , q_{x+2} and q_{x+3} recourse was had to Lagrange's formula.

For the sake of convenience we may substitute for $q_{[x]+n}$, $q_{[x]+1}$, $q_{[x]+2}$, q_{x+4} and q_{x+5} the symbols u_0 , u_3 , u_5 , u_8 and u_{10} respectively. In conformity with this notation $q_{[x]+1}=u_2$, $q_{[x]+2}=u_4$ and $q_{[x]+3}=u_6$. The values of q_{x+4} and q_{x+5} were taken from the ultimate table. After several tentative trials, it was decided to assume second differences as constant. The formulæ adopted, therefore, were as follows :

$$u_2 = \frac{(2-3)(2-5)}{(0-3)(0-5)} u_0 + \frac{(2-0)(2-5)}{(3-0)(3-5)} u_3 - \frac{(2-0)(2-3)}{(5-0)(5-3)} u_5 \\ = \quad \cdot 2u_0 \quad + \quad u_3 \quad - \quad \cdot 2u_5$$

Similarly $u_4 = \cdot 4u_3 + \cdot 6u_5 - \cdot 06u_8$

and $u_6 = \cdot 53u_5 + \cdot 6u_8 - \cdot 2u_{10}$

These formulæ were readily applied by means of an arithmometer, and their adoption ensured the requisite smoothness of progression through the years of assurance investigated, and produced a smooth juncture with the ultimate table.

31. The special advantage of operating upon E_x and θ_x separately, already alluded to, is manifest, for by this means the combinations required in order to obtain $q_{[x]+1}$ and $q_{[x]+2}$, and to pass from an ultimate table excluding five years to one excluding four years of assurance, were made with due regard to the weight of the data.

32. The data were only sufficient to allow of select rates being graduated for ages at entry from 20 to 65, inclusive. In the first instance, the new formula was applied to the full extent of the ultimate table by the simple device of assuming that E_x and θ_x each assumed zero values beyond the last age at which there was any data (age 99). This had the effect, however, of unduly increasing the rate of mortality in the final section of the table, q_x becoming equal to unity at age 96.

Mr. J. Spence's short 15-term formula was then employed with a much more satisfactory result, and a smooth juncture was effected at age 75, without any arbitrary treatment or blending whatever.

33. The analyzed select rates of mortality and the ultimate table are given in Table 6 (p. 150.).

Table 7 is a summary similar in form to that already described in paragraphs 20 and 21, and compares the expected and actual deaths in quinquennial groups of ages attained for the ultimate table. Table 8 (p. 152) is the corresponding analysis for the years of assurance 0 to 3. The total deviation for the combined select and ultimate tables is—2·7 on a total of 15,700 actual deaths.

The rates of mortality are plotted graphically in Diagram 1.

34. By way of anticipating criticism on the over-elaboration of the graduation, I may, perhaps, be allowed to say that the work was originally taken up merely for experimental purposes in connection with the general study of graduation methods.

The peculiar characteristics of the experience rendered the application of frequency curves a matter of extreme difficulty on the one hand, and various graphic graduations, on the other hand, differing widely from each other, could be made according to previously conceived ideas.

Mr. Spence's 21-term formula was also employed, but in view of the necessity of smoothness in progression, not only according to age, but also according to duration of assurance, it was found that a formula of even stronger smoothing power was required.

Osculatory interpolation was tried in its turn, but without satisfactory result.

ANALYSIS OF THE RATES OF MORTALITY.

Aggregate Tables.

35. For the purpose of comparison it is necessary to fix upon some standard of reference, and the O^M table was selected, because the graduated tables are, in each instance, based upon the full aggregate experience, and although the effect of selection may not be involved to the same extent in each case, it appeared to be preferable to make the comparisons with respect to a full aggregate male table, rather than with the O^M .

The ratios of q_x , by each of the O^F , O^{NF} , O^{JF} , and O^{EF} Tables to q_x by the O^M Table were computed, but it was not thought necessary to give them in detail.

In the following Table G, which is similar in form to those given by Mr. T. G. Ackland in his paper on the British Offices' Life Tables (*J.I.A.*, vol. xxxvii, p. 113), the maxima and minima values of the ratio curve are set out. The corresponding figures for the H^F Table are also given for the sake of comparison.

TABLE G.

Comparative Rates of Mortality.

Maxima and Minima values of the ratios of q_x (by the female tables specified) to q_x by the O^M Table.

Age	H ^F		O ^P		O ^{NF}		O ^{JF}		O ^{EF}	
	Maxima	Minima	Maxima	Minima	Maxima	Minima	Maxima	Minima	Maxima	Minima
20	...	1·703	...	1·386	2·121	1·540	...	1·114
22	1·155	...
25	1·927
26	2·276	...	1·514
51	·813
54	·617
59	...	·811	·710	...
60	·835
61	·844
63	·820
65	·891
66	·768
67	...	·886
72	·853
74	·844
76	1·036
77	1·060
79	1·043
80	...	·880
82	1·013
94	·823
99	1·875

The ratio curve passes through unity between ages

...	48-49	44-45	48-49	45-46	26-27
...	73-74	79-80
...	77-78	84-85	73-74
...	...	96-97

36. It will be observed that the value of q_{20} is greater in the case of each of the female tables than q_{20} by the O^M Table. The ratio curve $q_x^{\text{or}} \div q_x^{\text{OM}}$ commences at a minimum of 1·386 at age 20, increases to a maximum of 1·514 at age 26, thence decreases, passing through unity between ages 44 and 45, and reaches a minimum of ·835 at age 60. From this point the ratios again increase to a maximum of 1·013 at age 82, subsequently reaching a minimum at age 94 and at age 99, where $q_x^{\text{or}} = 1$, the ratio is 1·875.

The ratios for the other tables may be read in a similar manner.

37. The rates of mortality of females are thus shown to be higher than those of males of corresponding age according to the O^M Table, up to age 44 according to the O^F, 45 by the O^{JF}, and 48 by the O^{NF} and H^F Tables. After these ages the advantage lies with the females, except above age 73 in the case of the O^{NF}, and from age 80 to age 84, and above age 96 in the O^F.

It may perhaps be pointed out that the expected deaths according to the O^M Table are in defect of the actual from age 70 to 84 ("Principles and Methods", p. 152) and that in the age group 80-84 the defect is as much as 1·5 per-cent of the actual deaths. We may therefore conclude generally that the mortality of female assured lives is greater than that of males of corresponding age up to about age 45, after which the conditions are reversed.

38. The endowment assurance female experience is an exception, as in this table the rates of mortality are higher than those of the O^M only up to age 26, but this is probably accounted for, in part at least, by the very recent selection of a large number of cases included in the experience, and possibly also by the inclusion of a larger proportion of spinsters than in the Whole-life experience.

Further reference is made to this table in par. 58, where it is compared with the corresponding male endowment assurance experience.

39. Comparing the several female tables with each other, it will be noticed from Table I that the O^F shows a considerable improvement upon the H^F up to age 55, after which the tables are in substantial agreement. The O^{NF} table exhibits considerably higher values of q_x than the O^F at the early ages, the difference diminishing with increase in age until age 59, where the rates coincide, and from 60-69 the rates by the former are less than

those by the latter. There is not, however, any wide divergence between the rates after age 60. In view of the comparatively small data in the non-profit experience we may say that the rates, of mortality in the with and without profit classes practically coincide after age 60.

The joint-life experience shows a phenomenally heavy rate of mortality from age 24 to 39. From this age onwards until 46 the first differences of q_x are negative, showing therefore a rate of mortality diminishing as age increases, and from age 46 to age 59 the rates are somewhat less than those of the O^F Table. The data in this experience is slender as compared with that of the O^F, but as the lives involved are presumably, almost without exception, wives, it is thought that the comparison tends to show that the extra mortality of the early years of life is mainly due to the child-bearing risk.

40. It will be noticed that in diagram 3 a curve has been drawn showing graphically the values of q_x for teachers. The rates are given in Table 9, and were taken from the report by Mr. G. King on the first septennial valuation of the Elementary School Teachers' Deferred Annuity Fund.

The period covered by this experience was the seven years from 1 April 1899 to 31 March 1906, the years of life recorded being 258,169, and the deaths numbering 872.

In these rates of mortality, due allowance has been made for the fact that a number of teachers retire upon falling into ill health, which fact would have the effect of producing a light rate of mortality, and Mr. King observes that "if the rates err at all they probably rather exaggerate the mortality prevailing among the teachers."

41. The object in calling attention to this mortality experience is to emphasize the remark at the end of par. 39 with regard to the cause of the extra early mortality amongst female assured lives.

In the O^F Table spinsters, wives and widows are included in unknown proportions, the joint-life experience may be taken as that of married women, while on the other hand the teachers' experience is mainly that of spinsters. It is true that the experience of the male teachers also exhibits a light rate of mortality, and this may therefore be largely due to the conditions of life in their profession, but the female teachers have the benefit of the lighter mortality throughout life, and the conclusion already drawn is thought to be warranted.

Select Tables.

42. In Diagram 2, the q_x curves for year of assurance 0, and for the ultimate table, are set out graphically and compared with the corresponding O^{M^*} curves.

Except at the ages 51 and 52, and above age 61, the values of $q_{[x]+0}$ by the O^F Table are throughout greater than the O^M values, while in the ultimate table the curves cross between ages 29 and 30, and 44 and 45.

The mortality curve for the year following entry is of an essentially different character from that for males. It commences at a maximum at age 20, decreasing up to age 32, thence increasing steadily until age 45, after which for four years there is a slight decrease, and it assumes its final upward bend at age 49.

The inflections in the curve are thought to represent well pronounced features of the experience. A graduation of this particular year of assurance by Mr. J. Spencer's 21-term formula is given in Table Z, Appendix I (p. 143), where it has been inserted for the sake of comparison.

43. In comparing the male and female ultimate tables, it must be borne in mind that in the former, the first 10 years of assurance were excluded, while in the latter the first 4 years only were excluded.

If, however, it be conceded that the effect of selection has worn off in the female experience after the lapse of 4 years from entry, the comparison is legitimate.

It will be observed that the same general features are exhibited by the ultimate table as in the full aggregate table, except that the values of q_x by the latter are actually greater than those by the former up to age 31.

This characteristic of the experience is shown by the tables given by Mr. T. G. Ackland (*J.I.A.*, vol. xxxvii, p. 178), where the comparison is made between the O^F and O^{F^*} rates of mortality.

44. Turning now to Diagram 1, we observe the reason for the full aggregate table giving higher rates at these ages than the ultimate table, or the truncated table, excluding the first five years of assurance.

The special feature referred to is the very high mortality of years of assurance 1 and 2. The curves take a most unusual course, but reference to Table 8 (p. 152) will show that the rates have

not been exaggerated, since the expected deaths are less than the actual for these ages and durations.

At age 31, the curves begin to assume a course more in accordance with male select rates, but it is not until age 40 that the abnormally heavy mortality of years of assurance 1 and 2 is exhausted.

45. This characteristic of the select tables is of sufficient interest to bear investigation, showing as it does the incidence of the extra mortality, and providing a concrete example of the early extra mortality curve, which may be of service in subsequent investigations.

It has already been pointed out in paragraph 15 that one of the reasons given for the extra mortality of assured females is the less effective medical examination as compared with males.

This contention, speaking generally, appears to be justified, in view of the fact that the rate of mortality for year of assurance 0 is greater at all ages, with the exceptions referred to in paragraph 42, than that applicable to males, and that the benefit of selection is more rapidly exhausted.

46. Through the courtesy of a large number of Offices in sending me their medical report forms, I recently had the advantage of examining 54 such forms, applicable to female lives. In six cases, special additional forms to those applicable to males are used, and in every case, questions, more or less searching, are put to female proposers relative to disorders of the organs and functions peculiar to the sex, and in the case of married women as to the number of children borne, and the favourableness, or otherwise, of the confinements.

With regard to the analysis of urine, this would seem to be generally required, though in seven cases the medical referee is instructed to make the analysis only when thought necessary or desirable.

An effective medical examination is not inherently impossible, but the difficulty in obtaining a searching examination seems to arise from a sense of delicacy on the part of the referee in putting the necessary questions to the applicant, and the dislike on the part of the examinee to an examination by anyone other than her usual doctor.

Dr. Galabin goes so far as to suggest that in certain cases a thorough examination by a gynaecological specialist is desirable, but most actuaries would probably agree that in practice, if business is to be procured, it is impossible to obtain an examination of such a thorough nature.

In the opinion of Dr. Galabin, the medical examination is not less effective in the case of women than that of men, and he cites the favourable mortality of the later years of life in support of this view.

But it is thought that the analysis now made possible by the select table, shows that the medical examination is, in fact, less effective.

47. The incidence of the chief weight of extra mortality may be due in part to selection, adverse to the Offices, by the withdrawals.

In Table H, specimen rates of withdrawal in years of assurance are given, and compared with the corresponding male rates as furnished by Mr. King (*J.I.A.*, vol. xxxvii, p. 463).

TABLE H.

O^[F]*Rates of Withdrawal per-cent in Years of Assurance.*

n	100 $(wq)_{[x]+n}$				$(wq)_{[x]+n}^{O[F]} \div (wq)_{[x]+n}^{O[M]}$				n	
	Grouped Ages at Entry				Grouped Ages at Entry					
	23-27	28-32	38-42	48-52	23-27	28-32	38-42	48-52		
0	3.375	3.433	2.545	2.294	1.11	1.35	1.06	1.11	6	
1	13.757	10.938	8.015	6.328	1.60	1.56	1.32	1.26	1	
2	9.236	7.655	5.402	4.223	1.59	1.60	1.34	1.18	2	
3	6.544	5.770	4.412	3.005	1.55	1.59	1.38	1.08	3	
4	4.780	3.970	3.761	2.201	1.43	1.38	1.47	.95	4	
5	4.900	3.866	3.336	2.144	1.76	1.53	1.58	1.16	5	
6	3.431	3.971	3.304	2.250	1.49	1.38	1.77	1.52	6	
7	3.924	2.988	2.224	1.368	1.81	1.51	1.34	.94	7	
8	3.036	2.241	2.439	1.818	1.59	1.30	1.73	1.37	8	
9	3.096	1.885	1.769	1.511	1.84	1.27	1.35	1.39	9	

Since the days of grace were included in the period of exposure to risk, withdrawals for year of assurance 1, for instance, are mainly due to the lapsing of policies through the non-payment of the second annual premium, and thereby to some extent the incidence of the withdrawals is obscured. Since, however, the male rates were obtained on the same basis, the comparison can be legitimately made.

No attempt has been made to graduate the figures, but they clearly show that the withdrawals are proportionately much greater amongst women than amongst men, in the case

of whole-life assurances, particularly at the younger ages and at the shorter durations.

No categorical opinion is expressed as to the effect of the withdrawals, but it would seem that they may, in fact, have an important bearing on the rate of mortality.

CAUSES OF THE EXTRA MORTALITY.

48. Dr. Galabin attributes the extra female mortality mainly to the risks of pregnancy and childbirth, and he estimates that the general mortality from parturition cannot be reckoned much under 5 per 1,000 live births.

The Report of the Registrar-General for 1907 gives the mortality from puerperal septic diseases and childbirth per 1,000,000 persons living in successive quinquennia, and these have been reduced in the following Table K to the rates per 1,000,000 females living.

TABLE K.

Deaths from Puerperal Septic Diseases and Childbirth.

Period	Crude Annual Death Rates per 1,000,000 Females living
1881-1885	321·1
1886-1890	276·9
1891-1895	325·4
1896-1900	266·6
1901-1905	232·3

These figures appear to show a considerable improvement in mortality from this cause, but differences in age distribution and, perhaps of greater importance, the decline in the birth rate, are ignored.

49. In order to investigate the matter more closely, I extracted from the reports the number of deaths resulting from causes which may be classed as (1) puerperal septic diseases and (2) abortion, miscarriage and other accidents of pregnancy and childbirth, and compared them with the number of live births.*

* The causes of death included in these groups are:

(1) Puerperal Septicæmia, Puerperal Septic Intoxication.

Puerperal Pyæmia.

Phlegmasia Alba Dolens.

Puerperal Fever (not otherwise defined).

(2) Abortion, Miscarriage.

Puerperal Mania.

Puerperal Convulsions.

Placenta Praevia, Flooding.

Other accidents of Pregnancy and Childbirth.

This comparison is not entirely satisfactory, since there is on the one hand no registration of still-births nor on the other of multiple births.

The figures are as follows :

TABLE L.
Mortality per 1,000 live births, England and Wales.

Period	DEATHS RESULTING FROM—		Total
	Puerperal Diseases	Accidents of Childbirth	
1881-1885	2·87	2·07	4·94
1886-1890	2·50	2·03	4·53
1891-1895	2·60	2·89	5·49
1896-1900	2·12	2·58	4·70
1901-1905	1·95	2·32	4·27
1906-1907	1·68	2·25	3·93

It will be observed that in the first class, with the small break for 1891-1895, continued improvement is shown. In the second class, owing to a phenomenal increase of deaths arising from abortion and miscarriage, and under the heading of "other and undefined accidents of childbirth", which appear in the figures from 1890 onwards, the ratio of deaths to births, though showing a continuous decline from 1891-1895 to the present time, is still above the figure at which it stood in 1881-1885.

50. Taken in total the figures, however, appear to indicate an improvement in mortality from the combined causes. The figures relate to the general population, but it is thought that the improvement will be found to be at least as great amongst assured females, as a class, and for this reason alone we may anticipate that the next investigation into the mortality of female assured lives will show an improvement upon the British Offices' experience, in the same way as the O^F has already been shown to be more favourable than the H^F.

51. Dr. Galabin quotes figures which appear to indicate a mortality three times as great for primiparae as for multiparae. A very significant table bearing on this point is furnished by Mr. J. R. Hart in his paper "On an investigation into the mortality of the Married Females of the Peerage." (T.A.S.E., vol. iv, 42, Table IV), in which he shows the ratio of actual to expected deaths, as deduced from the aggregate experience of

his observations on the Married Females of the Peerage, according to the duration of marriage.

An extract from the table is here reproduced.

TABLE M.

Analysis of the Mortality of Married Females of the Peerage according to duration of marriage.

Year after Marriage	Actual Deaths	Expected Deaths according to aggregate experience of Married Females of the Peerage	Ratio
			(2) ÷ (3)
(1)	(2)	(3)	(4)
1	14	13·3	1·06
2	20	14·2	1·41
3	18	15·1	1·19
4-6	50	49·2	1·02
7-11	80	83·5	.96
12-19	108	113·9	.95
20-28	105	113·0	.93
29-34	71	77·3	.92
35-39	67	70·2	.95
40-51	210	203·3	.96
Totals . .	743	753·0	.99

52. If a large proportion of the female assurances included in the British Offices' experience were effected about the time of marriage, as would seem likely in view of the fact that policies are frequently required under marriage settlements, this would furnish, in the light of Tables H and M, an explanation of the high mortality of years of assurance 1 and 2.

The high mortality of the year immediately following entry is doubtless accounted for in part by the fact that a number of assurances are effected while pregnancies actually exist.

53. The other causes of deaths which operate with greater intensity at a younger age in the case of women than in the case of men are phthisis and cancer, while men are more exposed to deaths from accident and violence than the other sex.

EXPECTATIONS OF LIFE.

54. Turning now to the cumulative effect of the mortality as shown by the expectations of life, it is thought that a sufficient comparison is afforded by Table N, wherein the values of the curtate expectations of life are set out for quinquennial ages.

TABLE X.

Comparison between Curtate Expectations of Life by Various Mortality Tables.

x	H ^M	H ^F	O ^M	O ^F	O ^M	O ^F	x
20	41·561	40·315	43·182	42·523	42·387	42·093	20
25	37·905	36·909	39·083	38·782	38·745	38·212	25
30	34·181	34·003	35·067	35·249	35·070	34·833	30
35	30·516	30·945	31·159	31·764	31·388	31·647	35
40	26·890	27·753	27·360	28·293	27·736	28·327	40
45	23·292	24·487	23·668	24·779	24·155	24·847	45
50	19·806	21·116	20·107	21·210	20·696	21·402	50
55	16·462	17·692	16·722	17·709	17·419	17·955	55
60	13·330	14·351	13·571	14·371	14·380	14·732	60
65	10·512	11·272	10·716	11·262	11·630	11·720	65
70	7·995	8·582	8·208	8·524	70
75	5·876	6·425	6·079	6·197	75
80	4·219	4·950	4·337	4·377	80

In every case it will be observed that the expectations of females is at first less than, and subsequently greater than that of males. Comparing the full aggregate tables, it is seen that females have the advantage over males at an earlier age in the later experience, but that the differences are not so great between the sexes as in the earlier table. It is also to be noted that the expectations of females by the O^F Table fall below those of the H^F from age 65 onwards.

The female select expectations by the O^F Table show the greatest defect from the corresponding male figures at age 25, the defect being ·533 of a year, and the largest excess at age 50, the excess being ·706 of a year.

It would therefore seem that the 60 Offices' experience exhibits a much closer approximation between the mortality of the two sexes than does the 20 Offices' experience.

MONETARY TABLES.

55. Monetary values, at 3 per-cent interest, have been computed, and are given in Tables 10 to 12 (pp. 153, 4).

These Tables comprise the following functions :

(1) Full Aggregate Tables. O^F.

Whole-life annuity-values, $a_{x, \cdot}$, for each age from 20 onwards.

(2) Select Tables. O^(F).

- (a) Whole-life annuity-values, $a_{[x]}$ for each age from 20 to 65, as at date of entry, and a_x for each age from 24 to 69 for the ultimate table.
- (b) Annual premiums for whole-life assurances, $P_{[x]}$ for each age at entry from 20 to 65.
- (c) Whole-life annuity-values, $a_{[x]+n}$ for quinquennial ages at entry, and for values of $n = 0, 1, 2$ and 3 .
- (d) Commutation functions, $D_{[x]} \dots D_{[x]+3}$ and D_{x+4} for all values of x from 20 to 65, and complete tables of $\mathbb{N}_{[x]}$ and \mathbb{N}_x ultimate.

56. The following comparative Tables, O, P and Q, are thought to be of interest.

In comparing the male and female premiums for whole-life assurances as in Table O, it must be borne in mind that the extra premiums shown to be required up to the age 35 at entry are in the nature of a balance between the unfavourable mortality of early life and the subsequent favourable experience.

If death were the only cause for cancellation of policies the extra premiums would suffice to meet the extra risk, but in view of the withdrawals by surrender or lapse of those who have had the benefit of assurance during the years of life in which the extra risk is incurred, and who do not remain on the books sufficiently long to contribute to the favourable mortality experience in later life, it would seem upon theoretical grounds that the net annual extras should be higher than those indicated in Table O.

This consideration is intensified when the higher rate of withdrawals prevailing amongst females as compared with that of male assured lives is remembered.

The nature of the extra premium as a balance between the unfavourable and favourable mortality is clearly shown by comparing the extras required for whole-life assurances with those required for Endowment Assurances (Table P), and for Temporary Assurances (Table Q).

Comparing the extra premiums, as shown in Tables P and Q, it will be noticed that the extras required for Endowment Assurances at the younger ages and shorter terms are less than those required for Temporary Assurances for the same term on lives of the same age. With the increase of age and length of term these conditions are reversed.

TABLE O.

*Annual Premiums for Whole-Life Assurances of £100.**Select Tables.**Interest 3 per-cent.*

Age [x]	O(F)	O(M)	Extra Annual Premium (2)-(3)	Equivalent annual extra ceasing at age 50	Age [x]
(1)	(2)	(3)	(4)	(5)	(6)
20	1.423	1.365	.058	.073	20
25	1.644	1.551	.093	.126	25
30	1.857	1.785	.072	.107	30
35	2.094	2.081	.013	.023	35
40	2.408	2.457	-.049	...	40
45	2.847	2.940	-.093	...	45
50	3.423	3.564	-.141	...	50
55	4.227	4.377	-.150	...	55
60	5.306	5.444	-.138	...	60
65	6.807	6.853	-.046	...	65

TABLE P.

Annual Premiums for Endowment Assurances of £100. $100 \cdot P_{[x]n}^{o^{(1)}}$. $100 \cdot (P_{x,n}^{o^{(1)}} - P_{[x]n}^{o^{(m)}}).$ *Select Mortality.**Interest 3 per-cent.*

$n =$	10	15	20	25	30	35	40	45	$= n$
x									∂
20	8.817 .114	5.602 .095	4.039 .093	3.135 .096	2.558 .098	2.167 .095	1.895 .089	1.706 .082	20
25	8.893 .172	5.699 .167	4.144 .165	3.241 .159	2.667 .151	2.283 .140	2.022 .127	1.849 .116	25
30	8.924 .176	5.747 .177	4.197 .166	3.300 .150	2.738 .135	2.372 .119	2.133 .102	...	30
35	8.933 .144	5.761 .133	4.226 .114	3.349 .094	2.809 .071	2.472 .050	35
40	8.957 .102	5.805 .080	4.293 .054	3.417 .026	2.946 -.001	40
45	9.029 .070	5.913 .039	4.441 .003	3.646 -.031	45
50	9.146 .024	6.089 -.018	4.688 -.061	50
55	9.391 .012	6.431 -.043	55

TABLE Q.

Annual Premiums for Temporary Assurances of £100.

$$100 \ P_{[x]n}^{1-\theta^{(1)}}.$$

$$100 \ (P_{[x]n}^{1-\theta^{(1)}} - P_{[x]n}^{1-\theta^{(u)}}).$$

*Select Mortality.**Interest 3 per-cent.*

$n =$	5	10	15	20	25	30	35	40	$= n$
$x = 20$.702 .262	.678 .141	.735 .137	.798 .145	.842 .143	.879 .123	.924 .099	.981 .077	$x = 20$
$x = 25$.842 .378	.860 .283	.920 .261	.958 .232	.993 .195	1.042 .160	1.107 .127	1.189 .097	$x = 25$
$x = 30$.875 .345	.967 .324	1.012 .260	1.051 .201	1.110 .151	1.189 .114	1.291 .077	1.419 .051	$x = 30$
$x = 35$.892 .315	1.001 .240	1.063 .166	1.143 .101	1.246 .057	1.377 .013	1.539 -.012	...	$x = 35$
$x = 40$.932 .241	1.066 .131	1.186 .057	1.324 .003	1.492 -.054	1.701 -.085	$x = 40$
$x = 45$	1.049 .154	1.258 .064	1.447 -.025	1.665 -.104	1.933 -.136	$x = 45$
$x = 50$	1.241 .069	1.569 -.019	1.871 -.141	2.224 -.196	$x = 50$
$x = 55$	1.637 .065	2.125 -.100	2.610 -.203	$x = 55$
$x = 60$	2.268 .008	3.061 -.107	$x = 60$

It is thus clear that the extra vitality of females after about age 45 is a matter for consideration in granting Double Endowment Assurances.

Theoretically, if an extra premium be charged on account of sex in whole-life cases, when the age at entry is such that an extra risk is incurred, a reduction should be made from the rates of premium applicable to males, when the age at entry is such that a more favourable rate of mortality than that of males is expected.

57. Assuming an office to transact a volume of new business under whole-life policies in one year between ages 18 and 67, equivalent to the number of new assurances effected in the British Offices' whole-life with profit experience, females, for £100 sum assured in each case, it will be seen from the following table that the amount of the annual premium income for the first year under the policies, if O^{EM} net premiums be charged at all ages, would be more than £1,000 in excess of the premium income if charged on the basis of O^{EF} rates throughout.

TABLE R.

Whole-Life Assurances. Extra Premium Income.

Grouped ages at entry	New Entrants	$\frac{100 \times}{(P_{x,n}^{EF} - P_{x,n}^{EM})}$	$+ 2 \times 3$	Grouped ages at entry
(1)	(2)	(3)	(4)	(5)
18-22	1,366	.058	79.2	18-22
23-27	2,763	.093	257.0	23-27
28-32	3,913	.072	281.7	28-32
33-37	4,477	.013	58.2	33-37
38-42	4,654	-.049	- 228.0	38-42
43-47	4,181	-.093	- 388.8	43-47
48-52	3,456	-.141	- 487.3	48-52
53-57	2,366	-.150	- 354.9	53-57
58-62	1,437	-.138	- 198.3	58-62
63-67	518	-.046	- 23.8	63-67
Total	29,131	...	+ 676.1 - 1681.1 - 1005.0	...

58. It has already been remarked that the O^{EF} Table exhibits a light rate of mortality, and the effect of this is shown in Table S, by way of comparison between the O^{EF} rates of premium and those for endowment assurances on male lives, according to the O^M and O^{EM} Tables. The values of $P_{x,n}$ for the O^{EM} table were computed for this purpose from the tables furnished by Mr. T. G. Ackland (*J.I.A.* vol. xxxvii, pp. 142 and 148). It is unnecessary to comment upon the results.

TABLE S.

$$100(P_{x_0}^{(O^F)} - P_{x_0}^{(O^M)}) \text{ and } 100(P_{x_0}^{(O^F)} - P_{x_0}^{(O^M)}).$$

Full Aggregate Experience.

Interest 3 per-cent.

20		25		30		35		x
OEF - OM	OEF - OEM	n						
.022	.054	-.011	.058	-.054	.048	-.080	.046	10
.015	.063	-.026	.061	-.068	.050	-.102	.037	15
.002	.066	-.039	.064	-.085	.048	-.133	.021	20
-.009	.075	-.054	.061	-.109	.037	-.169	.001	25
-.023	.067	-.073	.054	-.136	.023			
-.040	.062	-.096	.044			-.209	-.036	20
-.059	.055			-.277	-.075	-.164	-.010	15
		-.328	-.136	-.219	-.063	-.120	-.015	10
								n
		50		45		49		x

CURRENT LIFE OFFICE PRACTICE.

59. With the results of this investigation in mind, it is interesting to briefly consider the current practice of Life Offices in regard to assurances on female lives.

Having this object in view I ventured in September last to address a letter to all the principal Offices transacting Life Assurance business in Great Britain. Replies were received from all the 72 Companies to whom the letter was addressed, and I would here tender my best thanks to each one of them for the ready way in which the response was made. The enquiries were made in a similar form to that adopted by the Treasurer of the Actuaries' Club in November 1889, in order to facilitate comparison between the results.

The enquiry made by the Treasurer of the Actuaries' Club was addressed to 80 British and Irish Companies; of these, two stated that they had no fixed rule, one did not accept female lives, and one based its premiums for female lives on the H^F Table throughout.

The following is a summary of the practice of Life Offices in regard to Female Risks, under Whole-Life Assurances, with premiums payable for the whole term of life.

		Number of Companies.	September 1909.	November 1889.
(a) <i>For Unmarried Women:</i>				
No extra charge	35			
Generally, but special consideration if about to marry	4	47	46	
But 10s., £1, or £1 1s. per-cent if about to marry	8			
Single extra of £1 per-cent	3			
Of 10s. per-cent if 45, or under	1	8	6	
Of £1 on marriage	4			
Annual extra of 5s. per-cent throughout life	1			
Of 5s. per-cent throughout life, if thought necessary	1	16	24	
Of 5s. per-cent throughout life, if under 35	1			
Of 5s. per-cent, discontinued at age 50	13			
No fixed rule; extra (if any), by rating up	1			
Total	72	76		
(b) <i>For Married Women who have borne no Children:</i>				
No extra charge	28			
Generally, but special consideration if recently married	3	40	41	
But £1 1s. per-cent on first confinement	2			
But 10s. or £1 per-cent if recently married	7			
Single extra of £1 per-cent	9			
Of £1 per-cent unless married for several years or past childbearing	3			
Of £1 per-cent if 45, or under	1	15	12	
Of £1 per-cent and annual extra of 5s. per-cent afterwards, ceasing at 50	1			
Of £1 per-cent or 5s. per annum, with no fixed rule as to discontinuance	1			
Annual extra of 5s. per-cent throughout life	1			
Of 5s. per-cent throughout life, if thought necessary	1			
Of 5s. per-cent throughout life, if under 35	1	16	23	
Of 5s. per-cent, ceasing at age 50	12			
Of 10s. per-cent, removed after change of life	1			
No fixed rule; extra (if any), by rating up	1			
Total	72	76		

(c) *For Married Women who have borne Children:*

		Number of Companies.	
		September 1909.	November 1889.
No extra charge	48		
Generally, but special consideration if pregnant	7	55	52
Single extra		0	1
Annual extra of 5s. per-cent throughout life	1		
Of 5s. per-cent throughout life, if thought necessary	1		
Of 5s. per cent throughout life, if under 35	1	16	23
Of 5s. per-cent, ceasing at age 50	12		
Of 10s. per cent, removed after change of life	1		
No fixed rule; extra (if any) by rating up		1	...
Total		72	76

For whole-life assurances with limited payments of premium, 73 Offices charge the same extra premiums as under whole-life assurances with premiums payable throughout life. Of the other two Companies, each of which charges an extra annual premium of 5s. per-cent, ceasing at age 50, under whole-life assurances with premiums payable throughout life, one makes an equivalent extra charge, while the other charges no extra, when the premiums are limited to 20 or a smaller number of payments.

For short-term assurances the practice of 57 Companies would appear to be the same as for whole-life cases, while the remaining 15 either decline or discourage this class of business, or charge a higher extra premium.

For endowment assurances, 66 Offices apply the same rules as to the whole-life business, while the 6 other Offices, each of which charges an extra in whole-life cases, either make no charge, or treat endowment assurances, particularly for short terms, more leniently.

The practice in regard to the acceptance of risks, when the person proposing is pregnant at the time of the examination, may be summarized as follows :

	Number of Companies.
Defer acceptance in all cases.	17
Defer acceptance generally, but consider specially in certain cases.	16
Defer acceptance in case of first pregnancy, no charge for subsequent pregnancies	4
Defer acceptance in case of first pregnancy, and charge 10s. to £1 per-cent for subsequent pregnancies	6
Charge £1 per-cent in case of first pregnancy, and 10s. to £1 for subsequent pregnancies	7
Charge £1 per-cent in case of first pregnancy, and no charge for subsequent pregnancies	7
Charge 10s. or 10s. 6d. per-cent in all cases . .	3
No charge	5
No charge, but endorse policy to the effect that no amount, or a reduced amount only, will be payable in the event of death resulting from the confinement	2
No general rule, each case treated on its merits	5
Total	72

60. It would appear, therefore, that the tendency of change in Office practice in the course of the last 20 years has been in the direction of reducing or dispensing with an extra premium for female lives on the ground of sex.

This leads naturally to the question whether an extra premium is or is not required.

It cannot be gainsaid that the British Offices' Experience shows that there is additional risk in the early years of life.

Looking at the matter broadly, however, it has, I think, been shown that the reason of the extra mortality is to be found in the risks attendant upon childbirth. This risk there is reason to believe is a diminishing one.

If these conclusions be correct, it would seem to follow that spinsters, with no immediate prospect of marriage, should not be charged an extra premium; and married women who have borne children, and whose confinements have been favourable, and those who, having borne no children, have been married several years or are past child-bearing age, should also be exempt from extra charge.

An extra annual premium of 5*s.* per-cent for whole-life assurance, payable throughout life, has no ground for justification, and is inequitable, not only as between the sexes, but also in an aggravated form as between women of different ages at entry. It is also inequitable as between women of the same entry age in view of the withdrawals (paragraph 47).

The same remarks apply in a modified degree to an annual extra premium of 5*s.* per-cent ceasing at age 50.

A single extra premium has more justification on theoretical grounds, but it is doubtful if business on a large scale could be transacted, if it were the practice to make such a charge, even when it might be imposed on lives of not more than (say) 40 years of age at entry. Furthermore to ensure equity such extra premium would have to be graduated with respect of age.

A single extra premium, payable only in cases where the assured was recently married, or, in the case of spinsters, upon marriage, appears to meet the requirements more effectively. In the latter case, however, it may be a matter of difficulty for the Office to obtain payment of the extra premium as it becomes due, and the deduction of the extra charge, with interest thereon, in the event of a claim arising does not recoup the Office for the extra premiums which it fails to collect on lapsed policies.

The most suitable way of meeting the extra mortality, in my opinion, is by placing a heavy initial and rapidly diminishing debt on the policy. In the case of policies effected on recently married women, or on spinsters about to be married, this exactly meets the requirements. As measured by the death-strain the extra risk, arising upon the marriage of a spinster who has been assured some years, is not so great as in the case of a new assurance. It is not essential, though it may be desirable, that the deduction should follow a fixed scale, and the circumstances of each individual case should, in my opinion, be closely considered in applying any scale.

This paper has already so far exceeded the limits prescribed by custom that it would be unwise to load it still further with figures relating to diminishing debts. It may be remarked, however, that such a system, if adopted both for whole-life and endowment assurances, would eliminate the chief weight of adverse selection, and the Offices might find it to their advantage to dispense with medical examination in the case of females. The opinion has been expressed in paragraph 46 that the medical examination is less effective in the selection of women than in the

case of men, and the medical examination is itself, in many instances, the reason given by women, otherwise favourable to assurance, for refusing to assure their lives.

Some disadvantage might accrue from the system in the case of policies required for financial purposes, but an extra policy for a decreasing assurance equivalent to the diminishing debt could doubtless in such cases be procured, and the Office would be recouped for the extra risk by the premium for the additional policy.

POLICY-VALUES.

61. The question of policy-values is intimately connected with that of extra risk, and although the matter cannot now be treated exhaustively, I give in Table T specimen ratios of whole-life select policy-values by the O^(F) Table to the corresponding values deduced from the O^(M) Table, at 3 per-cent interest.

TABLE T.

$${}_n V_{[x]}^{(F)} \div {}_n V_{[x]}^{(M)}$$

Select Tables.

Interest 3 per-cent.

$x =$	20	25	30	35	40	45	50	55	60
1	.667	.811	.915	.910	.857	.875	.942	.940	.950
2	.690	.721	.812	.852	.862	.886	.939	.940	.973
3	.691	.684	.786	.838	.860	.883	.926	.931	.972
4	.731	.708	.785	.828	.854	.879	.917	.924	.963
5	.785	.752	.784	.816	.849	.878	.910	.922	.953
10	.900	.817	.797	.836	.881	.907	.934	.953	.970
15	.906	.827	.833	.879	.915	.941	.969	.984	.995
20	.887	.849	.871	.909	.942	.968	.989	1.000	...
25	.890	.879	.902	.936	.966	.986	1.000
30	.907	.905	.928	.960	.983	.997
35	.924	.930	.953	.977	.994
40	.943	.953	.971	.989
45	.961	.971	.985
50	.976	.983
55	.987

These ratios are of a striking character, and although direct comparison is impossible, it may be remarked that they agree closely with those deduced from the ungraduated tables, at $3\frac{1}{2}$ per-cent interest, as given by Mr. Chatham (*T.F.A.*, vol. i, p. 157, Table 16).

The question is unimportant from the point of view of reserves, as no distinction is made between male and female lives for valuation purposes, but it may be worth attention from the point of view of surrender-values.

62. The subject is a large and interesting one, and is by no means exhausted, but hope of a full discussion would fail were the paper further protracted.

APPENDIX I.

ANALYSIS OF THE NEW FORMULA.

The formula—

$$\begin{aligned} u'_0 &= \frac{1}{385} [5][7][11] \{ [3] + [5] - [7] \} u_0 \\ &= \frac{1}{385} [5][7][11] \{ u_0 + \gamma_1 - \gamma_3 \} \\ &\quad (\text{where } \gamma_n = u_{-n} + u_n), \end{aligned}$$

this being the form in which it is most easily applied, and in its expanded form may be written—

$$\begin{aligned} u'_0 &= \frac{1}{385} [45u_0 + 44\gamma_1 + 41\gamma_2 + 36\gamma_3 + 30\gamma_4 + 22\gamma_5 + 13\gamma_6 + 5\gamma_7 \\ &\quad - \gamma_8 - 5\gamma_9 - 6\gamma_{10} - 5\gamma_{11} - 3\gamma_{12} - \gamma_{13}] \\ &= .1169u_0 + .1143\gamma_1 + .1065\gamma_2 + .0935\gamma_3 + .0779\gamma_4 + .0571\gamma_5 \\ &\quad + .0338\gamma_6 + .0130\gamma_7 - .0026\gamma_8 - .0130\gamma_9 - .0156\gamma_{10} \\ &\quad - .0130\gamma_{11} - .0078\gamma_{12} - .0026\gamma_{13} \end{aligned}$$

It will be observed that 27 terms are involved, whereas Mr. J. Spencer's formula involves 21 terms, and the Karup-King formula (*J.I.A.*, vol. xli, p. 554) 29 terms.

The formula is correct to third differences, since the operation,

$$[5][7][11]u = 385(1+8b)u$$

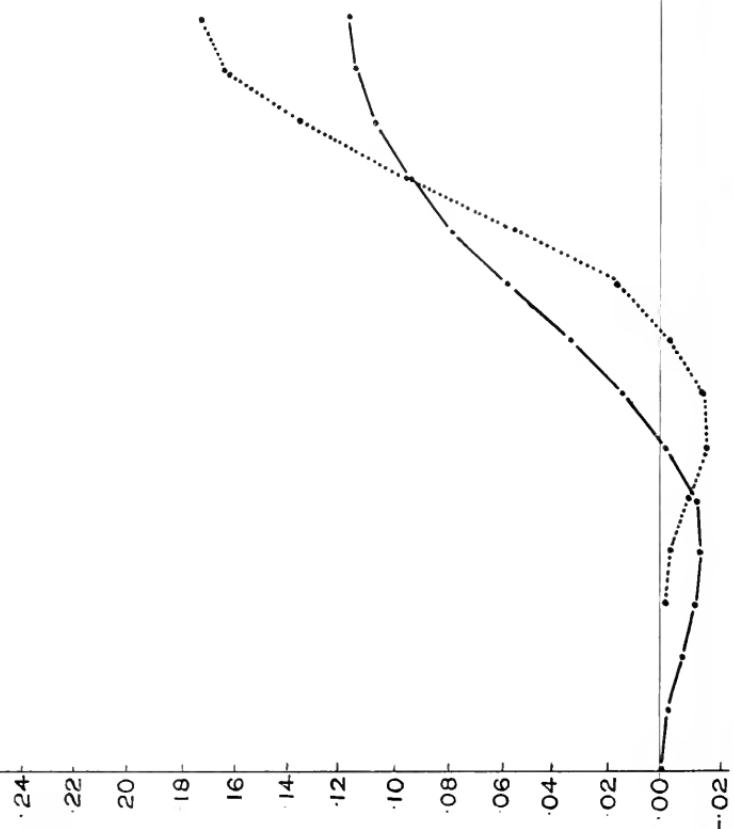
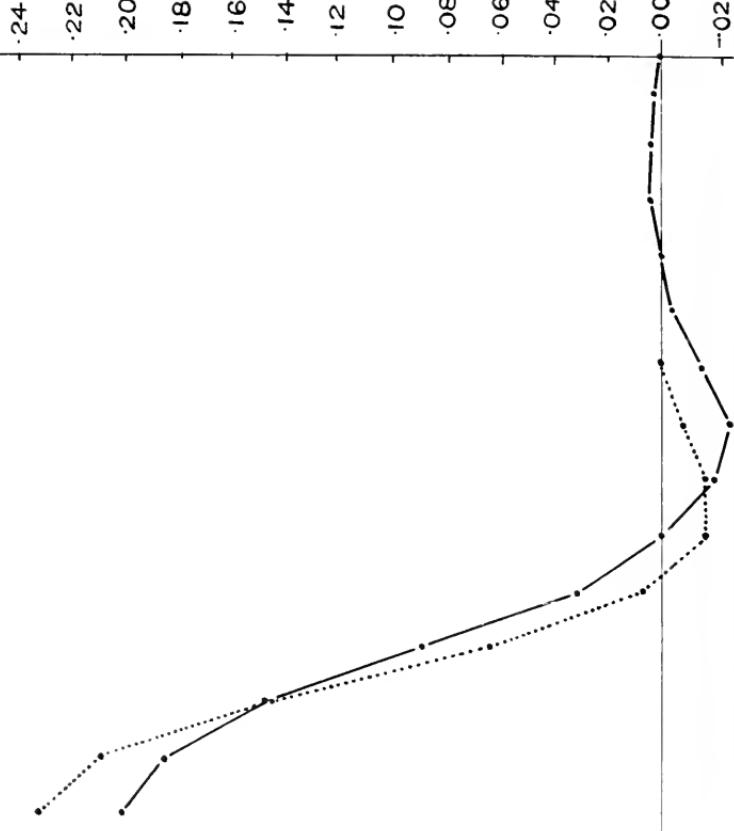
J. Spencers 15 term Formula/α

Karup-King 29 "

J. Spencers 21 term Formula/α

New

27 "



and the operand,

$$\{[3]+[5]-[7]\}u = \left\{ 3+5-7 + \frac{27+125-343-(3+5-7)}{24} b \right\} u \\ = (1-8b)u$$

The smoothing coefficient* is $\frac{\sqrt{140}}{3850} = \frac{1}{325}$ approximately, as compared with $\frac{1}{160}$ by Mr. J. Spencer's 21-term formula.

The following comparison, similar to that given by Mr. Spencer (*J.I.A.*, vol. xli, p. 390), is thought to be useful :

TABLE U.

Formula	Coefficient of central term	SUM OF COEFFICIENTS		Sum of 3rd differences irrespective of sign of coefficients entering into formula	Fourth difference error. Coefficient of $\frac{d^4}{dx^4} u_0$
		3 central terms	5 central terms		
Karup-King.	.200	.574	.865	.159	Nil
Spencer (21-term).	.171	.497	.766	.115	12.6
New117	.345	.559	.062	44.8

The coefficients of the formulae are shown in the appended diagram, in which, for sake of comparison, Mr. J. Spencer's 15 and 21-term formulas, and the Karup-King formula are also plotted.

It should be noted that if the formula is applied directly, one column less is required than in the similar application of Mr. Spencer's 21-term formula, because the operand is more simple in form. On the other hand, the longer range of the summations would make the work more laborious than that involved in the application of Mr. Spencer's formula.

The operation of continuous summation becomes tedious when the number of terms to be summed is more than five. Even when the summation extends to five terms only, considerable labour is involved in separately summing each successive group of five values, and for the purpose of verification it is necessary to compare the total of the column with five times

* The simplest way in which to obtain the smoothing coefficient is by means of the device suggested by Mr. G. J. Lidstone (*J.I.A.*, vol. xlii, p. 116).

the sum of the column from which it is derived, adjusted on account of the initial and terminal values not included five times in the summations. Should an error be disclosed there is no ready means of locating it.

I suggest, however, that the work is facilitated by the employment of subsidiary working columns.

$$\text{Since } [2n+1]u_x = u_{x-n} + u_{x-n+1} + \dots + u_{x+n-1} + u_{x+n}$$

$$\text{and } [2n+1]u_{x+1} = u_{x-n+1} + u_{x-n+2} + \dots + u_{x+n} + u_{x+n+1}$$

$$\text{therefore } [2n+1]u_{x+1} = [2n+1]u_x + (u_{x+n+1} - u_{x-n})$$

Thus if a column of $(u_{x+n+1} - u_{x-n})$ be constructed, the values being written against $[2n+1]u_x$, the former are the first differences of the latter, and the column $[2n+1]u_x$ can be completed by the addition of the differences.

The columns $(u_{x+n+1} - u_{x-n})$ are easily constructed with the aid of a card or a slip of paper, cut in such a way as to disclose only the terms to be differenced. If this method is adopted it is only necessary to make two actual summations for any one column, namely, those applicable to the first and last groups. The sum of the differences added to the initial summation, must, of necessity, reproduce the final summation, and as the work is carried out by a continued process, verification is automatic. It is advisable, however, to insert verification values in the summation column at, say, every fifteenth term, to ensure accuracy as the work proceeds. Although the method suggested seemingly adds to the number of figures to be recorded, it saves considerable time and trouble, especially when the summations, as in the case of the new formula, are in 7's and 11's.

I have, throughout, deferred the division by 385 until the last stage; but when the columns are completed in the manner above suggested, it is possible to dispense with the last summation column, because, the reciprocal of 385 can be set up as a constant multiplier on the fixed plate of an arithmometer, and we have only to multiply by the initial value of the column headed (11), and then continuously, without clearing the slide, by the successive differences which appear in column (10). This also allows an automatic correction to be made for the last place of decimals retained, by setting up five in the first place of decimals rejected, in the product holes of the arithmometer, before commencing the work.

The following table is a specimen of the actual work involved :

TABLE W.

Specimen working Schedule for formula $u_0 = \frac{1}{385} [5][7][11]^{\frac{1}{2}} - u_{x+3} + u_{x+2} - u_{x+1} + u_x + u_{x-1} - u_{x-2} + u_{x-3}$

Example taken from O^{MF} Table.

x	y_x	[3]	$q_{x-3} + q_{x+3}$	(3) - (1) = $u_x - u_{x+3} + u_{x-3}$	[5] $u_x = r_x - r_{x+4}$	[7] $r_x = r_x - r_{x+5}$	[11] $w_x = w_x - w_{x+6} + w_{x-5}$	y_x	(1)
(1)	(2)	(3)	(1)	(5)	(6)	(7)	(8)	(9)	(10)
30	.0038								
1	1276	-02924							
2	810	3178							
3	1092	3217	-02190	-01057					
4	1345	3531	2101	1133					
5	1097	3794	1057	1837	-00269	-05499			
6	1352	3574	2719	855	-416	5768			
7	1125	3624	3007	617	1035	6184			
8	1147	3899	2573	1326	1657	7219	-02070	-50942	
9	1627	4436	2857	1549	149	8876	442	53012	
10	1662	4765	1803	2872	654	9025	2122	52570	
11	1476	4673	2161	2512	802	8371	2978	50418	
12	1535	3779	3013	766	2243	7569	4410	47450	
13	768	3317	2615	672	1264	5326	4415	43660	10377
14	1014	3168	2121	747	179	4062	2893	38615	10441
15	1386	3383	2751	629	225	4241	2010	35752	8325
16	6	983	3311	1218	111	4466	884	33742	4927
17	945	3147	2202	915	808	4610	2813	31626	5084
18	1219	3162	2565	897	81	5178	3126	37439	9052
19	1298	3705	2814	891	651	5559	20065	10565	11284
20	1188	3665	2168	1497	665	6210	1674	42571	20615
21	1179	4198	2869	1329	492	6875	1276	14245	33089
22	1831	4238	2637	1596	895	7367	2533	-45521	5332960
23	1223	1704	3142	1562	188	6172	4058	-48054	
24	1659	4212	2829	1383	470	6284	5817	52112	
25	1339	4943	4311	602	1338	6754	4138	57929	
26	1954	4913	3802	1141	2176	8092	4164	62367	
27	1650	6114	4018	2066	2124	-10268	66831		
28	2510	6739	3839	2960					
29	2579	7187	3928	3559	887	12692			
30	2398	7177	4151	3026					
31	2500	6872	6618	254					
32	1974	7275	6078	1197					
33	2804	8883							
34	4108	-10108							
35	3499								

Note.—Negative values which in the actual working are written in red ink are indicated in this Schedule in heavy type.

The formula can be applied equally well to the graduation either of q_x or $\text{colog } p_x$, or to E_x and θ_x separately.

Herr Julius Altenberger, in a letter to the Editor of the *Journal (J.I.A.)*, vol. xli, p. 46, suggests the application of summation formulas to E_x and θ_x separately, and the subject is also dealt with by Mr. G. F. Hardy in his "Lectures on Graduation" (p. 21.) It is contended that the method not only produces very smooth results, but that, being based upon the correct weights, the graduated rates of mortality more nearly reproduce the actual deaths than rates obtained by graduating the unadjusted q_x would do.

The method has other advantages, particularly when applied to Select Tables (*see* par. 31), and it is to be noted that in a complete experience, by considering the numbers exposed to risk to assume zero values before the initial term and after the final term, a ready means is available for completing the graduation of the extremities of the table. There may be some doubt, however, as to the propriety of applying the formula in this manner in the case of an endowment assurance experience, in which there may be well marked breaks in the continuity of E_x due to the maturity of endowments.

When the formula is applied to E_x and θ_x separately, it is unnecessary to perform the division by 385, since this factor is common to both the numerator and denominator of q_x .

An important criticism of summation methods of graduation in general is that, if there be a succession of errors of observation of the same sign, these errors are not eliminated but are retained and made to appear as a feature of the graduated table.

Owing to the combination of the two summations of such widely divergent range as [5] and [11], the new formula belongs to the group described as the "wave cutting" class, the characteristic of which is that a succession of errors of the same sign are not retained to the same extent as in formulæ with summations of more nearly equal range. This characteristic is exemplified by the following comparison (Table Z) between the graduations made by Mr. J. Spencer's 21 term formula, and the new formula, both applied to E_x and θ_x separately, for "year of assurance 0." The "wave cutting" is clearly shown in column (6), in which the differences between the values of q_x , as graduated by the two formulas, are tabulated.

The opinion has already been expressed (par. 18) that a large theoretical fourth difference error is of little consequence throughout the main section of most mortality tables, and this statement is substantiated by an examination of the columns headed "Sum of $10^5 \times \Delta^3 q_x$ irrespective of sign" in Tables 2 to 5 and Table 7.

TABLE Z.

Comparison between Mr. J. Spencer's 21-term formula and the new formula applied to O^(F) Table, year of Assurance 0.

x	Spencer $q_{(x)}$	New $q_{(x)}$	$10^5 \times \Delta q_{(x)}$		(Col. (2) - Col. (3)) $\times 10^5$	EXPECTED - ACTUAL DEATHS		ACCUMULATED DEVIATIONS		x
			Spencer	New		Spencer	New	Spencer	New	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
20	.00863	.00711	- 60	- 14	152	- .8	- 1.2	- .4	- .7	20
1	.00803	.00697	- 74	- 14	106	- 2.3	- 2.6	- .4	- .5	1
2	.00729	.00683	- 71	- 21	46	...	- .2	2.7	3.1	2
3	.00658	.00662	- 57	- 20	- 4	- 1	- 1	2.7	3.3	3
4	.00601	.00642	- 34	- 16	- 41	1	.3	2.8	3.4	4
5	.00567	.00626	- 17	- 21	- 59	3.3	3.6	2.7	3.1	5
6	.00550	.00605	1	- 22	- 55	1	.4	- .6	- .5	6
7	.00551	.00583	12	- 20	- 32	- 2.6	- 2.4	- .7	- .9	7
8	.00563	.00563	11	- 22	...	2.1	2.1	1.9	1.5	8
9	.00574	.00541	2	- 19	33	- 1.1	- 1.3	- .2	- .6	9
30	.00576	.00522	- 11	- 11	54	- 3.3	- 3.7	- .9	- .7	30
1	.00565	.00511	- 25	- 3	54	3.3	2.9	4.2	4.4	1
2	.00540	.00508	- 30	4	32	7	.4	.9	1.5	2
3	.00510	.00512	- 24	16	- 2	- 1.5	- 1.5	.2	1.1	3
4	.00486	.00528	- 6	19	- 42	- 8	- 5	1.7	2.6	4
5	.00480	.00547	18	22	- 67	4	1.1	2.5	3.1	5
6	.00498	.00569	43	28	- 71	- 1.6	- 1.0	2.1	2.0	6
7	.00541	.00597	60	31	- 56	3.8	4.3	3.7	3.0	7
8	.00601	.00628	68	32	- 27	2.5	2.8	- .1	- 1.3	8
9	.00669	.00660	66	34	9	7	.8	- 2.6	- 4.1	9
40	.00735	.00694	53	29	41	- 2.8	- 3.3	- 1.9	- 3.3	40
1	.00788	.00723	32	22	65	- 1.1	- 1.7	.9	...	1
2	.00820	.00745	7	19	75	7	- 1.3	2.0	1.7	2
3	.00827	.00764	- 15	11	63	- 1.1	- 1.6	2.7	3.0	3
4	.00812	.00775	- 33	2	37	3.2	2.9	3.8	4.6	4
5	.00779	.00777	- 34	2	2	- 1.2	- 1.2	.6	1.7	5
6	.00745	.00775	- 28	7	- 30	9	1.2	1.8	2.9	6
7	.00717	.00768	- 12	- 11	- 51	- 6	- 2	.9	1.7	7
8	.00705	.00757	8	- 2	- 52	- 6	- 2	1.5	1.9	8
9	.00713	.00755	24	10	- 42	4	7	2.1	2.1	9
50	.00737	.00765	38	29	- 28	1.2	1.4	1.7	1.4	50
1	.00775	.00794	55	53	- 19	25	...	1
2	.00830	.00847	73	71	- 17	3	2	7	...	2
3	.00903	.00918	88	83	- 15	9	9	1.0	.2	3
4	.00991	.01001	110	94	- 10	- 2.7	- 2.6	.1	- .7	4
5	.01101	.01095	121	96	6	9	9	2.8	1.9	5
6	.01222	.01191	124	94	31	1.1	1.0	1.9	1.0	6
7	.01346	.01285	118	97	61	5	2	.8	...	7
8	.01464	.01382	105	93	82	33	- .2	8
9	.01569	.01475	80	85	94	- 1.0	- 1.3	...	- .2	9
60	.01649	.01560	62	88	89	- 2.6	- 2.9	1.0	1.1	60
1	.01711	.01648	28	91	63	1.7	1.6	3.6	4.0	1
2	.01739	.01739	12	94	...	5	5	1.9	2.4	2
3	.01751	.01833	20	112	- 82	- 5	- 4	1.4	1.9	3
4	.01771	.01945	74	112	- 174	- 1	1	1.9	2.3	4
65	.01845	.02057	- 212	2.0	2.2	2.0	2.2	65

APPENDIX II.

TABLE I.
Female Rates of Mortality. Full Aggregate Tables.

<i>a</i>	H.F. q_x	O.F. q_x	O.M. q_x	O.P. q_x	O.E.F. q_x
(1)	(2)	(3)	(4)	(5)	(6)
20	.00688	.00560	.00857	.00622	.00450
21	.00729	.00577	.00858	.00680	.00478
22	.00792	.00604	.00853	.00735	.00498
23	.00882	.00639	.00817	.00802	.00511
24	.00984	.00680	.00843	.00872	.00518
25	.01051	.00721	.00845	.00927	.00520
26	.01138	.00757	.00849	.00962	.00520
27	.01182	.00787	.00856	.00981	.00519
28	.01201	.00810	.00863	.00994	.00517
29	.01195	.00826	.00876	.01005	.00517
30	.01185	.00843	.00893	.01020	.00520
31	.01171	.00864	.00910	.01045	.00528
32	.01157	.00887	.00922	.01050	.00539
33	.01166	.00911	.00931	.01122	.00558
34	.01172	.00935	.00935	.01179	.00551
35	.01176	.00957	.00941	.01217	.00603
36	.01190	.00978	.00949	.01259	.00626
37	.01204	.00998	.00966	.01294	.00646
38	.01219	.01017	.00994	.01319	.00667
39	.01234	.01036	.01031	.01328	.00695
40	.01255	.01057	.01075	.01321	.00729
41	.01275	.01077	.01125	.01300	.00767
42	.01294	.01094	.01174	.01271	.00801
43	.01315	.01113	.01221	.01241	.00825
44	.01338	.01131	.01260	.01214	.00839
45	.01362	.01151	.01291	.01187	.00844
46	.01376	.01175	.01318	.01165	.00840
47	.01388	.01205	.01345	.01152	.00843
48	.01401	.01244	.01375	.01154	.00863
49	.01416	.01292	.01414	.01177	.00902
50	.01445	.01348	.01461	.01227	.00956
51	.01496	.01413	.01516	.01297	.01016
52	.01562	.01488	.01578	.01382	.01061
53	.01641	.01572	.01650	.01479	.01111
54	.01739	.01665	.01730	.01580	.01183
55	.01827	.01767	.01822	.01655	.01300
56	.01907	.01875	.01919	.01799	.01430
57	.01989	.01991	.02022	.01923	.01580
58	.02074	.02116	.02133	.02066	.01740
59	.02180	.02254	.02258	.02234	.01919

TABLE I—(continued).

Female Rates of Mortality. Full Aggregate Tables.

<i>x</i>	H ^F <i>tx</i>	O ^F <i>tx</i>	O ^{NF} <i>tx</i>	O ^{IF} <i>tx</i>
(1)	(2)	(3)	(4)	(5)
60	.02368	.02411	.02399	.02423
61	.02587	.02592	.02562	.02619
62	.02816	.02799	.02745	.02810
63	.03137	.03033	.02955	.02980
64	.03451	.03297	.03196	.03130
65	.03738	.03592	.03472	.03282
66	.04018	.03020	.03783	.03181
67	.04340	.04279	.04135	.03761
68	.04698	.04667	.04540	.04136
69	.05099	.05090	.05012	.04611
70	.05564	.05552	.05573	.05143
71	.06195	.06056	.06231	.05685
72	.06917	.06610	.06955	.06213
73	.07708	.07225	.07852	.06700
74	.08576	.07919	.08766	.07211
75	.09485	.08704	.09695	...
76	.10405	.09572	.10625	...
77	.10976	.10526	.11530	...
78	.11505	.11565	.12426	...
79	.11934	.12690	.13326	...
80	.12179	.13890
81	.12358	.15148
82	.13170	.16447
83	.14526	.17768
84	.16280	.19095
85	.18564	.20410
86	.21214	.21730
87	.23696	.23125
88	.25822	.24611
89	.27255	.26167
90	.27504	.27666
91	.26901	.28858
92	.23200	.29782
93	.17969	.30870
94	.18571	.32938
95	.20858	.36972
96	.25616	.43892
97	.33775	.54336
98	.50000	.70048
99	1.00000	1.00000

TABLE 2.—(F).

Summary of data, and Comparison between Expected and Actual Deaths.

Age Group	Exposed to risk	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation	Expected Deviation \pm	$10^5 \times \Delta^3 q_x$ irrespective of sign	Age Group
0- 9	250	0- 9
10-19	4,750	...	21	10-19
20- 4	8,951	55·7	61	- 5·3	5·6	5·95	16	20- 4
25- 9	17,952	141·0	147	- 6·0	10·9	9·46	14	25- 9
30- 4	29,650	264·5	252	12·5	16·9	12·95	6	30- 4
35- 9	42,354	423·3	432	- 8·7	4·4	16·38	8	35- 9
40- 4	53,348	584·5	579	5·5	13·1	19·23	15	40- 4
45- 9	61,019	741·0	743	- 2·0	7·6	21·65	6	45- 9
50- 4	64,062	959·3	963	- 3·7	9·6	24·59	6	50- 4
55- 9	61,870	1,235·7	1,232	3·7	13·3	27·84	18	55- 9
60- 4	55,515	1,560·5	1,576	- 15·5	9·6	31·16	9	60- 4
65- 9	44,865	1,914·7	1,923	- 8·3	25·1	34·25	23	65- 9
70- 4	31,382	2,061·7	2,064	- 2·3	33·4	35·10	52	70- 4
75- 9	18,763	1,945·4	1,895	50·4	35·7	33·38	47	75- 9
80- 4	8,820	1,409·0	1,432	- 23·0	- 14·7	27·43	140	80- 4
85- 9	2,813	632·3	632	- 3	8·3	17·64	...	85- 9
90- 4	609	178·0	166	12·0	8·0	8·94	...	90- 4
95- 9	69	31·0	35	- 4·0	- 4·0	3·01	...	95- 9
..	502,042	14,137·6	14,132	5·6
Total	507,042	...	14,153	Total

TABLE 3.—O^{NF}.

Summary of data, and Comparison between Expected and Actual Deaths.

Age Group	Exposed to Risk	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation	Expected Deviation \pm	$10^5 \times \Delta \ddot{q}_{x_e}$ irrespective of sign	Age Group
0- 9	154	0- 9
10- 19	781	...	7	10- 19
20- 4	1,725	14·6	11	3·6	5·2	3·04	17	20- 4
25- 9	3,426	29·4	30	- .6	1·6	4·32	20	25- 9
30- 4	5,271	48·5	51	+ 2·5	2·2	5·55	18	30- 4
35- 9	7,676	75·2	71	- 4·2	4·7	6·90	14	35- 9
40- 4	9,994	117·5	115	- 2·5	5·5	8·62	15	40- 4
45- 9	11,713	158·3	159	+ .7	+ 2·0	10·0	11	45- 9
50- 4	13,275	211·0	209	- 2·0	- 1·3	11·53	17	50- 4
55- 9	14,044	285·2	300	+ 14·8	+ 3·3	13·37	18	55- 9
60- 4	13,568	375·2	380	+ 4·8	11·5	15·28	21	60- 4
65- 9	11,925	496·2	502	+ 5·8	16·3	17·45	65	65- 9
70- 4	9,096	634·5	606	- 28·5	22·1	19·43	131	70- 4
75- 9	5,674	640·6	647	+ 6·4	+ 6·4	19·04	90	75- 9
80-100	107,387 3,788	3,086·2 ...	3,081 669	5·2	80-100
Total	112,010	...	3,757	Total

TABLE 4.—O^{1F}.

Summary of data, and Comparison between Expected and Actual Deaths.

Age Group	Exposed to Risk	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation	Expected Deviation \pm	$10^5 \times \Delta^3 q_x$ irrespective of sign	Age Group
-19	320	-19
20- 4	1,425	11·3	9	2·3	5·4	2·68	51	20- 4
25- 9	4,915	48·1	50	- 1·9	3·1	5·52	26	25- 9
30- 4	8,492	92·8	92	- 8	5·0	7·66	17	30- 4
35- 9	10,603	136·2	135	- 1·2	4·2	9·27	17	35- 9
40- 4	11,644	147·7	150	+ 2·3	3·0	9·66	23	40- 4
45- 9	11,579	135·2	135	- 2	5·3	9·25	30	45- 9
50- 4	10,225	141·8	144	+ 2·2	5·1	9·46	17	50- 4
55- 9	7,888	151·7	155	+ 3·3	7·3	9·76	45	55- 9
60- 4	5,291	146·0	142	- 4·0	10·6	9·53	118	60- 4
65- 9	2,863	107·5	113	+ 5·5	6·6	8·14	133	65- 9
70- 4	1,266	76·1	64	- 12·1	12·1	6·76	...	70- 4
Total	76,191	1194·4	1,189	5·4	Total
75-96	567	...	66	75-96
Total	77,078	...	1,255	Total

TABLE 5.—O^{EF}.

Summary of data, and Comparison between Expected and Actual Deaths.

Age Group	Exposed to Risk	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation	Expected Deviation \pm	$10^5 \times \Delta^3 q_x$ irrespective of sign	Age Group
15- 9	826	...	4	15- 9
20- 4	2,758	13·6	13	.6	3·7	2·94	7	20- 4
25- 9	5,576	28·9	26	2·9	3·1	4·29	8	25- 9
30- 4	7,479	40·8	42	-1·2	.2	5·10	20	30- 4
35- 9	7,858	50·8	48	2·8	1·4	5·68	21	35- 9
40- 4	7,128	56·4	55	1·4	-1·4	5·98	23	40- 4
45- 9	5,515	47·3	54	-6·7	-2·8	5·48	46	45- 9
50- 4	3,278	34·5	25	9·5	3·9	4·67	99	50- 4
55- 9	1,631	25·4	31	-5·6	-5·6	4·00	...	55- 9
Total $\int (0-14) \&$ $(60 \text{ and } \text{upwards})$	41,223	297·7	294	3·7	Total $\int (0-14) \&$ $(60 \text{ and } \text{upwards})$
Total	42,646	...	304	Total

TABLE 6.

*British Offices' Experience—Females. O^[F].**Whole-Life Participating Assurances. Rates of Mortality in Years of Assurance.*

x	$q[x]$	$q[x+1]$	$q[x+2]$	$q[x+3]$	$q[x+4]$	$x + 4$		
20	.00711	.00773	.00841	.00707	.00571	24		
21	.00697	.00839	.00926	.00767	.00606	25		
22	.00683	.00896	.00989	.00811	.00638	26		
23	.00662	.00945	.01032	.00843	.00669	27		
24	.00642	.00982	.01060	.00867	.00703	28		
25	.00626	.01009	.01077	.00888	.00742	29		
26	.00605	.01027	.01081	.00909	.00795	30		
27	.00583	.01031	.01073	.00933	.00857	31		
28	.00563	.01025	.01058	.00957	.00919	32		
29	.00541	.01015	.01041	.00975	.00975	33		
30	.00522	.01001	.01019	.00991	.01025	34		
31	.00511	.00983	.01000	.01004	.01066	35		
32	.00508	.00967	.00984	.01015	.01097	36		
33	.00512	.00950	.00976	.01028	.01120	37		
34	.00528	.00934	.00975	.01042	.01136	38		
35	.00547	.00919	.00978	.01055	.01147	39		
36	.00569	.00904	.00981	.01068	.01156	40		
37	.00597	.00890	.00984	.01079	.01162	41	70	.05542
38	.00628	.00880	.00986	.01088	.01168	42	71	.06046
39	.00660	.00872	.00988	.01096	.01175	43	72	.06596
							73	.07197
40	.00694	.00868	.00991	.01107	.01187	44	74	.07850
41	.00723	.00872	.01002	.01121	.01202	45		
42	.00745	.00883	.01022	.01145	.01227	46	75	.08561
43	.00764	.00899	.01052	.01178	.01262	47	76	.09335
44	.00775	.00918	.01087	.01220	.01306	48	77	.10184
							78	.11130
45	.00777	.00933	.01127	.01268	.01359	49	79	.12237
46	.00775	.00954	.01169	.01323	.01419	50		
47	.00768	.00973	.01209	.01379	.01485	51	80	.13492
48	.00757	.00994	.01249	.01437	.01560	52	81	.14876
49	.00755	.01026	.01295	.01501	.01642	53	82	.16306
							83	.17727
50	.00765	.01072	.01351	.01573	.01731	54	84	.19095
51	.00794	.01135	.01423	.01657	.01827	55		
52	.00847	.01210	.01510	.01752	.01931	56	85	.20410
53	.00918	.01291	.01604	.01855	.02041	57	86	.21730
54	.01001	.01373	.01701	.01965	.02163	58	87	.23125
							88	.24641
55	.01095	.01454	.01799	.02083	.02300	59	89	.26167
56	.01191	.01524	.01884	.02206	.02456	60		
57	.01285	.01586	.01964	.02338	.02636	61	90	.27666
58	.01382	.01653	.02057	.02491	.02841	62	91	.28858
59	.01475	.01734	.02171	.02667	.03071	63	92	.29782
							93	.30870
60	.01560	.01837	.02317	.02874	.03329	64	94	.32938
61	.01648	.01970	.02504	.03116	.03615	65		
62	.01739	.02131	.02725	.03389	.03931	66	95	.36972
63	.01833	.02327	.02981	.03694	.04281	67	96	.43892
64	.01945	.02558	.03275	.04034	.04663	68	97	.54336
							98	.70048
65	.02057	.02831	.03585	.04391	.05082	69	99	1.00000

*Ultimate Table—
(continued).*

TABLE 7.

*O^[F].—Ultimate Table, excluding first 4 years of Assurance.**Summary of data, and Comparison between Expected and Actual Deaths*

Age Group	Exposed to Risk	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation	Expected Deviation \pm	$10^5 \times \Delta^3 q_x$ irrespective of sign	Age Group
20-4	4,115	20·9	23	- 2·1	3·3	3·65	31	20-4
25-9	8,933	60·9	63	- 2·1	5·4	6·22	29	25-9
30-4	17,331	160·8	149	11·8	7·5	10·10	12	30-4
35-9	29,021	324·1	343	- 18·9	- 4·3	14·32	10	35-9
40-4	40,990	479·8	466	13·8	14·6	17·42	14	40-4
45-9	51,381	654·5	656	- 1·5	·8	20·34	7	45-9
50-4	58,123	912·5	908	4·5	2·3	23·98	5	50-4
55-9	59,940	1,229·7	1,216	13·7	- 2·2	27·77	19	55-9
60-4	56,955	1,627·5	1,625	2·5	- 15·9	31·81	9	60-4
65-9	48,598	2,078·8	2,101	- 22·2	- 18·4	35·69	16	65-9
70-4	34,951	2,290·0	2,274	16·0	3·8	37·00	29	70-4
75-9	21,390	2,154·4	2,149	5·4	- 12·2	35·19	201	75-9
80-4	10,248	1,617·3	1,648	- 30·7	- 17·6	29·43	227	80-4
85-9	3,358	754·9	754	·9	13·1	19·27	513	85-9
90-4	698	204·0	190	14·0	12·2	9·57	3,792	90-4
95-9	85	39·2	41	- 1·8	- 1·8	3·38	...	95-9
Totals	446,117	14,609·3	14,606	3·3	Totals

TABLE 8.
O[F]. Summary of data, and Comparison between Expected and Actual Deaths.

Years of Assurance										3						2						1						0									
Grouped Ages at Entry		Exposed to Risk		Expected Deaths		Actual Deaths		Exposed to Risk		Expected Deaths		Actual Deaths		Exposed to Risk		Expected Deaths		Actual Deaths		Exposed to Risk		Expected Deaths		Actual Deaths		Exposed to Risk		Expected Deaths		Actual Deaths		Exposed to Risk		Expected Deaths		Actual Deaths	
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)		
20-24	1,069	132	17	-3.8	1,665	15.1	13	-2.1	1,497	14.8	17	-2.2	1,364	11.2	11	-2	-3.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
25-29	3,181	184	16	-2.4	2,794	28.5	34	-5.5	2,518	26.8	29	-2.2	2,358	22.1	22	-1	-5.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
30-34	4,183	216	21	-2.4	3,764	36.0	42	-6.0	3,453	34.3	36	-1.7	3,225	32.9	29	-3.9	-6.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
35-39	4,547	274	21	-6.4	4,152	37.1	39	-1.9	3,854	38.0	36	-2.0	3,631	39.1	36	-3.1	9.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
40-44	4,174	339	38	-5.0	4,107	36.4	10	-3.6	3,853	39.6	38	-1.6	3,635	42.0	50	-8.0	-15.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
45-49	3,945	303	33	-3.3	3,655	35.5	35	-5.5	3,408	41.1	36	-5.1	3,260	45.0	44	-1.0	6.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
50-54	2,974	257	26	-7.5	2,750	33.0	19	-14.0	2,582	38.9	52	-13.1	2,417	42.8	38	-4.8	-7.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
55-59	1,054	248	24	-8	1,846	29.0	37	-8.0	1,717	34.2	29	-5.2	1,682	39.2	32	-7.2	5.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
60-64	989	169	18	-1.1	938	19.1	12	-7.4	907	21.0	29	-5.0	859	28.3	34	-5.7	-4.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
65	109	22	...	2.2	108	3.1	6	-2.9	96	3.4	2	-1.4	88	3.9	3	-9	1.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
Total	28,322	2133	214	-7	25,779	2731	277	-3.9	23,915	29.51	304	-8.9	22,549	30.65	299	-7.5	-6.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE 9.

Mortality Table for Teachers, 1899-1906. Females. (See page 120).

x	q_x	x	q_x	x	q_x	x	q_x
20	.00270	35	.00325	50	.00758	65	.02480
21	.00289	36	.00326	51	.00826	66	.02699
22	.00300	37	.00327	52	.00903	67	.02915
23	.00308	38	.00330	53	.00982	68	.03224
24	.00312	39	.00335	54	.01065	69	.03540
25	.00315	40	.00343	55	.01152	70	.03898
26	.00316	41	.00357	56	.01245	71	.04303
27	.00317	42	.00377	57	.01342	72	.04763
28	.00318	43	.00403	58	.01446	73	.05303
29	.00319	44	.00436	59	.01558	74	.05905
30	.00320	45	.00476	60	.01678	75	.06577
31	.00321	46	.00522	61	.01809	76	.07314
32	.00322	47	.00574	62	.01952	77	.08109
33	.00323	48	.00630	63	.02109	78	.08955
34	.00324	49	.00692	64	.02284	79	.09847

From age 80 onwards the rate of mortality is that of the British Offices' Ultimate Annuity Tables for Females.

TABLE 10.

*Whole-Life Annuity-Values.**Full aggregate Table.*O^F.*Interest 3 per cent.*

x	a_x	x	a_x	x	a_x	x	a_x
20	22.232	40	17.760	60	10.916	80	3.868
21	22.028	41	17.488	61	10.522	81	3.627
22	21.821	42	17.209	62	10.126	82	3.403
23	21.612	43	16.921	63	9.730	83	3.195
24	21.403	44	16.625	64	9.335	84	3.002
25	21.196	45	16.319	65	8.943	85	2.821
26	20.990	46	16.004	66	8.555	86	2.651
27	20.785	47	15.681	67	8.171	87	2.489
28	20.578	48	15.318	68	7.792	88	2.334
29	20.369	49	15.008	69	7.419	89	2.190
30	20.155	50	14.660	70	7.051	90	2.056
31	19.936	51	14.306	71	6.690	91	1.927
32	19.713	52	13.947	72	6.335	92	1.790
33	19.486	53	13.582	73	5.987	93	1.572
34	19.255	54	13.213	74	5.646	94	1.391
35	19.020	55	12.840	75	5.316	95	1.136
36	18.780	56	12.463	76	4.998	96	0.857
37	18.534	57	12.082	77	4.692	97	0.573
38	18.283	58	11.697	78	4.402	98	0.293
39	18.025	59	11.309	79	4.127	99	0.000

TABLE 11.
British Offices' Experience—Females.
 $O^{[F]}$.

Whole-Life Participating Assurances.

Select Tables.

Interest 3 per-cent.

x	$a_{[x]}$	$P_{[x]}$	Ultimate a_x	x	$a_{[x]}$	$P_{[x]}$	Ultimate a_x
20	22.067	.01423	..	45	16.365	.02847	16.224
21	21.827	.01469	..	46	16.058	.02951	15.914
22	21.597	.01513	..	47	15.750	.03058	15.596
23	21.376	.01557	..	48	15.436	.03172	15.269
24	21.159	.01600	21.371	49	15.116	.03293	14.935
25	20.947	.01644	21.139	50	14.785	.03423	14.595
26	20.738	.01687	20.906	51	14.441	.03562	14.249
27	20.536	.01731	20.671	52	14.092	.03713	13.898
28	20.341	.01773	20.435	53	13.734	.03874	13.542
29	20.151	.01816	20.196	54	13.371	.04046	13.181
30	19.965	.01857	19.958	55	13.006	.04227	12.816
31	19.778	.01900	19.721	56	12.641	.04419	12.445
32	19.587	.01945	19.489	57	12.278	.04619	12.071
33	19.391	.01992	19.260	58	11.912	.04832	11.692
34	19.186	.02041	19.033	59	11.545	.05059	11.310
35	18.975	.02094	18.806	60	11.173	.05302	10.924
36	18.757	.02149	18.580	61	10.798	.05563	10.534
37	18.530	.02208	18.349	62	10.421	.05843	10.144
38	18.293	.02270	18.113	63	10.042	.06143	9.754
39	18.048	.02338	17.871	64	9.664	.06464	9.365
40	17.792	.02408	17.621	65	9.289	.06807	8.978
41	17.526	.02485	17.362	66	8.594
42	17.248	.02568	17.093	67	8.214
43	16.960	.02655	16.814	68	7.839
44	16.665	.02749	16.524	69	7.469

TABLE 12.
British Offices' Experience—Females.
 $O^{[F]}$.

Whole-Life Participating Assurances.

Select Tables.

Interest 3 per-cent.

x	$a_{[x]}$	$a_{[x]+1}$	$a_{[x]+2}$	$a_{[x]+3}$	$a_{[x]+4}$
20	22.067	21.862	21.724	21.566	20
25	20.947	20.712	20.550	20.396	25
30	19.965	19.671	19.466	19.257	30
35	18.975	18.652	18.389	18.128	35
40	17.792	17.453	17.135	16.825	40
45	16.365	15.988	15.624	15.276	45
50	14.785	14.316	13.937	13.551	50
55	13.006	12.544	12.111	11.703	55
60	11.173	10.691	10.218	9.774	60
65	9.289	8.769	8.294	7.861	65

TABLE 13.

<i>Select Tables.</i>	$D_{[x]}$	$D_{[x]+1}$	$D_{[x]+2}$	$D_{[x]+3}$	$D_{[x]+4}$	$x + 1$
20	55,368	53,373	51,417	49,499	47,719	24
21	53,554	51,631	49,706	47,812	46,064	25
22	51,758	49,907	48,029	46,159	44,151	26
23	49,980	48,204	46,359	44,543	42,882	27
24	48,235	46,529	44,729	42,967	41,353	28
25	46,522	44,884	43,138	41,430	39,867	29
26	44,844	43,274	41,582	39,935	38,418	30
27	43,190	41,687	40,055	38,172	37,003	31
28	41,565	40,127	38,559	37,041	35,617	32
29	39,971	38,597	37,093	35,638	34,262	33
30	38,413	37,101	35,659	34,267	32,939	34
31	36,899	35,641	34,263	32,932	31,653	35
32	35,434	34,227	32,959	31,637	30,403	36
33	34,023	32,862	31,602	30,382	29,194	37
34	32,666	31,547	30,342	29,171	28,027	38
35	31,361	30,281	29,129	28,004	26,901	39
36	30,104	29,061	27,960	26,879	25,818	40
37	28,897	27,889	26,836	25,798	24,776	41
38	27,740	26,762	25,754	24,757	23,775	42
39	26,626	25,680	24,714	23,758	22,813	43
40	25,558	24,612	23,716	22,797	21,888	44
41	24,533	23,647	22,758	21,871	20,999	45
42	23,551	22,694	21,839	20,987	20,142	46
43	22,607	21,781	20,957	20,132	19,315	47
44	21,695	20,900	20,105	19,307	18,516	48
45	20,810	20,047	19,281	18,508	17,742	49
46	19,953	19,221	18,483	17,735	16,991	50
47	19,117	18,418	17,708	16,984	16,262	51
48	18,305	17,638	16,953	16,254	15,551	52
49	17,519	16,881	16,221	15,544	14,865	53
50	16,761	16,148	15,510	14,855	14,195	54
51	16,031	15,441	14,821	14,185	13,543	55
52	15,328	14,756	14,153	13,533	12,909	56
53	14,616	14,089	13,502	12,899	12,291	57
54	13,982	13,438	12,868	12,281	11,689	58
55	13,333	12,804	12,250	11,679	11,103	59
56	12,696	12,179	11,644	11,092	10,531	60
57	12,068	11,566	11,051	10,519	9,973·6	61
58	11,156	10,969	10,473	9,958·9	9,427·8	62
59	10,557	10,386	9,908·3	9,410·9	8,893·2	63
60	10,274	9,819·1	9,357·9	8,875·0	8,368·9	64
61	9,707·3	9,279·2	8,822·1	8,350·6	7,851·7	65
62	9,153·6	8,732·5	8,297·6	7,836·3	7,350·2	66
63	8,612·9	8,208·8	7,754·1	7,332·1	6,855·7	67
64	8,085·0	7,696·8	7,281·5	6,838·0	6,371·0	68
65	7,565·5	7,191·0	6,786·9	6,352·9	5,897·0	69

TABLE 14.

Select Tables. O^F .*Interest 3 per cent.*

x	$\bar{I}^x_{\{x\}}$	$\bar{N}_x^{\text{ultimate}}$	x	$\bar{I}^x_{\{x\}}$	$\bar{N}_x^{\text{ultimate}}$
20	1,277,166	...	60	125,070·4	125,570
21	1,222,493	...	61	114,524·7	115,039
22	1,169,570	...	62	104,540·8	105,065·4
23	1,118,361	...	63	95,108·5	95,637·6
24	1,068,853	1,067,509	64	86,216·2	86,744·4
25	1,021,014	1,019,790	65	77,843·2	78,375·5
26	974,808	973,726	66	...	70,520·8
27	930,159	929,275	67	...	63,170·6
28	887,044	886,393	68	...	56,314·9
29	845,434	845,040	69	...	49,943·9
30	805,313	805,173	70	...	44,046·9
31	766,669	766,755	71	...	38,612·6
32	729,488	729,752	72	...	33,628·9
33	693,747	694,135	73	...	29,082·9
34	659,410	659,873	74	...	24,960·4
35	626,432	626,934	75	...	21,246·0
36	594,760	595,281	76	...	17,922·9
37	564,358	564,878	77	...	14,972·8
38	535,175	535,684	78	...	12,376·0
39	507,165	507,657	79	...	10,111·6
40	480,287	480,756	80	...	8,157·7
41	454,498	454,938	81	...	6,492·9
42	429,758	430,162	82	...	5,094·7
43	406,022	406,387	83	...	3,939·1
44	383,237	383,574	84	...	3,000·21
45	361,360	361,686	85	...	2,250·21
46	340,364	340,687	86	...	1,661·08
47	320,208	320,545	87	...	1,205·85
48	300,869	301,230	88	...	859·92
49	282,330	282,714	89	...	601·73
50	264,574	264,972	90	...	412·82
51	247,583	247,981	91	...	277·41
52	231,332	231,719	92	...	182·32
53	215,789	216,165	93	...	116·63
54	200,931	201,300	94	...	71·85
55	186,739	187,105	95	...	41·80
56	173,181	173,562	96	...	22·23
57	160,243	160,653	97	...	10·26
58	147,921·9	148,362	98	...	3·73
59	136,199·8	136,673	99	...	·84

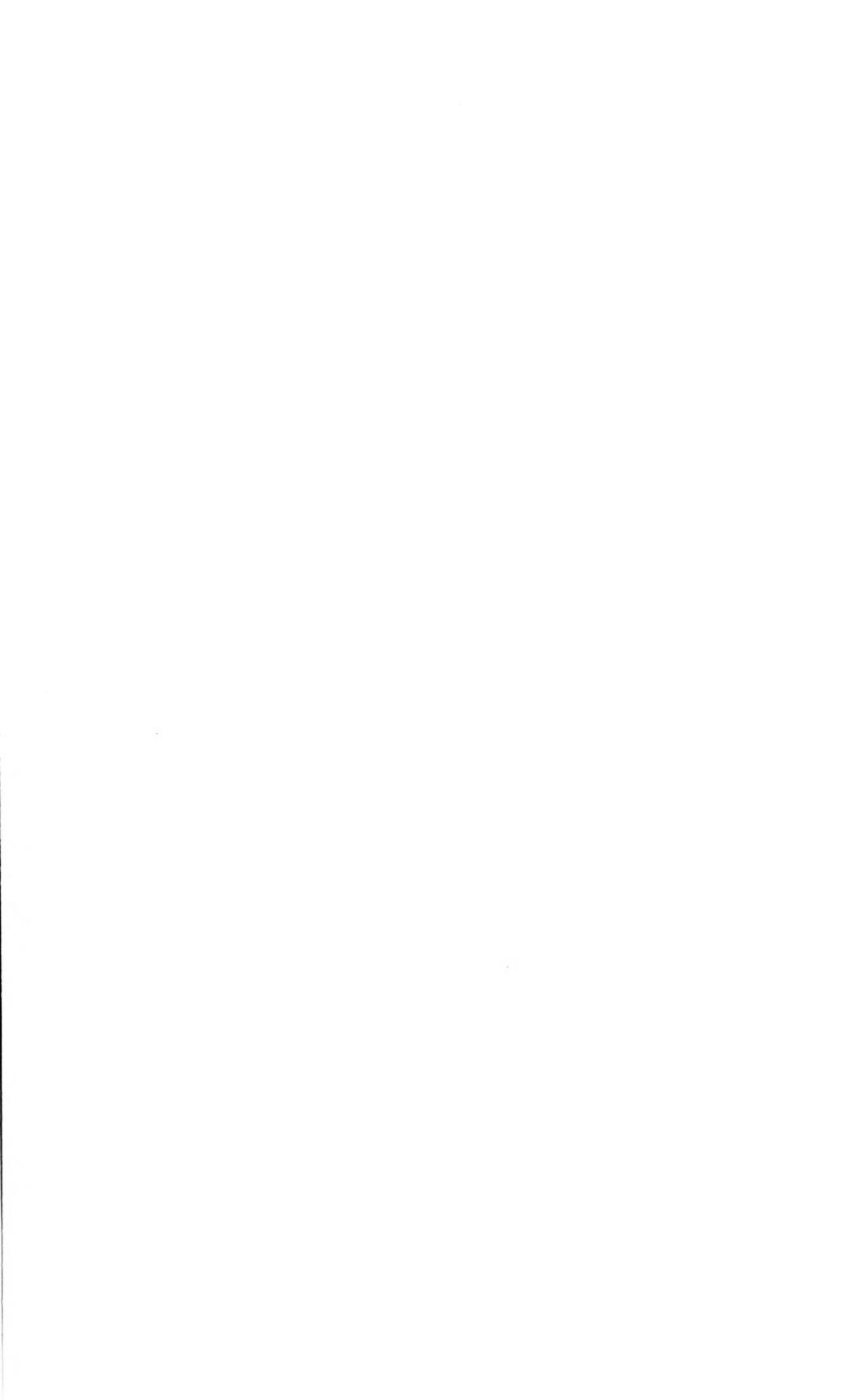


Diagram I.

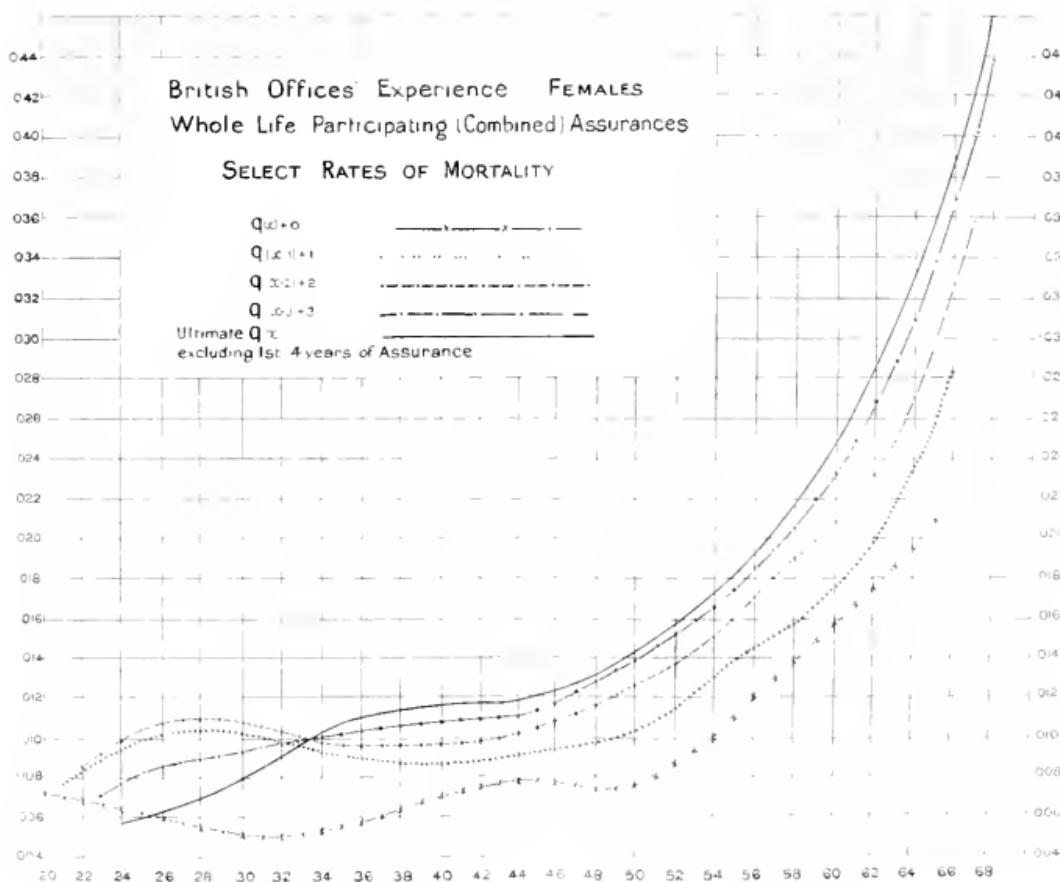
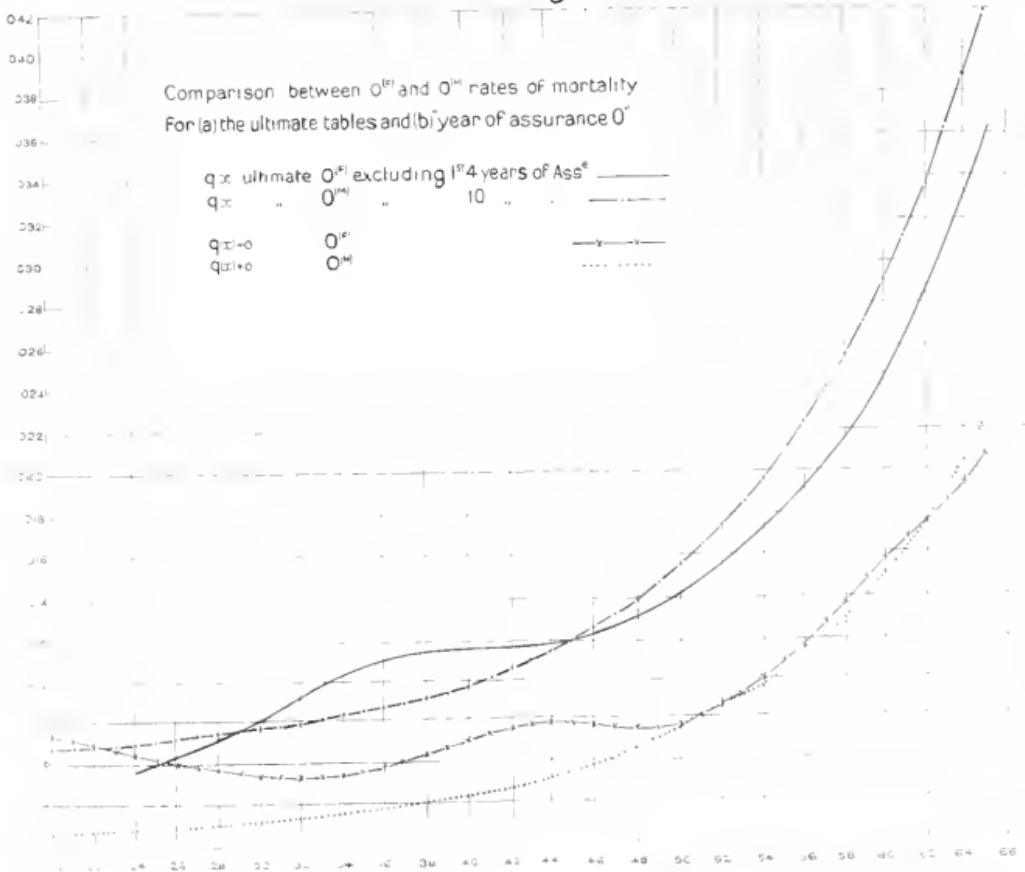




Diagram 2.



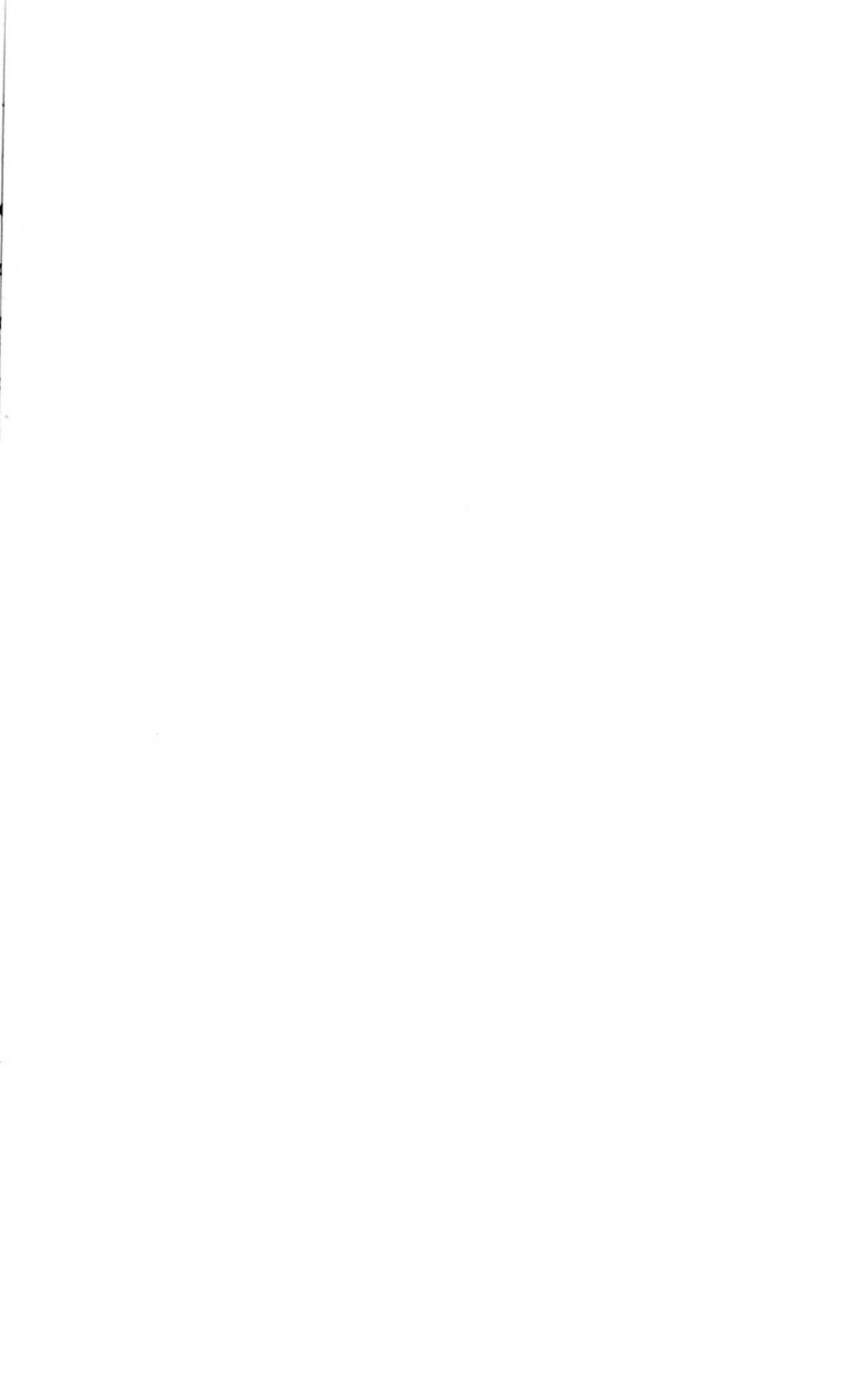
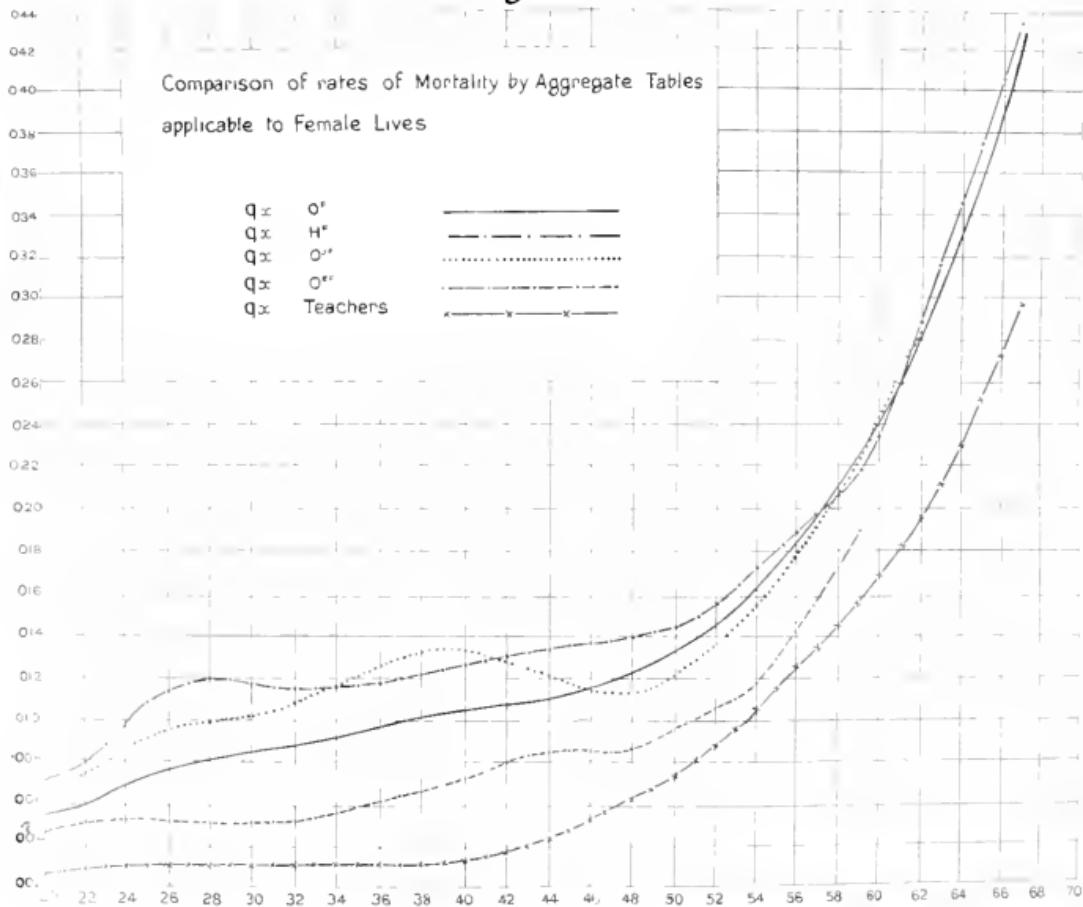


Diagram 3.



ABSTRACT OF THE DISCUSSION.

MR. H. E. W. LUTT said he was sure he might welcome most heartily, on behalf of all the members present, a new contributor to the Institute's proceedings, and congratulate him on the admirable way with which he had dealt with a subject of great interest to the profession. It was to be regretted that the information afforded in connection with female mortality by the Institute Experience of 1893 must be regarded as already growing old. Population statistics, which he thought might be legitimately used for comparative purposes, were certainly indicative of improved female mortality. Taking the figures for the three decennia 1851-1860, 1871-1880, and 1891-1900, and basing the figures on the age distribution revealed by the last census, the percentage of female mortality to male mortality decreased at ages 20-24 from 96·6 to 92·5 and 88·1; at 25-34, from 103·7 to 92·2 and 89·9; and at 35-44, from 97·4 to 84·3 and 83·4. Distinct evidence was there seen of the greater improvement of female mortality over that of males, although it was true that in the years 1881-90 there was a temporary check to that decreasing ratio. From the Census Report of 1901, the last returns available, it would appear that female mortality was only in excess of the male rates at ages 10-20, which was also shown in the case of the Selected Healthy Districts: whilst in the London Life Table 1891-1900, published by the London County Council in 1902, it was less from age 13 according to the graduated table.

In paragraphs 48 and 49 of the paper, Mr. Kenchington commented upon the causes of the extra female mortality, which he attributed principally to the risks of childbirth, and, secondly, to phthisis and cancer. Table K showed the progressive diminution in the deathrate in recent quinquennia from diseases, &c., attributable to child-birth. The age, however, had a very great effect upon the rate. Comparing the average rates for the three decennia preceding 1891, given by Mr. Baker in his paper read before the Berlin Congress of 1906, with the rates for 1891-1900 taken from the Registrar-General's Returns, the child-birth mortality per thousand females living at certain ages was as follows:

Ages	Average (1861-1890)	1891-1900
15-19	.153	.099
20-24	.640	.543
25-34	.922	.879
35-44	.862	.756
45-54	.052	.011

The improvement was manifest at every group of ages, but, as the author pointed out, changing conditions of life might account for the difference, and it must be remembered that the average marriage rate calculated on the unmarried and widowed population aged over 15, had fallen from 51·5 per thousand in 1880-1882 to 47·8 in 1903.

Moreover, the proportion of married women in the total female population over 20 was decreasing, while the birth-rate had almost continuously decreased, from 35·3 per thousand of the population in 1870-1872, to 28·6 in 1900-1902, and 28·4 in 1903; or, from 153·7 per thousand women aged 15-45 living in 1870-1872, to 114·8 in 1900-1902, and 113·8 in 1903. They must turn, therefore, to the mortality from child-birth per thousand births, which was given in Table L, and there the figures served to confirm the opinion that that particular danger to female life was becoming less.

There was no excess in female mortality from consumption after age 20, according to the last Census returns, but the risk of that disease was lessened by medical examination in the case of assured lives, and the general improvement might have lost some of its effect. Cancer was a continually increasing danger, but the rate of increase seemed to have been much less for females between the ages of 20 and 45 than for males, and there again medical information could be of assistance to assurance companies. On the whole he thought Mr. Kenchington was justified in his conclusion that the decrease in the mortality of female assured lives shown by the O^F Experience over the H^F was likely to be confirmed by a more modern investigation, while the comparative improvement of the general female mortality over that of males, disclosed in the Census Returns, might be similarly anticipated from the experience of assurance companies.

He had extracted a small experience of female assured lives from 1886 to the present date, but he was afraid it did not give much information in the earlier ages, where it was most required. Its growth had been fairly uniform, so that the question of too recent selection would hardly arise. As an indication of its nature, 42 per-cent of the business in 1897-8 consisted of endowment assurances; and in 1907-8 the proportion of endowment assurances was just over one-half of the whole. In 1897-8, 62 per-cent of the endowment assurances and 15 per-cent of the other policies were on the lives of unmarried women; in 1907-8 the percentages were 66 and 28. He took out the experience in two parts, from 1886-1895 and from 1896-1908, for the purpose of seeing if there was any difference, but unfortunately it was not large enough to give an indication for the younger lives. As a matter of fact, the percentages of actual to expected deaths by the O^M Table for the first experience were, up to age 35, 65·5 : 36-50, 75·9 : 51-65, 82·2 : 66 and upwards, 73 : whereas for the later experience, 1896-1908, they were 79·6, 61·2, 80·8 and 70·3 for the same respective age groups. Although very much reliance could not be placed upon those figures, so far as the earlier mortality of female assured life was concerned, he thought they went to show that, by the exercise of careful selection, it was possible to transact assurance on female lives on the same basis as for male lives.

It was unfortunate that the results of the O^F Experience did not discriminate between married and unmarried females. The fact that the O^F Table as a whole indicated an early extra mortality must mean,

if it was assumed as caused by the risks of child-birth, that in cases where the extra existed a much heavier extra charge should be imposed than was indicated by the figures given in the paper. If, however, the O^{EF} Experience was regarded as more likely to be that of spinsters, and the joint life (O^{JF}) experience as more likely to be that of married women, the conclusion seemed to be arrived at that there was very little extra risk for spinsters while they remained so; but, on the other hand, the extra for married women was somewhat heavier than that shown by the O^F Experience.

Dr. Galabin's statistics showed the largely increased risk from the birth of the first child compared with that of subsequent confinements, namely, in the ratio of 3 to 1. Those statistics, however, could not be applied, because there was nothing to guide actuaries as regards the actual risk, nor had any actual risk statistics been submitted with regard to the first pregnancy of female assured lives. It appeared that, though the number of companies tabulated by the author was four less than were included in 1889 by the Actuaries' Club, the number charging a single extra premium to unmarried women, or to married women who had borne no children, had increased. He thought that in that way lay the solution of the difficulty. Each case required careful scrutiny, and, as the circumstances pointed to the risk in question, it should be assessed either as an immediate charge or as an addition payable on marriage as the case might be. To consider the point, a comparison of the single premiums calculated upon Mr. Kenchington's O^F, and the O^M Tables was necessary. Where ordinary annual premiums were to be charged, the differences between the single premiums were not strictly comparable, but, on the same valuation basis, he thought it allowable to take the actual differences between the true single premiums as an indication of the assumed extra mortality to be met by a single additional payment. In that case the single extra premium for a whole-life policy worked out at about 20s. per-cent at age 20; at 26s. at age 25; and 18s. at age 30. He might here refer to a statement made at the Berlin Congress of 1906, that one of the German companies charged an extra of 3s. per-cent for females until they arrived at age 39; and after age 55, as a general rule, they charged them 6s. per-cent less. That was certainly a logical way of carrying out the conclusions.

As regards the graduation formula employed, the only point to which he wished to draw attention was the want of elasticity in all mathematical formulas. The smoothing power of a formula involving 27 terms of a series was doubtless great, but the regularity of the figures where the power of the formula was most apparent was apt to obscure the nature of the possibly scanty experience, and to lead to too much reliance being placed upon the manufactured results. Further, to complete the table, recourse must be had to other machinery, and the results require blending to secure a smooth connection. The graphic method, however, although possibly leading to different interpretations, did admit of a complete graduation, and if any preconceived ideas were derived from other

reliable experiences, and not from mere assumptions, it seemed to him that their introduction into the graduation was not an unmixed evil. The adjustment of the results derived from the graphic method to secure the desired smoothness of progression could generally be effected by means of a formula involving only a very few terms of the series. He should, however, add that Mr. Kenchington's formula had made the expected deaths to coincide in a remarkable manner with those actually experienced.

MR. W. PENMAX said that it seemed to follow from the various tables presented by the author that in a female mortality experience the rates actually experienced below age 45 would depend very largely upon the relative proportions of spinsters and married women included in the experience. As the tables contained nothing to indicate that any extra premium was necessary in the case of spinsters, the greater portion of the extra mortality being attributable to first pregnancies, it appeared that the most satisfactory method of dealing with female risks was to charge only a single extra premium when the circumstances of the case warranted it. The practice of meeting the extra risk by a rapidly diminishing debt was equally sound, and corresponded fairly closely with a single extra premium. The author's figures indicated that in some cases the customary charge of £1 per-cent might be rather low, for, if the extra premiums shown in tables O and P were commuted, the equivalent single extra premiums would in some cases exceed 40s. He did not agree that the difference between a single premium by the O^(F) Table and a single premium by the O^(M) was a correct measure of the single extra premium. He thought the correct measure was the annual extra premium multiplied by its appropriate annuity, and if that plan were adopted, heavy extras (compared with the customary charge of £1) were obtained, particularly in the endowment assurance section.

The trend of the curves of mortality brought out did not confirm the view that double endowments on female lives were particularly suitable from the point of view of the offices. The comparatively high mortality between ages 25 and 40 tended to increase the premiums, and the very rapidly increasing rate of mortality required to counterbalance this in the last third of the term was not shown in the tables. He had compared the premiums brought out by the O^(F) ultimate table with the corresponding premiums by the O^(M) ultimate table, and the actual figures were as shown on the next page. The annual extra premiums brought out, which never exceed 2s. per-cent, would not justify the imposition of an annual extra premium in practice, although they might justify a single extra premium in some cases, as for instance, when the age at entry was young and the term comparatively long. It should be borne in mind, however, that the premiums by the O^(M) Ultimate Table were themselves larger than the premiums by the O^(M) Select Table by a shilling or two, so that in comparing the O^(F) Ultimate (on account of the absence of medical examination), with the O^(M) Select Premiums, a substantial extra premium was obtained.

*Annual Premiums for Double Endowment Insurances of 100
Interest 3 per cent.*

Term	Age at entry	O ^(F)			EXCESS OF O ^(F) ULTIMATE		
		Ultimate	O ^(M) Ultimate	O ^(M) Select	Over (O ^(M)) Ultimate	Over (O ^(M)) Select	
15	20	10·471*	10·483	10·416	-·012	·055	
	30	10·485	10·465	10·389	+·020	·096	
	40	10·492	10·429	10·324	+·063	·168	
20	20	7·280*	7·306	7·242	-·026	·038	
	30	7·344	7·286	7·211	+·058	·133	
	40	7·321	7·253	7·151	+·068	·170	
25	20	5·429*	5·438	5·378	-·009	·051	
	30	5·494	5·415	5·344	+·079	·150	
	40	5·456	5·393	5·295	+·063	·161	
30	20	4·236*	4·221	4·163	+·015	·073	
	30	4·291	4·197	4·131	+·094	·160	
	40	4·243	4·205	4·108	+·038	·135	

* The O^(F) Table commences at age 20 (see tables 13 and 14) and consequently the premiums given above for age at entry 20 are really on a select basis.

On the thorny subject of graduation, he ventured to say that it did seem that the unadjusted rates at age 27, and age 53, could have very little actual effect on the rate which ought to be brought out at age 40. But, on the other hand, if the operator were graduating a mortality experience graphically, and he was dealing with the section between 30 and 40, he would undoubtedly be influenced by the trend of the curves from 20 to 30, and 30 to 50. It seemed to him that summation formulas, including a large number of terms, were in a way analogous to the graphic method. The terms at the extreme limit might, he thought, be regarded as feeling the shape of the curve.

MR. H. P. CALDERON said it seemed to him that the important question was, what was the practice of the companies whose business was included in the returns of the sixty offices from 1863 to 1893. No doubt in the circular issued by the combined companies they were told to include the cases taken at ordinary rates with the £1 or 5s. charged under a certain age; but, if they had been treated by way of rating-up in the ordinary way, very possibly those cases were not included, and therefore a proper and full mortality table had not been obtained.

With regard to the 27-term formula, such formulas always looked very beautiful and gave good results, but the actuary did not know where he was in the same exact manner as when he used the graphic method. He was very much obliged to Mr. Hardy for allowing him to make mention of a graduation he attempted on that very table some seven years ago, but which was not made use of at the time. The formula ran right

through the table : and, taken in fifteen-year groups, he obtained no departure greater than 5 between the expected and actual deaths. The formula adopted was a Makeham formula, with a frequency curve starting at about age 20, coming to a maximum at about age 42, and finishing at age 70. That seemed to him to represent the actual incidence of female mortality, seeing that the truncated table and the full table agreed in each case ; there was one deviation of 5 in each case, and nothing else above 2. He compared his own with the author's figures, and found that in the main part, the centre of the table, the premium values did not differ by more than a penny. Therefore, he thought that, valuable and interesting as Mr. Kenchington's formulas were for making summations, it was possible, by the application of a mathematical formula, providing care was taken in choosing it, to obtain something which would give a more definite law of mortality than the summation formulas. He had also found that selection did not go over more than three years, and he graduated the first three years : and when he applied to the years of assurance 4 to 9 the truncated 3 years rates, in groups of ages of assurance, the difference was 6 expected deaths out of 2,050, and the deviations of actual and expected deaths showed ten changes of sign.

MR. H. J. RIETSCHEL said that the female mortality curves followed their normal sequence for any age at entry above 33, but under that age that order was not followed, and there were certain anomalies in the select rates at young ages. To ascertain whether that was due to the graduation, he took out the total ungraduated exposed to risk and deaths for the groups of ages 20 to 24, 25 to 29, 30 to 34 and 35 to 39, and assumed that the rates of mortality obtained therefrom were those for the central age in each group. The general characteristics pointed out by Mr. Kenchington and shown in the diagram appeared in every case, but the fall in the mortality commenced with $q_{[x]-2}$, and became less pronounced as the age at entry increased, and after age 37 at entry the mortality took its normal course. It would be observed that the peculiarity was practically confined to the child-bearing period of life, from age 20 to about age 40. In order to compare the female rates of mortality with those of other tables, he examined the ungraduated rates of the O^{NM} experience for the before-mentioned groups of ages, and found the same fall in the rate of mortality, but when he examined the O^M experience the peculiarity had disappeared. He thought that pointed to the fact that there was a factor, common to both the O^{NM} and the female experiences, which accounted for the course of the rates of mortality in the early years of assurance.

The author stated in paragraph 15, that "assurances effected on female lives for financial purposes bear a larger proportion to the total assurances on lives of their own sex than is the case with male lives." He suggested that the word "participating" should be inserted before the last two words of the paragraph. The peculiar form of the O^{NM} and female curves was probably due to the

presence of a large proportion of assurances effected for financial purposes, which had a very unfavourable effect on the mortality for year of assurance 1, but which adverse selection was considerably accentuated amongst the female lives, owing to the risks attending child-birth. If this contention were correct, the mortality experience which the author had investigated did not give any indication of the mortality experience which might be expected by offices transacting ordinary thrift business. In any case, if the tables were to be employed in practice, he thought it was absolutely necessary that the waves should be smoothed away, and he was not at all convinced that the waves in the first year of assurance should be retained.

Turning to the monetary tables, Table O, which gave the extra premiums for whole-life assurances, justified the present-day tendency of charging no extras for female lives ; but for endowment assurances and for temporary assurance it seemed, according to the tables, to be necessary to charge an extra premium. If the female mortality was largely influenced by the risks attending female lives assured for financial purposes, then offices transacting ordinary thrift business would probably find that they could accept female lives on the same terms as males, even for endowment assurances and temporary assurances. He certainly thought, however, that those offices which were adopting abnormally low without-profit premiums, must very seriously consider the class of the risks they were taking. A heavy initial and rapidly diminishing debt on the policy was probably a scientific way of treating the extra mortality, and he believed in former times a good many offices used to adopt the method, but there were now few offices which used the plan. That was due, in his opinion, to the very large reduction necessary to balance even small extra premiums, which rendered the method unworkable in practice.

MR. E. A. RUSHER, in closing the discussion, said the author must be congratulated on producing, by an investigation of the system of which Mr. Spencer's 21-term formula was a type, a formula which had proved its value by the two-fold standard of smoothness and of close adherence to the original facts, and still further on having the courage to put the formula, not in its neatest and most elegant manner, but in an elongated form which, whilst it looked cumbersome on paper, gave a means of considerably shortening the actual process of calculation. New graduation formulas and methods would continue to be produced, so long as there were mortality tables requiring graduation, for the reason that no set formula could be applied to all tables alike, without regard to the peculiar incidences inherent in each. The author had kindly pointed out to him an illustration of that in the Austro-Hungarian Experience, published during the present year. Incidentally, he might state that there seemed to have been some alteration in the method of acceptance of female lives in the Austria-Hungarian offices, as the percentage of policies upon the total assurances was, for the old experience, no fewer than 27, but in the new it was only 11½. The

female life tables had been graduated by Makeham's formula, with the result that, whilst it was true that the totals of the actual and the expected deaths agreed closely, yet in the important group of ages from 20 to 45 the expected were considerably less than the actual in each quinquennial age group. On the other hand, during the age period 45 to 64, the difference was very much greater in the opposite direction. The result was, of course, to under-estimate the premiums at the ages at which assurances were usually effected. Though he had a great preference for the graphic method owing to its elasticity, yet he had no hesitation in speaking of the value of Mr. Kenchington's formula, after seeing it applied to mortality statistics, limited in quantity and with very marked natural peculiarities.

On the question of female lives, the author had dealt mainly with the child-bearing risk. The risk of general selection against a company on account of defective medical examination was, however, also a very real one, and had received much more attention, it seemed to him, on the other side of the Atlantic than here, perhaps because the conditions of life there had made it more likely that the gentle sex would assure in larger proportions. Medical examiners in America had given the subject special prominence, and from a perusal of the many interesting papers on the subject which could be found in their official publication, "The Medical Examiner," it appeared that they considered the chief element of risk to arise in the necessarily-defective medical selection, and though the child-bearing risk was considered of almost, if not quite, equal importance, it was discussed principally from the point of view of diseases arising therefrom, of a nature to elude superficial examination; so that the main object of the papers seemed to be how those diseases, or weaknesses which might ultimately tend to produce them, could be discovered before acceptance. It might be added that many of the suggestions were such as would be considered quite impracticable for medical examiners in this country, and he thought one must admit that the examination could never be so effective as that for males. In the English Life Tables, the female mortality was lighter than the male at practically all ages. If it could be shown that the proportion of married females was greater amongst assured lives than amongst the general population, it might lead one to suppose that child-bearing was the cause of the excess mortality in the former. From an enquiry into a very large number of assurances, however, whilst not speaking positively, he inclined to the belief that at the present time the class of women effecting assurances was largely made up of the unmarried. In fact, one had found in practice that, not infrequently, a policy was lapsed upon marriage.

In the discussion which followed the reading of Mr. Hardy's paper on the mortality experience of Assured Lives and Annuitants in France, Mr. Adlard dealt very ably with the question of Medical Selection. The proportion of female lives in the French Experience was 1 in 5, against 1 in 9 in the Twenty Offices, and 1 in 14 in the Sixty Offices. The French Experience

showed very light mortality. Mr. Adlard came to the conclusion that "The only possible solution was that there might be some difference in the measure of severity with which the proposals for assurance on female lives were scrutinised by the respective assurance companies of England and France" (*J.I.A.*, vol. xxxiii, p. 508). Mr. Kenchington referred to what might be called a lack of commercial morality on the part of female proposers. Whilst admitting its probability, he (Mr. Rusher) doubted whether the difference in that respect between male and female proposers was of sufficient importance in itself to call for much remark, except in so far as it was combined with the defective medical examination already referred to. Passing to the more general aspect, actuaries had to bear in mind that the *OE* experience took them only up to the year 1893. It was safe to say that, since that date, the whole outlook of womanhood had so completely changed, that many of the deductions from those facts had become almost useless as guides to the future. The change was clearly shadowed in the author's tables in paragraph 59, from which it would be seen that the tendency of the past 20 years had been to lessen the restrictions upon female entrants, if not to remove them altogether. Mr. Kenchington warned them very clearly that, owing to the increase of the number of females engaged in professional, commercial and similar occupations, who should prove to be thrifty and likely to propose for assurance, the rate of mortality in the future would depend upon considerations widely differing from those of the past. In that connection, it was interesting to note that that increase had probably occurred entirely within the present century. A comparison of the figures of the Census for 1901 with those for 1891 showed that the number of employed females of the class indicated was practically the same at both periods, the only alteration being a drop of some 50,000 in the number of teachers, which was compensated for by an increase of 45,000 in the number of female clerks. That showed that up to 1901 there had been merely a transference from one occupation to the other. No doubt when the next Census fell due, a large increase, both actual and relative, in the number would be shown. An investigation which he had made into a very large experience of assured lives showed that of assurances where medical examination was required, the number of policies issued on female lives in 1893 was 10 per cent of the whole issue, whereas in 1903 it was 12 per-cent, and in 1908 16 per-cent. It might be added that of those that were issued in 1903, over half the lives were still unmarried in the middle of 1909.

Taking the view that past experience was likely to be repeated in the future, there were several ways open by which the offices could be secured, without making any addition to the premium charged. Mr. Kenchington mentioned the diminishing debt system, which appeared exactly to meet the facts of the case in theory; but of course the difficulty was always how to make the initial debt sufficiently great, without frightening off proposers. Another method, a modification of the first, was already in use

amongst some companies for other purposes, under which only a small proportion of the sum assured was payable in the event of death during the first year, the proportion increasing rapidly till the full sum assured was payable in the third or fifth year. That had the advantage of overcoming much of the difficulty caused by early lapses. A third method that might be employed was to issue only non-profit policies on female lives, when ordinary commercial considerations would come more prominently into view. In such cases the author's warning words in paragraph 61 as to surrender values would have to be carefully borne in mind, but it might be pointed out that a smaller ratio of profit on a larger number of non-profit policies might quite conceivably result in a greater general benefit to the company. It would have been helpful to have had some figures bearing on the average sum assured on female lives. In offices where it was large, it might generally be assumed that the assurances were required in connection with some financial transaction, and might result in the introduction of an undesirable class of risk, but he believed that small policies on female lives might be taken, speaking generally, without differentiation of rates, and that where that was done a remunerative return for the extra trouble involved would result.

THE PRESIDENT, in proposing a hearty vote of thanks to Mr. Kenchington for his interesting and important paper, said he was rather surprised to find that the Institute has not formally dealt with the subject of the mortality of female assured lives since 1875, when Mr. Walford read his paper on the subject. It was possibly due to the fact that so many years had elapsed since the subject had been thoroughly discussed at the Institute, that so little change was to be noted in the practice of offices with regard to extras for female lives. He was rather struck with the similarity between the two sets of figures which the author gave in section 59, representing the practice of offices in 1909 and 1889, which seemed to show that, with regard to this question, they had hardly moved with the times. It was certainly a matter for regret that so long a period usually elapsed before the valuable material which was collected from time to time by the Institute or by other bodies became fully available for every-day practice. If some means could be adopted whereby, either the data could be collected at more frequent intervals, or its analysis could be more quickly effected, so that the experience upon which they based their practice was always fairly up-to-date, it would be of great advantage to actuaries generally.

Several interesting suggestions had been made on the practical subject of extra premiums for female risks, and in that connection he wished to refer to the conclusions Mr. Lutt had deduced from the Returns of the Registrar-General, as these seemed to show that possibly at the present time, if sufficiently recent data were available to deal with the problems, it would be found that very little extra premium was required in average cases. The fall in the general mortality of females, as revealed by the returns during the last

20 or 30 years, had been so much more than in the case of the male population, that it was possible it had by now practically obliterated the theoretical extras at which the author had arrived. Assuming, however, that some extra was still thought to be necessary for safety, the most practical suggestion, it seemed to him, was that of a single extra, as proposed by the opener of the discussion; but he agreed with many speakers that, for assurances upon the lives of spinsters or widows, and probably also in the case of married women where the whole surroundings of the case appeared to be perfectly satisfactory, no extra need be charged. One advantage of the single premium extra was that it practically got rid, at the outset, of the difference between the reserve values, or the surrender values, under these assurances and those on male lives.

The resolution was carried unanimously.

MR. C. W. KENCHINGTON, in reply, said that he was glad to find that Mr. Lutt was in agreement with himself, as to the improvement in female mortality during recent years. He was interested to hear that Mr. Lutt had brought out the extra single premiums, which he himself had also computed, and he was in substantial agreement with Mr. Lutt on that point. With regard to the question of charging a single extra premium, in the place of an annual premium or a diminishing debt, the point which occurred to him was whether it was possible for the assured to pay such a single premium, and whether an office having a large business in small cases would be able to collect it. Turning to the remarks of Mr. Penman, he thought it would be found that extra premiums for double endowment assurances on the basis of select tables could be obtained from Tables P and Q, without further investigation.

Through the kindness of Mr. Ackland, he had had the advantage of seeing the figures obtained by Mr. Calderon some years ago, and the agreement between those figures and his own was very close indeed. It appeared to him, however, that there was one great disadvantage in the method there adopted, namely, the very large number of constants required in working out the frequency curves. He had personally made an attempt at graduating the experience by frequency curves, purely for the purpose of experiment, but he gave it up. He was very much interested in Mr. Rietschel's remarks, comparing the progression of the rates of mortality in the early years of assurance as shown by O^{NM} Table with the O^{EI} rates. That was a matter which would bear further investigation. With regard to Mr. Rietschel's remarks on the year of assurance 0, he wished to point out that, dealing with this year of assurance by means of a graphic graduation, Mr. Noir, in the "Transactions of the Faculty of Actuaries" (vol. i. p. 25), brought out a curve almost identical with the one that he (Mr. Kenchington) gave. The adjustment of the select rates, described in paragraph 29, to which Mr. Rietschel referred, was an arbitrary method adopted with the curves before him resulting from the first application to the original data, and the method appealed to him as being one which would bring out the adjustment in the way desired.

With regard to the question of the large number of terms in the graduation formula, reference to the diagram giving the coefficients of the formula showed that the only values which had any great influence on the final graduated value were seven terms on either side of the central point. He agreed with Mr. Penman that the final terms were in the nature of a feeling of the curve from point to point. A comparison had been made between his summation formula of graduation and the graphic process. Having tried both, he confessed that the summation formula appealed to him as being by far the easier method. It was necessary, in a graphic method, first of all to plot certain fixed points; difficulty was then experienced in drawing the curve, and perhaps the greatest difficulty with the reading off of the values, and the smoothing-out process. It took about three hours or three and a half hours to graduate a complete table by a summation formula, and only in exceptional circumstances could a satisfactory adjustment be made so readily by means of a graphic process. He was not therefore in entire accord with those who advocated the graphic method.

*Some points of interest in the operations of Friendly Societies,
Railway Benefit Societies and Collecting Societies.* By
ALFRED W. WATSON, F.I.A., F.S.S.

[Read before the Institute, 31 January 1910.]

NEARLY twenty-two years have elapsed since the masterly essay of our President gave to the Institute an opportunity for comprehensive examination of the British friendly society system. During the intervening period, phases of the subject have been occasionally before the Institute, but the period has been one of great activity amongst the friendly societies, and developments of interest to actuaries culminating, it may be suggested, in the co-operation of the State and the societies proposed by the Chancellor of the Exchequer, have emerged. In the circumstances it has seemed to me that the Institute might again survey this wide field of provident effort with interest to its members and profit to the community.

I propose accordingly to examine the more important groups of societies, paying particular regard in respect of the leading affiliated orders to the machinery by which it is sought to equip the constituent parts for the effective recognition of their duties; thence passing to some questions of special interest to ourselves. For the purpose of connected description it will be necessary to mention particular organizations, but I trust that nothing of what I have written may seem to depart from the excellent custom of neutrality which governs our debates. I venture

indeed to say that so far from being unduly sensitive, all the societies which are striving for improvement (and I think this may be said of most of them) are receptive towards the stimulus of informed criticism.

There is apparently no authoritative statement in existence showing the number of members of permanent friendly societies in the United Kingdom. The last summary published by the Friendly Societies Registry Office gives returns from 26,917 societies as at 31 December 1905, with 5,899,918 members, and this number is apt to be quoted as showing the numerical strength of the societies (*vide J.I.A.*, xlvi, p. 250). The number in question includes, however, the membership of dividing societies, which are scarcely to be reckoned as permanent although they provide sickness assurance of a kind for very large numbers of the urban population, of local death benefit societies not ranking as collecting societies, of juvenile societies, and of societies confined to particular industrial organizations such as the great railway systems and great numbers of the members of which are to be found also in the affiliated orders. It is impossible to make any estimate of duplicate memberships, but the Report of the Registrar for 1906, Part A, Appendix N. Section XI, gives the means of separating the various types of society, and by its aid I have drawn up the following table which, I think, gives a more accurate idea than is obtainable from other sources of the true strength of the permanent societies assuring sickness benefits—which I take to be a fair definition of what is generally understood as a “friendly society.” In preparing this table, I have separated the 215 Societies of Miners and Quarrymen and the 73 Societies of Railwaymen from the general mass, the first, because I believe that in some cases the benefits are payable during disablement from accidents and not in time of ordinary sickness, and the second, because there is an element of compulsion in the membership, and as a rule the management is shared by the employers; both these bodies, therefore, rank as distinct groups.

My reason for giving the figures separately for the large affiliated orders is that, whilst the Registrar’s Report gives the number of “societies with branches” as over 150, with a membership of 2,673,246, more than one-half the entire number of members is accounted for by two only of the societies. Further analysis of the societies with branches shows a considerable membership of Dividing Societies, Juvenile Societies,

and Societies assuring death benefits only. These cases have been eliminated, as falling outside the adopted definition of a permanent friendly society. One known case representing, chiefly, duplicated membership is also eliminated. The Hearts of Oak Benefit Society comprises about three-quarters of the membership of the third group, the total of which is estimated for the purpose of separation from the aggregate given by the Registrar. The deposit societies (Group 7) are included as providing a form of assurance which, if rather less than "permanent", is distinct from the ephemeral membership of the Dividing Societies.

*Adult Members of Permanent Friendly Societies, 31 December
1905.*

1. Affiliated Orders without restriction on membership. Manchester Unity of Oddfellows	755,221
Ancient Order of Foresters	659,501
Loyal Order of Ancient Shepherds	124,638
Other Orders (6) with over 50,000 members in each	368,297
Orders with under 50,000 members in each	363,294
2. Affiliated Orders restricting membership to Total Abstainers	286,234
3. Centralized Societies	400,000
4. County and District Societies	39,291
5. Other Societies, mainly local (Urban and Rural)	694,181
6. Railway Benefit Societies	151,547
7. Deposit Societies.	241,760
8. Societies of Miners and Quarrymen	423,246
	<hr/>
	4,507,210

In comparing these figures with the published statistics of the societies themselves, it should be remembered that the latter commonly include colonial and foreign members, whose attachment to the British bodies is of a sentimental rather than legal character. This point is not unimportant; one small "Order" which appears in the returns of the National Conference of Friendly Societies with a membership of over 300,000, and so far as public opinion is influenced by such data, ranks possibly as a somewhat prominent body, returns only some 72,000 members to

the Registrar as its strength in this country. It is believed that negroes constitute a large element in its foreign membership.

It will be seen that the total adult membership of the societies, when deductions are made for the merely burial societies, for juveniles, and for the ephemeral dividing group, is nearly 1,500,000 less than published in the Registrar's Summary.

Until quite recent years, the number of members assured in the friendly societies was constantly increasing, though I am not sure that in the aggregate the increase was more than commensurate with the growth of the population. Such increases as those of the Manchester Unity of Oddfellows from 472,388 in 1880 to 751,431 in 1908, of the Ancient Order of Foresters from 517,529 in 1880 to 630,683 in 1908, and of the Hearts of Oak from 102,239 in 1884 to 298,404 in 1908 are of course conspicuous and satisfactory, in so far as they indicate the preference of the industrial population for some of the more efficient types.

Whilst for many years the increase of the larger bodies was proceeding with great regularity, it has to be recorded that circumstances have recently altered, and that this feature has disappeared from the operations of many of the societies. The most conspicuous case of numerical reversal is that of the Ancient Order of Foresters, which reached high-water mark in 1902, when it registered a membership (in the United Kingdom) of 676,974. In the following six years this number fell to 630,683, and the rate of decrease shows no sign of diminution. Vigorous financial reforms are probably the chief cause of this serious decline, in which the society pays a heavy price for the phenomenal progress of earlier years, when financial considerations were less regarded than at the present time. In smaller degree the Manchester Unity of Oddfellows is affected by the same trouble. The number of members of this Order in the United Kingdom in 1902 was 752,509 and in 1908, 751,431. Only by the increase of the colonial membership has the society been enabled to report continuous growth. The membership of nearly all the societies, large and small, connected with the National Conference exhibits this unsatisfactory feature of arrested growth or positive decline.

The decrease would be less unsatisfactory, if the youthful population were finding its way into other societies of an approved assurance type. Such, unfortunately, is not the case. The only instances of pronounced growth are in the "deposit"

group, in which the element of assurance is but slight, and in the nature of assessmentism; and the constitution of which to the actuary is further condemned by an extremely unscientific system of finance: insufficient regard, or none at all, being paid to the variation of liability with age, and no specific reserves being created in respect of liabilities actually accrued. When Mr. G. F. Hardy prepared his essay in 1888, these societies were so few and insignificant as to be worth nothing more than mere mention. At the present time the most prominent member of the group reports a strength of nearly 200,000.

1.—AFFILIATED ORDERS.

The more important members of this group exhibit, perhaps by the accident of circumstances, the most interesting features of the friendly society system. Established before provision against sickness and death was recognized by the working classes as a form of assurance—as shown by Rateliffe's frequent use of the term "sick gift" in his earlier reports to the Manchester Unity—the consequences of the financial defects by which they were beset, whether regard be had to the actual failures or to the measures taken to avert failure, would have destroyed organizations of a less sturdy type. It may almost be said of the best of them that their very faults have proved a virtue, in that they have attracted to their service a class to whom successful appeal was only to be made by serious constitutional or administrative problems. Of these societies the largest is the *Manchester Unity of Oddfellows*, which on 31 December 1908, consisted of 3,832 branches in the United Kingdom (of which 3,384 were in England) with 751,431 members, and of 1,216 over-seas branches, nearly all being in the British Dominions, with 131,167 members. The connection between the parent society and the colonial branches is in great degree sentimental, the society being self governing and subject to a separate Friendly Societies Act within the confines of each State. The colonial branches, notwithstanding their legal independence, render a great service to the parent Society in the agency facilities they afford to emigrant members.

A paragraph of the Report of the Royal Commission of 1870 describes the constitution of this Society so briefly and yet so well that I venture to quote it in full.

"The Board of Directors forming the central
"government of the Manchester Unity, which is fixed at
"Manchester, consists of nine directors, a grand master,

"deputy grand master, corresponding secretary, and the
 "last past grand master, thirteen persons in all, who
 "constitute also a supreme court of appeal or arbitration
 "court for the order. They are elected by the deputies
 "from the various districts of the Order, who meet in
 "what is called the Annual Moveable Committee (or
 "A.M.C.) at some town in the Kingdom named in the
 "previous year. In this committee is vested the power
 "of revising the rules of the order, as well as that of
 "making levies on the order at large. The business of
 "each district is transacted by the delegates of the lodges
 "of which it consists, at half-yearly, or more usually
 "quarterly meetings. Thus, the district meetings assess
 "the levies on the lodges for the funeral claims and for
 "district management, and appoint district auditors to
 "audit the lodge accounts.* The district also relieves
 "travellers, issues reports, and arbitrates between lodge
 "and lodge, or between members of one lodge and
 "another lodge or its officers, and generally also hears
 "appeals from lodge meetings. The lodge administers
 "its own sickness and management fund, and a
 "'summoned' lodge meeting is the court of first instance
 "in disputes."

The principal benefits provided by the Society are sickness allowances and payments on the death of members and their wives. Sickness benefits run throughout life; during the last 25 years great efforts have been made to spread the assurance of annuity benefits in lieu of sick pay in old age, but the results achieved in this direction have been extremely poor, and with the coming of State Pensions the direct assurance of annuities has been tacitly recognized as a lost cause. It is possible that, indirectly, small annuity benefits will hereafter enter into the transactions of the society; the question of commuting sick pay at 70 years of age for annuities of equivalent value has already arisen in connection with the difficulty of adequately supervising the sickness claims of members in receipt of old age pensions.

* This is not quite correct. Each lodge appoints its own auditors, but the districts appoint "lodge book examiners" whose duty is to see that an approved system of book-keeping is followed in each lodge, and that the accounts thereof are properly audited. It is worth mention that a class of "Qualified Auditors" is about to be created, examinations being held throughout the country and certificates of competency issued to such as satisfy the examiners. The subjects of examination are the financial and other important rules of the Society, arithmetic and accounts.

The original basis of the society permitted the lodges to pay what benefits they preferred, and to fix such contributions as they thought sufficient. Graduation of rates by age was unknown, and all the contributions were applied to one fund to which benefits and management expenses, the latter often excessive, were charged without discrimination. In 1845, the first step towards the alteration of this primitive system was taken in the separation of benefit and management funds, and in the adoption of a minimum rate of contributions for benefits. The latter, however, was not fixed upon any actuarial basis. At this early date the membership numbered about a quarter of a million, and the first effect of the modest reforms here mentioned was the wholesale secession of disaffected branches, resulting in a reduction of the membership by 10 per-cent in two years. In 1853, the effect of age on liabilities was partially recognized by the adoption of a scale of additional contributions, regulated by age at entry, but not taking account of variations in the scales of benefit; in 1864 a complete set of actuarially grounded tables of contributions was embodied in the rules. These, somewhat unfortunately, were net premiums, without any addition for possible excess of sickness claims or other contingencies. For many years after the adoption of the tables, the energies of the leaders of the Unity were absorbed in enforcing their acceptance by the branches, and particularly their application to existing members. This brief retrospect is sufficient to indicate the causes of the total deficiencies of £1,360,677, against the insignificant surpluses, amounting in all to £17,230, which the first valuation, taken as at 31 December 1870, disclosed. This valuation, which was undertaken voluntarily and in advance of the legal requirement, was based on the Society's Experience, 1866-70, without adjustment for extra risks of occupation or other causes, but with the universal use of interest at 3 per-cent. The second valuation showed an apparent reduction of the deficiency by nearly 60 per-cent; this was brought about by the valuation of a large proportion of branches on a 4 per-cent basis. As this high rate was adopted on the requisition of the branches themselves, without, so far as I have been able to learn, any evidence that it was then being obtained, or likely to be obtained in the future, it is clear that the apparent improvement was largely illusory. The third valuation was the first to take into consideration the actual probabilities as regards the future rates of interest, and the effects of extra risk of occupation. It resulted in the

restoration of the total amount of declared deficiencies to a sum approaching that exhibited by the valuation of 1870. Fortunately, the surpluses by this time amounted to over £600,000, so that the “net deficiency”—by which the society was accustomed, somewhat illogically, to measure its progress—was no more than £627,820. Subsequent valuations have proceeded on the lines laid down by Ratcliffe’s successor, the late Reuben Watson, for the third investigation, greater refinements being naturally introduced in the measurement of extra risks and in the discrimination as to interest rates. Between the third and the sixth valuations, the total deficiencies exhibited no signs of material reduction, although the surpluses, despite repeated appropriations, continued to grow under the influence of the factors which first brought them into existence, and when the sixth valuation was completed, the aggregate deficiencies came out at £1,307,621. It was thus borne in upon the society that the adoption of sound principles was not in itself sufficient to ensure a sound position, and that definite steps must be taken to make good the accumulated losses resulting from former insufficiencies.

The rules were then altered to give the “Districts” power to intervene in the affairs of “Lodges”, and in cases of serious deficiency to insist upon the adoption of reformative measures, either increase of contributions or reduction of benefits; at the same time the districts were authorized to soften the rigour of their requirements by granting assistance. This frequently takes the shape of the transfer from the lodge to the district of the sickness liabilities in respect of members over 60 years of age. In the worst cases the lodges are closed and their liabilities transferred to the districts. Behind the districts stands the Board of Directors, which authority, in addition to its ordinary functions of enforcing compliance with the rules, was required by the latter to examine all cases of serious or dangerously growing deficiency, and to insist upon the payment of hazardous occupation rates of contribution in all cases wherein it was evident that deficiency resulted from excessive claims incidental to members’ employments. (These hazardous occupation rates, it may be mentioned, are based upon Ratcliffe’s “Miners’ and Colliers’” Experience of 1866-70.) In its turn the Board of Directors was authorized to mitigate the severity of its demands, by recommending to the annual meeting the grant of Unity relief. As a rule this consists of the assumption by the

Unity of the death benefits of members over 60 years of age and their wives. Restorative measures thus frequently represent the joint effort of the lodge, the district and the Unity. The broad result is seen in the reduction of the gross deficiency on the eighth valuation to £849,166. The surpluses have increased (after writing off repeated appropriations) to £1,256,791, the resulting position thus being a net surplus of £407,625. The gross deficiency has been reduced to probably about £500,000 by the further efforts made since the issue of the eighth valuation reports.

The weak spot in the relief scheme is that the necessary funds are obtained by drafts upon the benefit funds of the lodges, and, in so far as they are derived from branches in deficiency, are adding to the deficiencies which it is desired to subdue. This objection perhaps loses some of its force from the consideration that the element of lapse is ignored in the valuations, and that the reserves released by lapses amount to many times the sums levied for relief purposes.

The incongruity of taxing surplus and deficiency lodges at an equal rate has not escaped attention, and by an alteration of rule lately adopted, all lodges which are appropriating surplus must apply a portion thereof, varying between 5 and 20 per-cent at their option, to the relief of lodges in deficiency within the bounds of their own districts. Many districts are entirely free of deficiency, and this tax will not apply in such cases. It is, however, a first step, and will doubtless be followed by a claim for the Unity relief funds in all cases wherein no local demand arises. The society has here indeed touched only the fringe of a serious problem affecting all the important affiliated bodies. Strong enough to survive their earliest faults (and supported in this respect by the popularity of the "Orders" to which they are attached), the deficient branches have not thrown off the consequences of those faults, and the burden which the present generation is called upon to bear is often one in the creation of which it has had little or no part. To soothe dissatisfaction whilst re-distributing the inherited load is a task evoking considerable diplomatic powers, and I am often glad to realize that it falls outside the professional duties of the Actuary.

The lodges are forbidden, whatever their financial position, to "close the funds" against sick members. The Society's experience, it may therefore be noted, embodies the whole duration of

permanent sickness without artificial limitation. This rule has pressed hardly upon some of the weaker branches, and to prevent evasion it has been found necessary to fix a limit below which the sick pay in protracted cases shall not be reduced. This limit is 2s. per week, a painfully small sum in itself, but not unfair in comparison with the tabular quarter pay of 2s. 6d., for which in many thousands of cases the members contribute. Small as it is, this minimum has been found beyond the capacity of a number of branches—those, of course, which offered the longest successful resistance to reforms—and the central body has found it necessary to undertake a part of the burden, the relative shares of the lodges and the Unity being actuarially apportioned.

The number of "districts" in the United Kingdom is 383. This number, which is excessive, results in great measure from the anxiety of past generations to obtain the distinctions of district office. The existence of small and ineffectual districts to-day is frequently a bar to resolute administration. Considerable powers of enforced amalgamation have been taken, but difficulty commonly arises in the unequal position, financially, of the branches of neighbouring districts. To overcome this difficulty, the central body again has to come forward, and in numerous cases has taken the responsibility of large deficiencies. Guarantees of future good management are exacted, as far as possible, in all such cases.

A system of separation of the claims for sickness and death is universal, the lodges undertaking the sickness liabilities, and the districts the death benefits. This plan was adopted at a very early date, in the belief that it was dangerous to allow the lodges to bear the dual liability, but, as Mr. G. F. Hardy has pointed out, this idea is fallacious, a heavy death rate being rather productive of profit by the release of sickness reserves than of loss by the premature payment of some of the death benefits. The system, however, is administratively good in providing a financial basis for the existence of districts, assuming always that that basis is sound in itself. Such formerly was not the case with the Manchester Unity, or, indeed, with any of the affiliated orders, the death claims being levied over the lodges in proportion only to their respective numbers of members without regard to ages. Great injury was thus inflicted on the younger branches, which in many cases unwittingly contributed through the "district funeral funds" to the large surpluses acquired by the senior lodges. This system is now forbidden.

Defective substitutes have taken its place in many cases, but these also are being gradually retired; districts have now the option of completely separating the sickness and death funds and the corresponding contributions, or of providing for current death claims by assessment on the lodges according to ages and amount assured. If the latter system is chosen, the assessment is made generally by such one of the mortality tables applicable to the three "areas" of the Society's new experience as is appropriate to the geographical situation of the district.

So many reforms could have not been instituted with hope of success had not effective compulsory powers been obtained. These have been acquired by the revision of the conditions of "secession" by lodges. Secession cannot be prevented if branches are determined to go, but it is now provided by the rules that the funds accumulated in respect of death benefits, whether held by the districts or the lodges, and either in separate accounts or in combination with sickness funds, are the property of the districts, and that seceding lodges are not entitled to any claim thereon. Thus members leaving individually have no claim upon the common sickness and death funds, whilst members leaving collectively are allowed to take their sickness funds but must surrender their shares in the common death funds. It is further provided that a seceding lodge shall pay any additional sum actuarially found to be necessary to leave the district as a whole in as good a position, after the secession, as it was (regarded as a whole) before the secession took place. The object is to prevent a rich lodge from leaving a poor district and so escaping the burdens which the Unity has always recognized that, sooner or later, its wealthier branches must bear. In order to prevent the possibility of lodges defeating this rule by showing such obstinacy as to court expulsion, it is further provided that a lodge in process of expulsion shall be taxed as though it were seceding.

I need only increase my references, perhaps already too numerous, to this organization by saying that at its last annual meeting it decided, with realization of the consequences on its apparent financial position, to pass from the 1866-70 standard of valuation to the Tables of 1893-97. Endeavour has been made to counteract the imperfections of the old standard by keeping the valuation rates of interest at the lowest practicable point (70 per-cent of the valuations being taken at 3 per-cent), but it is now agreed by all concerned that the change of basis has become necessary.

The Ancient Order of Foresters is constituted with similar objects to those of the Society just described, but has certain features peculiar to itself. The most prominent of these is the migratory system of government, the central office being moved every year, and the Executive body elected by and from the members in the locality to which the administration is about to be transferred. This system is perennially discussed, its supporters claiming that it enables a much larger number of members to participate in the government of the Society than would otherwise be possible, whilst the opponents contend that under its operation experienced administration is impossible. Whichever of the two parties be right, the system evidently commands the approval of the larger section of the branches.

The constitution of the Order as defined in its rules shows a marked degree of respect for the independent control by the branches of their own funds, express provisions vesting each such fund in trustees for the sole use and benefit of the members of the branch concerned, and protecting the branches from compulsory amalgamation. The sanctity of this constitution, which to the critic might appear at times to hamper the Executive in the prosecution of desirable objects, is safeguarded by the provision that it is not to be altered except with the consent of a nine-tenths majority of the representatives at the annual meeting.

The publications of the Order give evidence of keen interest in actuarial matters, but, somewhat curiously for such an important institution, the valuations are not conducted by one officer under the authority of the Executive, each district, or in many cases, each unit, termed "Court", being at liberty to employ its own valuer. As may be expected under such conditions, the services of actuaries are not generally sought. The Executive in publishing a summary of the last valuations expresses its opinion that the present system of the appointment of valuers is not satisfactory, and "does not always ensure efficient results."

The Society has endeavoured to counteract diversity of method amongst the "valuers" employed, by the formulation of conditions with which all valuations must comply. The valuations are to be based in all cases upon the Ancient Order of Foresters' Experience, 1871-75, with adjustment to the mean between the Standard Table and the Court's own experience.

This condition is understood to be satisfied by comparing the sickness claims, expected and actual, in aggregate, without regard to ages or periods of attack, whilst no reference is made to the variation of liability resulting from probable departures from the standard mortality rates. Thus a branch experiencing in one quinquennium 110 per-cent of the expected aggregate sickness cost would be debited in valuation with 105 per-cent of the total standard values. Should the experience percentage fall in the next quinquennium to 90, the valuation estimates would be revised and written down to 95 per-cent. It is easy to criticize this plan, e.g., the prevalence of a sub-standard death rate may add to the liabilities to a greater extent than a sub-standard sickness rate relieves them, and a deduction from the standard values in recognition of the latter experience only may be altogether fallacious. Considering also the comparatively small number of members included in individual branches, a constant flux of experience within fairly wide limits is inevitable, and the repeated variation of the valuation estimates in sympathy with the most recent change in the experience seems not only difficult to justify, but calculated to arouse distrust as to the worth of valuation.

The valuation rates of interest are also controlled by the rules which provide for the use of the nearest multiple of $\frac{1}{2}$ per-cent to the rate actually obtained in the past quinquennium, without either maximum or minimum limit. The purported valuation on the one hand, of 261 branches at $4\frac{1}{2}$ per-cent and upwards (3 being stated as 6 per-cent), and on the other hand of 370 branches at 2 per-cent and under (8 being stated as 0 per-cent), arrests attention.

The following are the published results of the last two valuations :

	Total Surpluses	Total Deficiencies
1902	£ 483,655	£ 2,208,417
1907	£ 898,489	£ 2,022,399

It is shown in the official compilation that the valuation rates of interest have been increased in many cases consequent on "a further amendment of General Law which provides for the rate used, in some cases, to be above the actual earnings

instead of being always below, as was formerly the practice." On this point the following summary is given.

Branches valued at 3 per-cent and under	1902	1907
	2,391	1,628
Branches valued at over 3 per-cent	1,521	2,140
	<u>3,912</u>	<u>3,768</u>

Against this relaxation of the valuation basis must be set increased additions to table values for excessive sickness claims.

The gross additions for this purpose are given as	£	
		991,181
Whilst the deductions from table values for assumed future light sickness are .		<u>323,453</u>
Giving a net addition of		<u>£667,728</u>

equal to nearly 5 per-cent on the sickness values all round.

On the whole it is probable that the average valuation basis has been strengthened, and that the progress of the last five years is consequently greater than the bare recital of the comparative amounts of deficiency would suggest.

This Order has been conspicuous for its efforts to assist unsound branches. In introducing the 1902 valuation summary, the executive body stated that the relief then given amounted to over £50,000, whilst the Order stood committed to a further sum of about the same total. As relief grants have continued to be made since 1902, it may be presumed that at the present time the aggregate amount of assistance rendered reaches a very considerable sum. The funds required are obtained from sources other than the benefit funds—probably, as a rule, from separate contributions, paid more or less *ad hoc*, and in this respect the relief system is undoubtedly more logical than that of the other large Order.

The remaining affiliated society, which is separately named in the Table of Membership (*Loyal Order of Ancient Shepherds*), does not present any features peculiar to itself, save that about 40 per-cent of its members are in Scotland. Its valuation summary is of some interest. It appears that the valuations are based on the Manchester Unity Experience, 1866-70, and that during the five years 1901-05 the claims exceeded this standard by 20 per-cent, pointing apparently to widespread extra risks of occupation. The excess is stated to have been 16 per-cent in England, 18½ per-cent in Scotland, 46 per-cent in Wales, and

41 per-cent in Ireland. The valuers purport to have used 22 different rates of interest, every rate at intervals of $\frac{1}{4}$ from $\frac{1}{2}$ per-cent to $5\frac{3}{4}$ per-cent being found. Such extensive variation seems inconsistent with the circumstance that the published tables based on the standard adopted do not comprise more than six rates of interest. This feature, discovered here and elsewhere, has led to the suggestion that the statutory valuation abstract should be enlarged, under the powers vested in the Registrar, to include specimen values of the tables used, when valuations purport to be made at rates of interest other than those at which tables have been published.

The aggregate deficiency in this Order at 1900 was given as £505,253, and at 1905, £253,476. In each case the table values of sickness benefit are increased on the average by about $6\frac{1}{2}$ per-cent for expected excess of claims. The apparent reduction of deficiency is stated to be due partly to the weakening of the valuation basis in respect of interest rates. No materials for judging the extent to which this factor has operated are given in the report, but it would seem that in face of an average excess of 20 per-cent in sickness cost, forestalled only to the extent of $6\frac{1}{2}$ per-cent in the valuation estimates, the standard must have been lowered materially for the ultimate result to be shown as a reduction of deficiency by over £250,000. The probability that reformative effort accounts for some part of the improvement should not, of course, be overlooked. The leaders of the Society appear to be keenly interested in financial matters and solicitous for the betterment of the organization.

2.—AFFILIATED ORDERS RESTRICTING MEMBERSHIP TO TOTAL ABSTAINERS.

The Independent Order of Rechabites is the largest of the Orders imposing total abstinence as a condition of membership. This Society has extended very rapidly in recent years, and the number of members was just under 177,000 at 31 December 1905. The number of branches is stated to be 2,260, and the average number of members would thus seem to be under 80. The benefit system is similar to that of the other Orders, save that the funds of some of the branches, called "Tents," are consolidated in districts for sickness assurances, whilst the "funeral funds" are in all cases separated from the sickness funds and maintained by regular contributions. The financial operations of the Society are regulated by tables based on the Society's own experience for the years 1878-87, as analyzed by

Mr. F. G. P. Neison, and published in 1889. From Mr. Neison's report, it appears that the number of lives under observation at the commencing date in the branches supplying valid returns was 6,748 only, and that 30,216 lives came into observation during the ten years. The number of exposures to risk was 127,269·5, the number of deaths 1,040, and the total weeks of sickness 180,210.

The tabulation was thus composed very largely of the experience of entrants during the decennium. Having in view the abnormally low death-rate exhibited, it is important to remember this, and the further circumstance that many of the entrants came in at somewhat advanced ages. On this point the following comparison is of interest :

Ages	PERCENTAGE OF ENTRANTS	
	Rechabites 1878-87	Manchester Unity 1893-97
Under 25	52·5	71·8
25-34	28·0	23·2
35 & over	19·5	5·0
	100·0	100·0

The number of exposures at ages 55-64 was 5,419 only, and at ages 65 and upwards 1,954 only. Mr. Neison explained that in consequence of the paucity of data the experience after age 70 had not been used, that of the Ancient Order of Foresters and the English Life Table No. 3 being substituted.

The complete expectation of life is shown by the following table, in comparison with the same function deduced from other friendly society experiences. In view of the immaturity of the experience, the apparent longevity can scarcely be said to prove anything with regard to the vexed question of the effect of total abstinence on the duration of life.

Age	COMPLETE EXPECTATION OF LIFE (\bar{e}_x)			
	x	Rechabites 1878-87	Manchester Unity 1866-70	Manchester Unity 1893-97
20	45·1	41·3	43·9	41·3
30	37·3	34·0	35·5	33·6
40	29·1	26·7	27·4	26·2
50	21·2	19·9	19·9	19·2

The comparative quantities of sickness obtained by summing the graduated rates in groups of ten years are as follows:

Ages	Rechabites 1878-87 Weeks	Manchester Unity 1866-70 Weeks	Manchester Unity 1893-97 Weeks	Friendly Societies 1876-80 Weeks
20-29	10·1	7·7	9·3	8·7
30-39	11·5	10·0	11·7	11·3
40-49	16·5	14·8	17·9	16·9
50-59	21·8	27·2	31·1	29·0
60-69	52·6	62·5	86·5	71·4

The high rate of sickness in the first group is worthy of observation (Mr. Neison suggests that it is possibly due to insufficient care in supervising admissions), but the abnormally light sickness rates at ages over 50 are of greater practical importance. These sickness rates are incorporated in the basis of the monetary tables, at least up to the point of blending with the Foresters' rates at an age approaching 70, but the policy of retaining such an abnormal experience, especially when resting on so small a body of facts, seems open to criticism. The circumstance that the experience was small does not account, of course, for the exhibition of a low sickness rate, but it may well have been that the comparatively few members who entered into the experience at the older ages were a select body, comprising many of the founders and promoters of the Order, whose position in life would be such as to free them from the necessity of drawing sick pay.

The valuation of the Order as at 31 December 1905, taken on the Order's standard tables at 3 per-cent in all cases, without adjustments for extra risk or other causes, showed aggregate surpluses of £172,307, and aggregate deficiencies of £138,836, nearly the whole of the latter being in the sick funds. The comparison of the sickness experience for the quinquennium 1901-5 showed that the claims of the first 26 weeks were equal to 103 per-cent of the expectation, those of the second 26 weeks 119 per-cent, and those of the remainder period 113 per-cent. In view of the composition of the standard table, it may be deemed singular that the excess is no greater, but it should be said that no comparison of the incidence of sickness as to age has been given, and it is impossible, therefore, to see how the experience at ages over 50 is emerging against the expectation. In view of the rapid increase in the membership a very great

majority of the lives at risk must be under 50 years of age, and even a serious excess in the claims above this age could exercise but a moderate influence over a comparison limited to aggregates.

An interesting light on the effect of local sickness experiences is given by a statement which purports to represent the position of each district, on the assumption that its sickness experience during the years 1901-5 will be permanently maintained. It is presumed that here also the ratio of the aggregate claims to the expectation, without regard to incidence by ages, has been employed. This comparison shows that on the basis named the net deficiency of about £55,000 in the Sick Funds is increased to a net deficiency of nearly £300,000. The extent of the change suggests that the results promulgated in many of the statutory valuations, all of which are based on the standard tables, are invalid. This is particularly the case in respect of some of the mining districts, the deficiency in Durham, for example, being increased from £9,996 on the Standard to £83,946 on "Own Experience", and the deficiency in East Glamorgan being similarly increased from £2,861 to £30,106.

The smaller bodies ranking as "affiliated orders" show nothing of actuarial interest, and the financial position of many of them will scarcely bear examination. In connection with these, and, indeed, nearly all the societies in this group, it is worthy of mention that the valuations are defective, in including no provision for the contingent liability in respect of a wife's death benefit for those members who are unmarried on the valuation date. As such benefit is definitely provided for in all tables of contributions (and whatever their shortcomings, the affiliated societies have as a rule adopted tables based on recognized standards), and as credit is always taken in valuation for the full contribution payable, it is difficult to understand the grounds on which the liability referred to is ignored. The remedy would appear to lie in the hands of the Registry Office, assuming (and the contrary appears to be beyond present realization) that the societies concerned will not avail themselves of the services of actuaries.

3.—CENTRALIZED SOCIETIES.

The bulk of the membership of this group is represented by two organizations, the *Hearts of Oak Benefit Society*, and the *Rational Association*. Upon the former of these it is necessary to say but little, in view of the full account of the Society's

constitution submitted to the Institute by Mr. R. P. Hardy (*J.I.A.*, vol. xxxi, p. 86). It appears that since 1892, the last year comprised within Mr. Hardy's tabulation, the number of members has increased from 153,595 to 298,404. This is extremely satisfactory taken as a whole, but there is some evidence that even this fine organization is being reached by the maleficent influences to which I have already referred, the annual rate of increase having fallen since 1899 from 4 per-cent to under 2 per-cent. The Society appears, moreover, to be affected by a curious and disquieting phenomenon which I have recently observed to be in operation throughout the whole extent of the largest affiliated order, namely, an increase in the average age of entrants. Although the total number of members of the Hearts of Oak Benefit Society increased during the year 1906 by 5,279, the number at ages under 35 fell by 2,191. This is the more serious in view of the fact, made clear by Mr. Hardy, that negative values arising under the uniform contribution system at youthful ages have hitherto supplied the means of correcting much of the excess of sickness cost over the valuation provision.

The financial position is not greatly different, according to the last valuation, from that exhibited in each of a series of years ending with 1892 by Mr. Hardy's Table XVI, but whereas a relatively small surplus was formerly disclosed, a relatively small deficiency now appears. It is not clear how far this seeming retrogression is due to variation of basis, but as with other changes the valuation rate of interest has been reduced from 4 to $3\frac{1}{2}$ per-cent, it is possible that the alteration is mainly attributable to this cause. The rate of sickness has recently risen to a serious extent above the estimates based on the Manchester Unity Experience, 1893-97, but whether this is due to the difficulty of supervision it is hard to say. This difficulty accounts, I do not doubt, for some of the recorded claims, but it is impossible to draw conclusions as to the experience of one institution, in comparison with the expectation based on the experience of another, without some knowledge of the effect of occupation risks on the two bodies. To the extent to which the "Hearts of Oak" rejects miners whilst the Manchester Unity admits them, the comparison should tell in favour of the former, but miners are certainly not the only hazardous class, and it is possible that the "Hearts of Oak" receives a greater proportion than the Manchester Unity of the persons engaged in other

unhealthy trades. So far as I am aware no statistics bearing on this point have been published by the "Hearts of Oak."

The Rational Association (94,000 members) is an interesting blend of the two principal types. For the purpose of collecting contributions and paying claims it is divided into local branches (which, however, are not separately registered), but all other administrative work is performed by the chief office. The branches do not retain their own funds, the whole of the accumulations being pooled in one fund under the control of the Executive. The system is thus open to criticism in assigning the important work of supervision to those who are not directly responsible for the moneys disbursed. It has been considered that much excess of sickness claims has arisen from this cause, especially in prolonged cases. I think this is probable, although the aggregate of sickness claims has latterly been but little in excess of the expectation derived from the latest Manchester Unity standard. As I have suggested above, it is difficult to draw conclusions from statistics as to the effect of any particular system of administration on sickness claims, unless the composition of the membership body in respect of occupations be known. It is understood that the membership of this association is largely agricultural, and although latterly the proportion of miners has considerably increased, I have an impression that Sutton's Experience, 1876-80, ought to measure the aggregate claims, under effective supervision. This has not been the case for many years. So far as the latest quinquennium is concerned the following are the published figures:

Rational Association. Sickness Experience, 1902-6.

Period of Sickness	Actual Sickness	Expected by		Expected by Manchester Unity Tables (1893-97)
		Sutton's Tables (1876-80)	Weeks	
First six months . .	517,034	469,901	471,196	
Second six months . .	73,836	56,008	74,587	
After twelve months . .	294,745	241,068	309,057	
Totals . . .	885,615	766,977	854,840	

I have referred above to the recent increase in the proportion of miners. This increase throws an interesting light on the influence of the various friendly societies on one another. For

many years past the Hearts of Oak Benefit Society has closed its doors to miners, and the Manchester Unity, whilst not going so far as this, has unceasingly pressed its mining branches to adapt themselves to their financial circumstances, with the result that the mining membership has largely decreased. It is probably more than mere coincidence that the miners in the Rational Association have rapidly increased in number. From the last valuation report it appears that, whereas in the Manchester Unity Experience, 1893-97, the mining group contributed only 5·8 per-cent to the exposed to risk, the miners in the Rational Association accounted for the following proportions :

1892-6	6·6 per-cent.
1897-1901	10·9 ,
1902-6	13·1 ,

Unfortunately for the society the increase has come from the worst districts, *i.e.*, those where the mining membership of the Manchester Unity has diminished to the greatest extent, the result being that the experience of miners has become progressively worse. On this point certain figures are published, of which the following is a summary :

Rational Miners Experience; pro-	1892-6, 72 per-cent. 1897-1901, 83 , 1902-6, 104 , Unity Experience, 1893-97.
portion of actual sick claims	
to expectation derived from	
mining group, Manchester	

The society is thus not only affected by an increased proportion of miners, but the rate of sickness amongst the miners is rapidly growing. The experience of this group as compared with the Manchester Unity Experience, 1893-97, *Whole Society*, is stated in the valuation report to have been, in the years 1902-6 :

- In respect of the first six months of sickness, 179 per-cent.
- In respect of second six months of sickness, 216 per-cent.
- In respect of remainder of sickness, 154 per-cent.

or for all durations 178 per-cent. The claims in respect of the entire Association have been no more than 104 per-cent of this standard, and it is thus clear that through the medium of the common fund much of the miners' risk is being carried by the agricultural labourers and others earning lower wages than the mining class. The Executive appears to have recently closed the

society to this class, and on the figures quoted it would appear to have had no alternative in equity to the other members.

The valuation as at 31 December 1906 is stated to have been based upon the sickness rates of Sutton's, 1876-80 Tables, with the society's own mortality experience and own lapse experience, the latter element being employed to a suitably restricted extent. The published result at 3 per-cent was a deficiency of £734,805, and at 3½ per-cent a deficiency of £587,458, the deficiency representing in either case about 25 per-cent of the value of all the benefits. The condition, therefore, is grave, and whilst the Executive appears to fully realize this, its opportunities of introducing remedial measures are restricted by the circumstance that the delegates of the members, in whom is vested the power of revision of rules, meet once only in each five years. It should be added that at the last meeting some remedial measures were adopted. The causes of the deficiency are the very usual ones of excessive claims for sick pay, and the insufficient contributions of the older members who (until 1909) have never been charged by the adequate scale that has been recognized as necessary in the case of incoming members.

4.—COUNTY AND DISTRICT SOCIETIES.

The records as to these societies show a numerical decline which is altogether inconsistent with the excellence of their financial position, taken as a group. In 1880, 29 of these societies showed a membership of 40,393, with funds amounting to £608,917. In 1905, the number of members was 39,291, and the funds amounted to £909,116. The last published valuations show surpluses in 24 cases amounting to £110,152, and deficiencies in five cases totalling £13,557. The most prominent member of the group is the Hampshire and General Friendly Society, with over 9,000 assurance members, £200,000 in funds, and £23,375 of surplus as at 31 December 1904.

The late Chief Registrar, discussing the causes of unprogressiveness in these societies, says that "they (*i.e.*, the people) "have the pardonable weakness of preferring to do things in their "own way to having things done for them by other people, even "in a better way." No doubt the element of outside control here referred to has militated against the progress of this group, but it is to be noted that of late years there has been a marked tendency towards placing the balance of power in the hands of the members themselves. Unfortunately the distinctive feature of annuity

assurances in old age is disappearing from most of these societies, the assurance of sick pay during life being almost everywhere preferred.

The valuations of the majority of societies within this group are actuarially prepared. There are exceptions, however, and in one case the Chief Registrar of Friendly Societies, when noting in his report a change of valuation result from deficiency to surplus, added: "In 1893, an increase in the deficiency again appeared, caused by the valuer having made a special reserve of £29,918 to meet probable future excess in sickness claims, on account of the heavy sickness experienced by the society during the period since the last previous valuation. Another valuer made the valuation for the five years ending 31 December 1898, and, although the sickness and mortality continued to be in excess of that provided for by the tables used, did not make any reserve on that account, and accordingly brought out a surplus instead of a deficiency. This result, therefore, cannot be considered satisfactory, as it is due not to an alteration in the facts, but to a difference of opinion between the two valuers." It need only be added that the first, but not the second, of the two valuations here referred to was made by a Fellow of the Institute of Actuaries.

At one time the county societies very generally provided continuous half sick pay, and their experience of this benefit became unfavourable in marked degree before the affiliated orders encountered the same disquieting feature. In several cases the continuous benefit has now been reduced to quarter pay, and this reduction having been followed by a general improvement in sickness rates, due partly, perhaps, to the reduction in question, but also to the adoption of more effective methods of supervision, the financial position of these excellent bodies has been improved to an extent that may be called remarkable.

The whole subject of protracted sickness allowances is of economic interest. Logically there is no defence to the charge that it is absurd to reduce the benefit as the duration of sickness increases and the impoverishment of the sufferer increases. Ansell asserted this view with considerable emphasis in his "Friendly Societies" (1835). Effective, however, as may be criticism of the practice from the philosophic standpoint, it fails to take into account one potent factor—human nature. The manual worker, if not indeed mankind generally, adapts himself

to his circumstances more frequently than he attempts to govern them. Six months of incapacity will generally—a longer period invariably—so far debilitate the habit of routine labour as almost to destroy it ; whilst it is remarkable how small an income can be made to suffice, when the urgings of financial responsibility have been silenced. For this reason, continuous full sick pay has nearly always had to be abandoned, and continuous half pay is fast following it. If it were not for the cases of undoubted permanent incapacity which exist, I should be impelled to advise a limit to the period of sick pay, but between two evils I choose the partial maintenance of those whose incapacity might thus be drastically cured as less than that of withholding support from the absolutely necessitous. It is deplorable, nevertheless, that it is impossible to maintain the weekly five to ten shillings of continuous benefit to the helpless because of the oppressive demands of the merely inert, and that the impossibility of segregation so frequently demands the reduction of the benefit to an ineffective pittance.

An important feature has been added recently to the operations of several of the county societies, in the re-arrangement of the contracts of members assured for sick pay to the age of 65 or 70 without subsequent pension. With old age remote, the low premiums of these classes of assurance invested them with a meretricious attractiveness, but the attainment of the terminal ages by the first entrants at once aroused dissatisfaction and unrest. It has been found expedient, in consequence, to prolong the assurances either at the full or at diminished rates for the whole period of life, and to this end considerable inroads for the benefit of a comparative few have been made upon the surpluses belonging properly to the whole body.

The warning thus expensively conveyed to the societies may be held to have lost some of its force by the institution of the national system of old age pensions. Again, to choose between evils, the danger of a truncated benefit system may have to be faced by the less wealthy societies, in order to protect the funds against the pressure of a sickness liability running concurrently with the receipt of a pension. In the case of even the wealthiest societies, the commutation of sick pay for its equivalent value in pensions may become inevitable.

5.—LOCAL SOCIETIES.

The progress of the larger societies has been accomplished partly at the expense of the small local societies—the village

clubs of earlier generations—which have diminished greatly in number during the past 30 years. I have endeavoured to form some estimate of the extent to which these interesting bodies have been superseded, but have found insuperable difficulty, not only in separating them from the mass of other societies on the register (though the latest publications of the Registry Office mark a great advance in the matter of classification as compared with the earlier volumes), but also in ascertaining which of the societies on the register in 1880 were actually in existence at that date. The names of some thousands of societies are entered, without particulars of annual returns or valuations, and whilst it is certain that in many cases the absence of such returns was due to disregard of the law, it is almost equally certain that in many other cases it indicated that the institutions concerned, although still on the register, had in fact ceased to exist. As a matter of possible interest, I have selected at random 1,000 active societies on the 1880 register (taking cases from every county of England and Wales), and have endeavoured to trace the same societies on the 1905 register. In the result I fail to find 550 societies, nearly all of which must be assumed to have gone out of existence, the chief exceptions being a few admitted as branches of the affiliated orders. These 550 societies in 1880 comprised 51,398 members, with funds of £294,217. In the remaining 450 societies the total number of members which, in 1880, was 60,199, had diminished to 55,465 at 1905, whilst the funds had increased from £477,802 to £672,017. On the whole, these figures confirm the results observed during a rather lengthy personal experience. The small local societies are everywhere diminishing in membership, but many of them are improving financially, and a large number present a quite respectable appearance from the actuarial point of view.

6.—RAILWAY BENEFIT SOCIETIES.

The societies in this group are not numerous, but, having regard to their constitution and to the amplitude of the data which their records generally yield, they are of particular interest to the Actuary. Nearly all of them are valued on special bases, deduced in great part or entirely from their own experience, and involving interest at high rates on the guarantees of the respective companies. There are some cases of serious deficiency, but, rightly or wrongly, the companies are believed to stand behind the societies, an impression to which colour is lent by the

generosity of the various boards of directors in their dealings with the societies or funds, and deficiencies are not regarded by the members in quite the same light as those existing on the valuations of independent organizations. No object of advantage in the present connection will be served by examination of particular members of the group, and I pass on to the consideration of points which actuarially differentiate the group from ordinary societies. These points are :

- (a) Sickness and Accident experiences.
- (b) Secession and Mortality rates.
- (c) The provisions as to Superannuation or Pension.
- (d) Retiring Allowances on withdrawal.
- (e) The provisions as to "out members" (ex-service contributors).
- (f) The provisions as to Transfer of Class.
- (g) The Company's Subsidy.
- (h) The provision (in rare cases) of Orphan Benefits.

(a) In a subsequent paragraph, I give examples of sickness and accident liability which show that considerable variations arise between the experiences of different societies of railway servants. It is well to enquire into the system of supervision, should an apparently high sickness rate appear. It will be found as a rule that the officials are conscious of certain defects, whilst able only to a limited extent to apply the necessary remedies. Railway servants, particularly if engaged in the manipulation of the traffic, are on the one hand a selected class of men, in physique superior, undoubtedly, to the general average ; on the other hand, much of the work is carried on under trying conditions, whilst the safety of the public, their fellow servants and themselves, imposes on the men a high standard of fitness for duty, precluding that early resumption of work, after periods of sickness, that is possible in the case of those engaged in less exacting occupations. On the balance of these contending factors, a sickness rate may emerge differing in considerable degree from that of any standard, and if the comparison should seem unfavourable it may be impossible, in view of the probable causes, to urge the adoption of repressive measures. In such cases, the society's experience is the only safe guide to the future, though it should not be assumed as a matter of course that this is the case.

With reference to the accident liability, it need only be said, having regard to the experiences hereafter given, that the average amount of claim (inclusive of full and reduced pay) should be deduced for each age group, when such average will sometimes be found to be in the nature of a constant, enabling the liability to be valued as the equivalent of an annuity.

(b) It is generally accepted that, in view of the quasi-compulsory character of the membership, it is possible to involve the element of lapse in the valuations of these funds with some degree of confidence. It is found, nevertheless, in actual working that the lapse rate varies considerably. The condition of the outside labour market exercises an important influence, particularly upon the unskilled class, such as labourers and platelayers, and upon the classes of skilled mechanics, e.g., engineers and carriage builders. This will explain some of the higher rates of secession frequently found amongst these classes, in comparison with those obtaining amongst the men engaged in working the traffic, who alone are entitled in strictness to the designation "railwaymen." Amongst the latter class, however, fluctuations are also found, and deviations in a particular direction persist as a rule sufficiently long to materially influence the secession rates of single quinquennial periods. An example of this is seen in the following figures, giving the aggregate secession rates per-cent experienced in three successive periods by a railway society consisting of some 30,000 members, engaged chiefly in traffic operations:

Ages	SECESSION RATES PER-CENT			Ages
	1891-95	1896-1900	1901-05	
15-19	27·04	24·54	19·12	15-19
20-24	14·88	13·14	11·21	20-24
25-29	5·47	5·43	4·64	25-29
30-34	2·05	3·32	1·99	30-34
35-39	1·15	1·48	.98	35-39
40-44	.72	.70	.59	40-44
45-49	.48	.37	.59	45-49
50 & over	.21	.21	.21	50 & over

Whilst aggregate experiences should not be used in valuation, they are useful as exhibiting the general tendency, particularly in view of the fact that few entrants come into the service at ages over 25. The same variability may be seen in the rates

obtained from the experience of another society with a membership of about 9,000, from which the entrants of the period under observation and the corresponding secessions have been excluded, on the plan suggested by me in *J.I.A.*, vol. xxxv, p. 313-4. These rates are as follows:

Ages	SECESSION RATES PER CENT			Ages
	1890-98 (Members of 1890 only included)	1899-1903 (Members of 1899 only included)	1904-08 (Members of 1904 only included)	
15-19	15·28	22·97	8·79	15-19
20-24	6·19	10·16	6·33	20-24
25-29	2·40	4·48	2·33	25-29
30-34	1·45	2·76	1·38	30-34
35-39	1·12	1·53	1·25	35-39
40-44	.90	.91	.56	40-44
45-49	.74	.71	.43	45-49
50-54	.29	.46	.35	50-54
55-59	.29	.50	.30	55-59

When such instances of diversity present themselves at the close of a valuation period, the task of discriminating between the permanent and the merely ephemeral is extremely difficult, and the want of some standard of reference is felt acutely.

The possibility of constructing such a standard, expressing in some degree the mollifying effect of consolidation upon the diverse experiences of different periods, may perhaps be questioned; but I have thought it well to make some tentative effort in this direction. I have accordingly consolidated sections of the experience of the two societies above referred to as follows:

For the first society :

The experience of the ten years 1891-1900 of the members existing at 1 January 1891.

The experience of the ten years 1896-1905 of the members existing at 1 January 1896.

The experience of the five years 1901-1905 of the members existing at 1 January 1901.

For the second society :

The experience of the nine years 1890-1898 of the members existing at 1 January 1890.

The experience of the five years 1899-1903 of the members existing at 1 January 1899.

The experience of the five years 1904-1908 of the members existing at 1 January 1904.

This plan involves some duplicating both of exposures and secessions; for example, the experience of the members of the first society admitted before 1891 appears twice (at the correct ages attained) for the five years 1896-1900, and 1901-1905 respectively. These duplications are unavoidable from the form of the data, but their existence is not altogether a disadvantage. Such lives as come in for a second quinquennium must by the nature of the case have contributed for over five years at the beginning of the second period, and their re-entry in the data is calculated to further modify the abnormal influence of recently admitted lives.

The following are (a) the table in abstract and (b) the graduated lapse rate at each age:

*Combined Lapse Experience, Railway Societies,
(New Entrants excluded.)*

Ages (last birthday)	Exposed to Risk	Lapses	Lapse-rate per-cent
15-19	6,532	1,239	18·97
20-24	58,969	5,194	8·81
25-29	115,054·5	4,051	3·52
30-34	129,621·5	2,584	1·99
35-39	106,190	1,237	1·16
40-44	90,063	637	.71
45-49	76,850	371	.48
50-54	56,706·5	191	.34
55-59	34,633·5	62	.18
Total	*674,620	*15,566	

* Including duplicates as explained *supra*.

Adjusted Lapse Rates deduced from the foregoing experience.

Age	Rate	Age	Rate	Age	Rate
16 $\frac{1}{2}$.2100	31 $\frac{1}{2}$.0220	46 $\frac{1}{2}$.0052
17 $\frac{1}{2}$.2500	32 $\frac{1}{2}$.0196	47 $\frac{1}{2}$.0048
18 $\frac{1}{2}$.2090	33 $\frac{1}{2}$.0175	48 $\frac{1}{2}$.0045
19 $\frac{1}{2}$.1740	34 $\frac{1}{2}$.0157	49 $\frac{1}{2}$.0042
20 $\frac{1}{2}$.1430	35 $\frac{1}{2}$.0141	50 $\frac{1}{2}$.0039
21 $\frac{1}{2}$.1160	36 $\frac{1}{2}$.0127	51 $\frac{1}{2}$.0036
22 $\frac{1}{2}$.0930	37 $\frac{1}{2}$.0114	52 $\frac{1}{2}$.0033
23 $\frac{1}{2}$.0740	38 $\frac{1}{2}$.0102	53 $\frac{1}{2}$.0030
24 $\frac{1}{2}$.0590	39 $\frac{1}{2}$.0092	54 $\frac{1}{2}$.0027
25 $\frac{1}{2}$.0480	40 $\frac{1}{2}$.0084	55 $\frac{1}{2}$.0024
26 $\frac{1}{2}$.0400	41 $\frac{1}{2}$.0076	56 $\frac{1}{2}$.0021
27 $\frac{1}{2}$.0344	42 $\frac{1}{2}$.0070	57 $\frac{1}{2}$.0018
28 $\frac{1}{2}$.0306	43 $\frac{1}{2}$.0065	58 $\frac{1}{2}$.0014
29 $\frac{1}{2}$.0274	44 $\frac{1}{2}$.0060	59 $\frac{1}{2}$.0010
30 $\frac{1}{2}$.0243	45 $\frac{1}{2}$.0056

The mortality experience will generally exhibit sufficiently marked variations from any standard to suggest at first sight the desirability of valuing on the society's own experience. The objection to this course is that the experience oscillates to a rather considerable extent from one quinquennium to another, and it is open to question whether in the long run some standard table, such, for example, as the Manchester Unity Experience, 1893-97, Area 1, would not more closely measure the liability. Whilst each case must be dealt with on its merits, it is advisable in respect of this element to obtain an exceptionally wide spread of facts.

(c) The provisions of the rules as to superannuation or pension exert, as would be expected, a vital influence on the financial position. The original bases of most of these societies contemplated nothing beyond the ordinary benefits of friendly societies, and whilst the occupation is invested with peculiar difficulty so far as old age is concerned, it was possible for a period to maintain the original basis, by reason of the consideration extended by the various companies to their aged employees, for whom the lighter posts were in great measure reserved. Circumstances, however, have altered; the number of the aged is gradually overtaking the number of posts within the capability of this class, and the increased pressure of the work and the tendency of public opinion is making it more than difficult for the various boards to retain the elderly men in active service. Retirement at or soon after the age of 65 is consequently becoming the rule, especially amongst those whom I have termed railwaymen as distinct from workmen employed by the railway companies. In certain cases, the ordinary reduced sick pay is claimed on retirement as a "pension", and allowed on medical certificates of a perfunctory character. In such cases, no standard rate of sickness is applicable; the facts must be accepted, and the valuation based on them.

In one case at least a definite superannuation benefit is payable on retirement at any age after 65. The following analysis of the retirement experience is interesting as showing, in regard to the year 1898, the effect upon voluntary withdrawal from the service of an increase of 2s. per week in the pension benefit, and in regard to the subsequent period the effect of the increased occupational pressure of which I have spoken.

Age last Birthday	RATES OF RETIREMENT PER-CENT UPON SUPERANNUATION CLAIMABLE AT OR AFTER AGE 65 (RAILWAY SOCIETY)			
	1896-7	1898	1899-1900	1901-5
64	13·3	23·5	26·2	36·6
65	14·1	16·7	14·3	28·5
66	5·2	19·4	16·5	27·4
67	6·1	20·8	13·1	20·8
68	10·4	14·3	20·0	19·8
69	11·2	13·3	25·0	24·5
70	10·7	...	21·7	35·1

It is evident from these figures that, where a variable age at retirement obtains, the greatest care must be used to avoid the under-estimation of the liabilities by the assumption, without investigation, that past experience will rule the future rates.

Within the ages at which retirement is wholly or partially optional, it may be expected that the mortality experience of the superannuated members will be different from that of the active class. On this point the following experience, drawn from the same case as the previous table, is instructive.

Age last birthday	ACTIVE MEMBERS			SUPERANNUATED MEMBERS		
	At Risk	Deaths	Rate of mortality per-cent	At Risk	Deaths	Rate of mortality per-cent
64	515	20	3·88	104	12	11·5
65-69	890	30	3·37	959·5	70	7·30
70-74	231	11	4·76	614	43	7·00
75-79	91·5	12	13·1	224·5	41	18·3

Combined with the low mortality of the "active" group a light sickness experience was also found. In such a case, it would be inexpedient to assume that standard sickness values would obtain with regard to the active, whilst it would obviously be improper to debit normal annuity values in respect of the retired. On the other hand, the experience will seldom be sufficiently large to permit of the construction of a valid mortality table for superannuated members only. The best course seems to be to apply to the whole body the aggregate rate of mortality at each age within the retirement period, and to value the entire liability within such period on the basis of

"total cost" at each age as experienced, including both the sick pay of the active members, and the superannuation allowances of the retired; effect is thus automatically given to all the elements in the case. The proportion of active lifetime at each age will be required, in addition, in the event of the contributions ceasing when the member retires. By the aid of this proportion the sickness liability can be separated from that of pensions, if desired.

(d) Retiring allowances, *i.e.*, surrender values on withdrawal from the service, are very usual in this class of society, and generally take the shape of a percentage (sometimes a high one) of the contributions paid by the retitant during his membership, subject to the deduction of any benefits he may have drawn. This principle of calculation is, of course, utterly unsound, the maximum amounts being payable to the healthiest class of retirants, whose withdrawal is rather a loss than a gain to the society, and the minimum—frequently nothing—to the unhealthy, who are thus induced to continue as "out-members" to the society's disadvantage.

When a rate of lapse is involved in the valuation, it will be necessary to get out the average retiring allowance at each age, and value it as a benefit payable on secession. Care must be taken to allow an adequate margin, having regard to the incidence of the benefit as affecting individuals, and the consequent danger that past experience may not be repeated. If the lapse element is not involved in the valuation, some investigation should be made as to whether in any material number of cases the retiring allowances exceed the net liability. In such cases, as the retirement benefit arises under the rules, some reserve should be made for possible losses in future under this head.

(e) The rules of most of these societies contain provisions enabling members who leave the service to retain their connection with the societies under prescribed conditions. In some cases the benefits are reduced, and in some the company's contribution in respect of such members is charged to the members themselves. The important point to the Actuary is the fact that a material change occurs in the conditions under which the benefits may be drawn, and that the risks may be sufficiently enhanced thereby to more than nullify the effects of any changes in the monetary conditions. For example, in an instance previously cited, the superannuation benefit is claimable (with cessation of contributions) on retirement from the service at any age after 65, and in the case of healthy men in the service

the natural disposition to postpone retirement as long as possible retards superannuation on the average for about two years beyond the minimum age. In the case of "out-members" no such influence comes into operation, and the superannuation benefit (payable at a slightly reduced rate because the member is out of the service) is invariably claimed at 65.

In other cases, the rules and the system of supervision which, taken in conjunction with the regulations of the company, are sufficient to confine the sickness rates within certain limits, are found to be inoperative in great measure as against the claims of "out members", and a marked excess of liability presents itself in this class. In one case at least this has necessitated recognition of the "probability of becoming an out-member" with appropriate functions in respect of the subsequent benefit-rights.

(f) The conditions and experience as to transfer of class should be closely examined. Owing to the exigencies of the service, the scales of contribution are adapted as a rule but loosely to the values of the benefits, *i.e.*, the contributions being deducted from wages and passed through the station pay-sheets, fractions of pence in the weekly rates must frequently be avoided for the convenience of the company's officials, whilst for the same reason a great degree of uniformity in the weekly deductions is urged as a necessity. Contribution scales are thus constructed to be as nearly as possible sufficient in the aggregate, due regard being had to the assistance from the company. This implies adequate rates, and perhaps the existence of some negative values, at the youngest ages, at which transfers seldom take place, and insufficiency at the higher ages, at which the transfers may be frequent. The following example from an actual case will make this clear.

Class	WEEKLY RATES OF CONTRIBUTION	
	Ages 16-26	Ages 27-34
A	d. 8	d. 9
B	6	7
C	4	5

A member transferring his class pays his original contribution increased by the difference at his present age between the old and

the new scales, but this difference will be seen in case of transfer from B to A to be 2*d.* per week at all ages under 35, and for transfers from C to A, 4*d.* per week at all such ages. The aggregate effect of the transfers which took place in the years 1904-08 was to increase the liabilities by £4,676 and the assets by £3,364, a further net liability of £1,312 thus being added to the contract reserves as they stood on 1 January 1904. In the case of some funds, it is almost an invariable custom for the members to enter in youth on a low scale, and to transfer to the maximum at a more advanced age. Transfers so operating, if they arise, under the rules, at the option of the members only, should obviously be recognized in valuation.

(g) The annual subsidy of the railway company may be either a grant *per capita* or may be a fixed amount irrespective of the number of members. If the former alternative obtain, it is necessary to ascertain the basis on which the grant is computed. It may follow the weekly contributions of the members, in which event its value is $(\sum n \bar{a}_x) \times S$, but it may be a uniform annual sum per member taken on the number of members on the books at the beginning of each year, but payable in instalments throughout the year. The value of the subsidy in this event will be, approximately, $[\sum r^{\frac{1}{2}}(1 + a_x) \times n] \times S$, but to fix the formula with precision the number and dates of the instalments will be required.

If the subsidy take the form of a fixed annual grant, and if the contributions of incoming members, taken either individually or in the mass, are sufficient for the benefits, the subsidy may be regarded as a provision for existing members. In this event, if the subsidy is unrestricted as to duration (a matter upon which enquiry should be made) it may be treated as an annuity-certain for a term equal to the probable lifetime of the survivors of the youngest group, thus approximating in value to a perpetuity. If the contributions of the members are not sufficient to support the benefits (which is the more probable case), a due proportion of the subsidy must be thrown off as appertaining to future members.

This raises the difficult question of probable increases or decreases in the membership, with consequent variability in the effectiveness of the grant. If a stationary membership can properly be assumed, such a subsidy should be valued as a life annuity of the average amount per member which it yields on the valuation date.

I have seen cases in which the value of a subsidy of fixed

amount has been taken as a perpetuity, i.e., at 25 years purchase on a 4 per-cent valuation, without regard to the interests of future entrants. I cannot conceive any practical case in which this course is likely to be justifiable.

(h) Orphan benefits are provided occasionally by railway societies. Two forms of this benefit have been investigated in papers read before the Institute or in communications to the *Journal*,* namely, a temporary annuity to each child under a specified age and a temporary annuity (approximating to an annuity for a term-certain) payable until the youngest child reaches the age limit. These are more simple problems than that sometimes found in the case of railway societies, wherein the annuity varies with the number of children, but is not at a uniform rate per child. I have recently had to value a benefit which comprised an allowance up to 14 years of age of 3s. 6d. per week for a family of one child, 6s. 6d. per week for a family of three children, and 9s. 6d. per week for a family of six or more children, with intermediate amounts for two, four, or five children. I do not propose to burden this paper with the details of a problem of this class, but hope at a subsequent date to submit a note thereon to the Editor of the *Journal*.

Another point that arises in connection with these benefits is the effect of including posthumous children. It is somewhat curious that this point has not been referred to in the various papers on orphan benefits. It may have been assumed that the birth-rate in relation to husbands dying is materially less than that in relation to husbands living, but even if this be so there must be some degree of liability. This is especially the case in funds connected with railway and other services, in which, at the comparatively youthful ages where the liability in respect of young children is at a maximum, the risk of death by accident of men in good health represents a material proportion of the total death-rate. In one society which I have valued, the deaths of members at the ages 20–40, at which ages the parents of the great majority of newly-born children would be found, were 351 from disease and 50 from accident. In respect of the second group, if not of the first, posthumous births would evidently correspond closely with the normal birth-rate.

With regard to the organizations in group 7 (Deposit Societies) and group 8 (Societies of Miners and Quarrymen) I do not propose to add to the references on pp. 171–2 and 169

* See *J.I.A.*, vol. xxx, p. 291, xxxviii, p. 108, xxxix, p. 337, and xl, pp. 196, 200.

respectively, except to say, as to group 8, that the registered societies of miners are probably far outnumbered by the unregistered societies of the same class. These societies are frequently attached to particular collieries, and their financial arrangements are primitive.

OCCUPATION RISKS.

Occupation may be asserted with confidence to exercise a paramount influence in the variation of sickness claims from a common standard. It is not only the first element to which attention should be directed when a variation of any importance is discovered, but its probable effect on the future working should be recognized, even if no material departure from the standard has characterized the past experience of a society when taken as a whole. In such a case, the excess of occupation risk in certain classes may have been balanced by an abnormally light experience in the normal group, but whilst the excess of the "hazardous" section is permanent and unavoidable, the sub-standard experience of the normal group may be, and generally must be presumed to be, purely fortuitous. It is not, however, always practicable to obtain the material for a precise division of the data. Occupations change, and normally the changes are not announced to the friendly society and recorded; there are practical difficulties in the way of detailed enquiries (*vide pp. 28-9 of the Report on the Manchester Unity Experience, 1893-97*) which neither the Actuary dealing with a small society nor the officials may be disposed to encounter if another course lies open. It is consequently advisable to ascertain whether occupation excesses can be reduced to a general expression, to which resort may be had in the case of societies that are normally constituted, so far as the ages of the members are concerned, whatever may be the distribution of the members amongst the several occupation grades. In order to obtain such an expression, use has been made of three sets of figures, obtained by blending a large number of actual cases, and purporting to show the age-distribution of a lodge after 20, 40 and 70 years of existence respectively. These figures were employed in the report of the Manchester Unity Investigation, and may be deemed to give the distribution by ages of normally constituted societies. On resorting to them, to ascertain the expected sickness claims under various benefit conditions and occupation grades, the following comparative results are obtained :

Comparative Expectation of Sickness.

Assumed Occupation Group	SICK PAY								
	1 —First 52 weeks .5—After 52 weeks			1 —First 52 weeks .5 —Second 52 weeks .25—After 104 weeks			1 —First 26 weeks .5 —Second 26 weeks .25—After 52 weeks		
	AGE OF LODGE (YEARS)								
	20	40	70	20	40	70	20	40	70
A H J	100	100	100	100	100	100	100	100	100
B C D	121	121	118	122	121	118	121	121	118
E F	146	141	136	147	141	135	146	141	136
G	187	182	168	188	181	166	187	180	167

The relative distributions by age producing these results are as follows :

Ages of Members	AGE OF LODGE		
	20 years	40 years	70 years
Under 20	34	34	21
20-29	453	292	257
30-39	337	265	207
40-49	138	203	204
50-59	31	141	165
60-69	7	58	101
70-79	...	7	38
80-89	7
...	1,000	1,000	1,000

A considerable degree of uniformity in the effects of occupation risk is exhibited by the first of these tables. The differences due to variation of benefits are scarcely perceptible, and those arising from variation in the distribution by ages are by no means formidable, especially when the matter is regarded with that breadth of view which the actuary ought always to exercise in problems of this class. It would appear, therefore, that if a similar age-distribution can be assumed to obtain in respect of each class of occupations, a fair estimate of the relation which the sickness cost should bear to the standard may be readily obtained,

without recourse to detailed analysis. Thus if a society, established over 70 years ago, consist of 600 members distributed as follows : Group A H J, 300 ; Group F, 60 ; Group G, 240 ; and if there is reason to presume that the distribution of these groups with regard to age is similar, an average excess approximating to 30 per-cent of the A H J standard might be expected, and as a first step in the investigation the experience might be examined in this light. In like manner, a society of about forty years standing, with a membership divided equally between groups B, F and J, each being similarly distributed as to age, might look for an average excess over the whole body of 21 per-cent of the A H J expectation. Within the limits here defined, this method of regarding the subject may be particularly useful in the case of small societies. In such cases, the paucity of the data will frequently divest the results of more formal enquiries of real significance.

Since the issue of the report on the investigation above referred to, I have had occasion to observe, though in less wealth of detail, the further experience of the Manchester Unity in respect of occupation risks. The following statement shows the results of a comparison of actual and expected sickness claims during the last quinquennial period. This begins and ends on different dates, according to the locality of the branch, but the earliest year included is 1900, and the latest 1907.

It will be understood that, as many lodges comprise members engaged in occupations attended by much variety of risk, it has been found impossible in this classification to obtain absolutely clear-cut distinctions between non-hazardous and hazardous cases, and that the formation of some "mixed" groups has been necessary. The reference letters after the description of each group identify them with the corresponding sections in the report on the experience of the period 1893-97.

1. Agricultural and Country Lodges; no special risks (A and H).
2. Town and City Lodges; no special risks (J).
3. Miscellaneous Occupations involving varying degrees of extra risk (B to G and J).
4. Agricultural and Hazardous Occupations (A and H, and B to G).
5. Mainly Iron, Steel and Chemical Work and other occupations of this type (F).
6. Mainly Quarry Work (E).
7. „ Mining (G).

Occupation Group	Expected Cost (1866-70 Standard)	Actual Cost	Percentage of Actual to Expected
	£	£	
Group 1	1,367,433	1,418,450	104
" 2	998,657	1,087,293	109
" 3	296,306	369,223	125
" 4	127,030	153,482	121
" 5	31,367	43,956	140
" 6	8,151	12,398	152
" 7	99,589	158,430	159
Totals	2,928,533	3,243,232	111

Some of these experiences evince an extent of liability in respect of which it is impossible to obtain corresponding contributions. Why this should be so it is difficult to explain, but there seem to be conventional limits beyond which in no circumstances will the industrial classes permit their contributions to friendly societies to pass. It may or may not be a question of ability to pay—doubts on this point are engendered by the circumstance that the best friendly society members are to be found in the rural districts where the wages of regular work are lowest—but the fact remains that under the conditions now prevailing it is impossible to establish a friendly society with hope of success in the mining, ironworks, or quarry districts, or to look for the maintenance, for a prolonged period, of most of the societies now operating in these industrially important localities. In circumstances of such gravity, I have been impelled to the conclusion that the only advice that can be given is that the sick pay be limited to cases of sickness, and that in cases of industrial accident the members be left to rely upon their statutory claim upon employers. To some extent this recommendation has been based upon surmise, because until very recently no effort was made by friendly societies to separate the sickness and accident claims. This defect is now being overcome, and I am enabled to submit certain data, not all, of course, in the form one would like to obtain it, which go far to show that the bulk of the extra sickness cost in the groups of heaviest liability is due to accidents of employment in respect of which compensation is paid. As the returns all tell much the same tale, I ought, perhaps, to say that so far as I am concerned they are not the result of selection, but represent practically everything of authority that has come into my hands. It is possible, of course, that an element of

selection lies in the tendency of secretaries to keep records of accident cases if they have reason to think that their societies are specially affected by such claims. This consideration, however, cannot apply in any degree to either of the two large railway societies in which the accident allowance has for many years been a separate benefit ; whilst generally, I think, having regard to the universality of accident claims disclosed by the experience of accident insurance offices, employers' mutual funds and "contracting-out" schemes, that there is very little force in it.

CASE A.—*Railway Society. Experience 1904-08.*

Ages	Years at Risk	Actual weeks of Sick Claim	Actual weeks of Accident Claim	Expected weeks of Claim, M.U. Experience 1893-97, Group A II J
15-19	760	611	386	651
20-24	4,749	4,028	1,822	3,890
25-29	6,657	5,821	2,934	5,818
30-34	6,838	7,326	3,075	6,694
35-39	5,955	6,650	2,750	6,926
40-44	5,114	5,992	1,914	7,548
45-49	4,736	6,657	1,925	8,786
50-54	3,114	7,241	1,658	8,233
55-59	2,278	9,487	1,610	8,912
Totals	40,201	53,813	18,074	57,458

Ages	PROPORTION OF MEMBERS CLAIMING		AVERAGE CLAIM FOR SICK PAY		AVERAGE CLAIM FOR ACCIDENT PAY	
	Sick Pay	Accident Pay	All Members	Sick Members only	All Members	Disabled Members only
15-19	.313	.138	.804	2.567	.508	3.676
20-24	.288	.126	.848	2.940	.384	3.042
25-29	.273	.127	.874	3.202	.441	3.460
30-34	.277	.114	1.071	3.862	.450	3.952
35-39	.265	.102	1.117	4.206	.462	4.516
40-44	.257	.082	1.172	4.567	.374	4.546
45-49	.259	.084	1.406	5.421	.406	4.861
50-54	.293	.083	2.325	7.931	.532	6.377
55-59	.360	.086	4.165	11.555	.707	8.173

- NOTE—(1) The experience after age 60 is materially influenced by pension arrangements of an exceptional character, and is therefore omitted from consideration.
- (2) The "expected weeks" have been calculated by the rate applicable to the central age of each group.

It will be observed that the sickness experience, apart from accident claims, is distinctly below the expectation of the A H J or normal group of the Manchester Unity; in this case, therefore, all excess is traceable to accident claims.

The composition of the accident rates is of interest. Although the average claim is very nearly constant at all ages under 50, it is seen to be the resultant of two factors which are distinctly variable with age. The proportion of members claiming accident pay is at a maximum at the youngest ages, and diminishes gradually to a minimum at about 40. This economic advantage, resulting apparently from the steady influence of increasing age and experience, is neutralized by the lengthening duration of the average period of incapacity in the case of those laid aside.

CASE B.—*Railway Society, Experience 1901-5.*

Ages	Years at Risk	Actual weeks of Sick Claim	Actual weeks of Accident Claim	Expected weeks of Claim, M.U. Experience 1893-97, Group A H J
-19	8,401	9,745	1,831	7,200
20-24	25,682	34,974	6,900	21,034
25-29	29,794	37,488	8,131	26,040
30-34	29,672	38,291	8,650	29,049
35-39	19,970	25,829	5,570	23,224
40-44	14,656	21,882	4,528	21,633
45-49	13,692	26,624	4,273	25,398
50-54	11,841	31,265	5,141	31,309
55-59	7,956	29,706	4,694	31,124
60-64	3,918	24,834	2,700	24,413
Totals	165,582	280,638	52,418	240,424

Ages	AVERAGE CLAIM (ALL MEMBERS)			Average Claim, M.U. Experience 1893-97, Group A H J
	Sick Pay	Accident Pay	All Pay	
-19	1.160	.218	1.378	.857
20-24	1.362	.269	1.631	.819
25-29	1.258	.273	1.531	.874
30-34	1.290	.292	1.582	.979
35-39	1.293	.279	1.572	1.163
40-44	1.493	.309	1.802	1.476
45-49	1.914	.312	2.256	1.855
50-54	2.640	.434	3.074	2.644
55-59	3.734	.590	4.324	3.912
60-64	6.338	.689	7.027	6.231

The "expected weeks" have been calculated by the rate applicable to the central age of each group.

The experience in this case is considerably different from that of case A, although both bodies of lives are drawn from persons engaged in the same class of occupations. Incidentally, this illustrates the importance of obtaining the actual experience of all large organizations as the basis of valuation. The sickness experience in case B is much above that of Group A H J of the Manchester Unity data at ages under 40, but after this age the experience is fairly close to Group A H J. As the sickness experience on the whole is so much above that of case A, an equal pressure, at least, from accidents might have been looked for. The contrary however is found, and it will be observed also that in this case an increase in the rate of accident claim from about age 40 is exhibited.

In view of the differences between the claims upon the two societies, I have made careful enquiry as to the respective systems of record, and am satisfied that in neither case have claims for accident or sickness been wrongly scheduled. It is worthy of mention, having in view my subsequent remarks as to the influence of the amount assured, that, in addition to their benefits from this society, the members assured in case A are also guaranteed in the event of accident a benefit from an accident fund of amount considerably exceeding, in many cases, the statutory half-wage.

The foregoing experiences may be compared with Group C of the Manchester Unity Experience. This group comprised railwaymen only, and excluded such as were not directly engaged in the transport service. The exposures amounted to barely 50,000 years; the experience of the group was thus insufficient for separate treatment, and was amalgamated with Groups B and D. It was accordingly not separately published, and the following abstract embodies figures not to be found in the official report.

Up to age 59, the excess over the normal expectation exhibits a marked tendency towards a constant, the average being .3 of a week. This value is almost identical with the average accident claim rate exhibited by the foregoing case B. The excess in question may, therefore, be assumed with some confidence to represent the actual accident experience included in the Manchester Unity Group C. The sudden increase in the excess after the age of 60 cannot be so regarded. Such increase sets in with severity at the precise age 60, and in my opinion is due to the retirement regulations of some of the large railway companies. It is of particular interest, not only as showing how the financial

GROUP C. M.U. EXPERIENCE, 1893-97					
Ages	Exposed to Risk	Expected weeks of Sickness, Group A H J	Actual Weeks of Sickness	Excess Weeks	Average excess per Member (Weeks)
-19	1336	1,121	1,462	341	.255
20-24	7002	5,743	8,154	2,411	.344
25-29	8210	7,137	9,461	2,324	.283
30-34	7832 ⁵	7,565	9,513	1,948	.249
35-39	6909	7,884	10,972	3,088	.417
40-44	5933	8,530	10,399	1,869	.315
45-49	4537 ⁵	8,200	9,132	932	.205
50-54	3168	8,023	8,774	751	.237
55-59	2228	8,329	8,779	450	.202
60-64	1426	8,335	11,311	2,976	2.087
65-69	637 ⁵	6,273	7,988	815	1.278
70-74	358	5,959	7,406	1,447	4.042
75-79	195	4,800	5,700	900	4.615
80-84	57	1,792	2,192	400	7.018
85-89	7.5	265	190	- 75	
Totals	49,837	89,956	110,533	20,577	...

arrangements of one class of society may operate on the experience and on the position of another class, but also as indicating the probable effect at the age of 70 of the Old Age Pensions Act on all friendly societies.

CASE C.—Branch composed, as to one-half of miners, and as to one-half of persons following miscellaneous occupations. Staffordshire. Experience, 1904-08.

Ages	Exposed to Risk	Expected weeks of Sickness, M.U., Group A H J	ACTUAL WEEKS	
			Sickness	Accidents
-19	301	256	321	171
20-29	1,285	1,080	1,055	1,038
30-39	1,251	1,306	947	980
40-49	542	857	667	646
50-59	260	785	949	253
60 and over	114	1,194	1,912	68
Totals	3,753	5,478	5,851	3,156

Accident Experience only, 1898-1908.

Ages	Exposed to Risk	Number of Claims	Number of Weeks	Proportion of Claimants to total Members	AVERAGE CLAIM		Average cost (all Members) allowing for reduced pay in protracted cases
					All Members	Disabled Members only	
-19	782	124	320	.159	409	2.58	.253
20-29	3,029	446	1,643	.147	542	3.68	.299
30-39	2,402	345	1,378	.144	574	3.99	.323
40-49	1,018	138	855	.136	840	6.20	.426
50-59	451	47	336	.104	745	7.15	.315
60 and over	212	12	84	.057	396	7.00	.123
Totals	7,894	1,112	4,616

This is a typical mining branch. It has been established for 75 years in an old coal-mining district, and it is safe to assume that the 50 per-cent of miners amongst the membership are distributed in normal proportion over all ages. If the accident experience be exclusively due to mining claims, the average rate of claim above shown should be doubled to show the accident liability in respect of miners. It is probable that a very large proportion of the accident claims does come from the mining group, though the data are not sufficient to establish this. It is clear that with the accident liability the claims are almost unmanageably high, but that without such liability they would present no financial difficulty.

The accident experience exhibits the same interesting feature as that shown in case A, the proportion of accident cases diminishing with advancing age, but the average duration of claim increasing. The gradual rise of the average claim to a maximum in the group 40-49 and its subsequent decline is probably not significant, despite its symmetrical appearance.

CASE D.—*Branch of about 600 members, 75 per-cent being miners. Staffordshire. Years 1904-08.* Weeks.

Weeks of Sickness, full pay	3,141
Weeks of Sickness, half pay	4,780
Weeks of Sickness, equivalent at full pay throughout to	5,531
Weeks of Accident, full pay	1,672
Weeks of Accident, half pay	1,766
Weeks of Accident, equivalent at full pay throughout to	2,555
Total	8,086
Percentage of total cost arising from accident claims	31.6

The total sickness cost of the quinquennium, 1902-06, was £3,160, as compared with an expectation by Group A H J of £2,140. The excess here found is 32·3 per-cent of the total cost, a proportion which corresponds very closely with that of the accident cost for the years, 1904-08.

CASE E.—Branch of about 160 members, about 40 per-cent being miners. South Wales. Years 1905-08.

Weeks of Sick Pay	1,850
Weeks of Accident Pay	379
Total	<u>2,229</u>

Percentage due to accidents, 17.

The total sickness cost of the quinquennium, 1903-7, was £993, as compared with an expectation by Group A H J of £768. If 17 per-cent (£169) be deducted from the total cost for accident claims, the excess over the expectation is reduced to about 7 per-cent of the total.

CASE F.—Branch of about 250 members, 50 per-cent of whom are engaged in mining and occupations connected therewith. Leicestershire. Years 1907-08.

Weeks of Sick Pay	431
Weeks of Accident Pay	213
Total	<u>644</u>

Percentage due to accidents, 33.

The total sickness cost of the quinquennium, 1901-5, was £670, as compared with an expectation by Group A H J of £557. It may be inferred, therefore, that apart from accidents, the sickness cost was within the expectation.

CASE G.—Branch of about 200 members, whereof 25 per-cent were pottery workers and 20 per-cent miners. Staffordshire. Years 1907-08.

Total Expected Weeks of Sickness by Group A H J	756
Actual Weeks, Sickness only	856
Actual Weeks, Accidents only	170
Total	<u>1,026</u>

CASE H.—Branch of about 500 members, employed as iron-workers, railway employees, general labourers, and in the textile industry. Manchester. Years 1906-08.

Weeks of Sickness Claim	1,397
Weeks of Accident Claim	448
Total	<u>1,845</u>
Percentage due to accidents, 24.	

The total sickness cost of the quinquennium, 1901-5, was £1,565, as compared with an expectation by Group A H J of £1,384. If, in agreement with the experience of the three following years, about one-fourth of the total claims arose from accidents, the balance is well within the expectation, thus corresponding with the usual experience of friendly societies composed largely of textile workers.

CASE K.—District of 9 branches with about 3,000 members, iron smelting locality. North Yorkshire. Years 1903-07.

Exposed to risk 14,793	Sickness Claims £10,042
	Accident Claims 2,649
Total	<u>£12,691</u>

Percentage due to accidents, 21.

The expected sickness cost in this period was 75 per-cent of the actual cost. Little excess would remain, therefore, after the elimination of accident claims. The division of the lives at risk in the 1893-97 investigation was as follows:

Groups B, C, D	419
Group F	1,047
Group J	1,408
Total	<u>2,874</u>

It may be assumed that the occupation liability of iron smelters and others in Group F accounted for the bulk of the accident claims.

CASE L. — *Branch of about 350 members, largely dock labourers, watermen, goods porters, and general labourers. Cheshire. 4½ Years—January 1905—June 1909.*

		Weeks	Cost
Total Sickness Claims	.	4,478	£1,429
Total Accident Claim	.	1,118	473
Total	.	<u>5,596</u>	<u>£1,902</u>
Percentage due to accidents	.	20	25

The pressure of numerous accident claims for short periods, chiefly at the full pay rate, is here illustrated. Although the weeks of accident claim represent but 20 per-cent of the total "time sick", the cost is equal to 25 per-cent of the total claims.

In the quinquennium 1900-04, the expected sickness cost of this branch was about 80 per-cent of the actual cost. If the same proportion obtained in the years following 1904, the difference between actual and expected cost would be fully accounted for by the accident claims.

CASE M. — *Three branches of a district in an old coal-mining locality near Manchester. Total number of members about 1,300. Years 1906-8.*

Class of Occupation	Exposed to Risk	Weeks of Accident Pay	Weeks of Sick Pay	Average weeks Accident Pay per annum	Average weeks Sick Pay per annum
Colliers . . .	1,478	1,912	2,394	1·29	1·62
All other classes	2,311	492	2,782	·21	1·20

Average annual claim (all causes) : Colliers . . . 2·91 weeks.
 " " " " Other classes . 1·41 "

The distribution by age may be assumed to be similar in both groups. No comment on the figures is necessary. It should be said that the separated experiences comprise full pay and half pay covering the first two years of claim. Quarter pay claims represented 3,868 weeks in all, but owing to lack of uniformity in tabulation these cannot be analyzed completely.

It is found, however, as to one case of the three, that accidents in all occupation groups accounted for 311 weeks, against 507 weeks arising from sickness, whilst in another of the three cases, about 80 per-cent of 1,540 weeks of quarter pay went to the colliers who, as a group, did not contribute more than 46 per-cent of the exposures to risk. It is safe to assume therefore that nothing in the quarter pay experience would tend to invalidate the conclusions arrived at on the experience of full and half pay.

The total accident *cost* comprised by the above figures is £1,049, and the total sickness cost £2,495, in all £3,544. Accidents thus represent about 30 per-cent of the total claims. The actual claims (full and half pay) from all causes in the years 1901-5 were £5,931, against an expectation of £3,903, the excess over the latter thus being 34 per-cent of the total. On the analyzed experience in respect of the succeeding years, it may safely be assumed that accidents brought nearly the whole of the excess. Such excess as did not arise from accidents seems to be traceable to the relatively high *sickness* claims of colliers.

The foregoing cases, although insusceptible of summarization for the purpose of obtaining general accident rates, are sufficient to indicate the heavy burden of this liability. In the main, they afford ample support to the conclusion that if the accident benefit were eliminated, the remaining cost, although, on the whole, somewhat above the normal sickness rate, would be provided for by such contributions as generally are obtainable from the members of friendly societies.

Information of a character confirming the above conclusions has been obtained by the Ancient Order of Foresters, which society applied for a return of claims from each branch that in the last quinquennium exhibited an excess of sickness cost, to the extent of 30 per-cent or upwards, over the expectation of Neison's Foresters' Tables, 1871-75. The number of returns asked for is not stated, but the number obtained is given as 115. These showed an aggregate exposure to risk of about 105,000 years. Claims for ordinary sickness numbered 13,492, and for accidents 3,305. The disbursements were: for ordinary sickness £89,621, and for accidents £20,829, the respective percentages being 81 and 19.

OTHER ELEMENTS AFFECTING SICKNESS.

Whilst accidents are shown to be a potent element in the sickness experience of many societies, they do not by any means account for the whole of the excess discovered. The claims in old age, compared with the expectations grounded on the early standards, are known to be remarkably heavy. This is a universal experience; it must be accepted, and can only be met by the adoption of tables applicable to present circumstances. There are other causes at work, however, and some of these ought to be susceptible to remedial measures. In a paper submitted to the Institute some years ago (*J.I.A.*, vol. xxxv, p. 291), I showed that in a large experience the numbers of cases of high sickness varied directly with the size of the society, so that, amongst branches composed of fewer than 80 members, the cases of excess were 41 per-cent of the whole, whilst among branches of 500 members and over, the cases of excess were 82 per-cent; and I showed, further, that this applied not only to the whole body, but to practically all of eight divisions into which the 3,581 branches were separated with reference to occupations. The inference drawn was that claims varied with the efficiency of supervision, a self-evident proposition perhaps, but one which was worth demonstration from such a conclusive body of material. There is another factor which supervision can only partially reach, but which is responsible, I am convinced, for much mischief to the friendly society system. I refer to excessive insurance. On this point I have obtained some interesting evidence. In one case (K in the foregoing list) the sick pay is in three scales, of which the full benefits are respectively 10*s.*, 15*s.*, and 20*s.* per week. The following are the respective percentages of actual to expected sickness cost, the latter being based on the experience of 1866-70:

Scale 1 (10 <i>s.</i> per week, &c.)	128
Scale 2 (15 <i>s.</i> per week, &c.)	184
Scale 3 (20 <i>s.</i> per week, &c.)	203

Scales 2 and 3 have been introduced in recent years, and include very few assurances at ages over 50. Any excess due to pressure of old age claims tells therefore against Scale 1 only, and renders more significant the super-excess of Scales 2 and 3. Going from a mineral-working and manufacturing district in the north of England to an agricultural district in the south, the following experience of a large county society shows precisely the same features :

Full Sick Pay per week :

- 6*s.* and under—full pay cost 75 per-cent of expectation, half pay 36 per-cent.
- 7*s.* and 8*s.*—full pay cost 77 per-cent of expectation, half pay 36 per-cent.
- 9*s.* and 10*s.*—full pay cost 85 per-cent of expectation, half pay 158 per-cent.
- 11*s.* to 13*s.*—full pay cost 90 per-cent of expectation, half pay 96 per-cent.
- 14*s.* to 16*s.*—full pay cost 100 per-cent of expectation, half pay 150 per-cent.
- 18*s.* and over—full pay cost 134 per-cent of expectation, half pay 196 per-cent.

Such experiences as these afford full justification for the attempts made by some societies to limit the maximum assurance to a fixed proportion of the average weekly wage. It is matter for regret that this practice is not more general. It is useless to think that a sickness assurance can, with success to the assuring body, be made a complete indemnity against loss of income. If the claims are to be kept within reasonable limits, the financial inducement towards recovery must be maintained. On this point the statement of Mr. Tidd Pratt, the then Registrar of Friendly Societies, made in the course of evidence given by him in the enquiry of the Poor Law Commissioners of 1834, remains true—which indeed is only to say that human nature remains unaltered. Mr. Tidd Pratt was asked: “In these institutions, is the condition of a member receiving relief, or living without work, ever allowed to be as eligible, on the whole, as the condition of a member living by his work”? He answered: “In most cases the allowances made by the societies are so adjusted as to make it the interest of every member not to receive relief from the society so long as he can earn his usual wages. The average allowance which they make is about one-third of what a member can earn. Thus, if the average earnings of the members of a benefit society were £1. 4*s.* a week, the allowance in the case of sickness would be, on an average of the whole time of the sickness, about 8*s.* a week. During the last Session, Mr. Slaney brought in a Bill for the purpose of sanctioning the formation of societies for the relief of members when out of employment. At his instance

" I made enquiries amongst some of the most intelligent and
 " respectable of the labouring classes as to what should be the
 " extent of allowance to those who were out of work. I
 " suggested to the parties that one-half the usual wages might
 " be a proper allowance. The unanimous reply of all the
 " operatives with whom I conversed on the subject was that an
 " allowance of one-third would be ample, and that more than
 " that would only induce the members to continue on the society
 " rather than endeavour to find work."

There is sometimes found another type of excess which requires and is amenable to suitable treatment. This is illustrated by the three following cases, the first being the North Yorkshire district designated "K" above, the second a branch of such district, and the third a section of a large district in South Wales, with full pay assurance of 20s. per week:

Class of Sick Pay	PERCENTAGES OF 1866-70 EXPECTATION		
	(1)	(2)	(3)
Full Pay	141	111	140
Half Pay	276	495	182
Quarter Pay	84	35	79

In each of these cases, it may be assumed that the compulsory period "off the funds" after a member has exhausted the benefits at the higher rates, before he is eligible to resume such benefits, is too short, leading to partial avoidance of the quarter pay and undue recurrence of claims at the higher rates. The remedy may be found generally in such an extension of the "off" period as to make it a matter of financial indifference whether the member continues on the funds at quarter pay, or indulges in a series of intermittent claims for the larger benefits.

Case "K" well illustrates the complexities inseparable from the administration of a sickness risk in some urban localities. Not only is the pressure from accidents heavy, but there is much insurance of disproportionately high benefits, and undue resort to full and half pay, in defiance of the undoubted intention that the benefit at these rates shall only be drawn once for the prescribed period in respect of each attack.

VARIATIONS OF STANDARD VALUES.

The sickness experience of small friendly societies must of necessity be in a constant state of oscillation, and it may be useful

to review the principles on which the adjustment of standard values in particular cases should proceed. It has always been recognized by actuaries that a standard table ought not to be applied automatically to every society, without regard to its actual experience; but, excepting one of its phases, namely, the "probable error" or average deviation investigated in Mr. G. F. Hardy's chapter on Fluctuations in Death and Sickness Rates (*J.I.A.*, vol. xxvii, p. 321), the question of variations from the standard does not appear to have evoked formal consideration. I have an impression that at times adjustments have been made in consequence of past excess, without special regard to the causes of that excess and the possibilities as to its avoidance in future. In this view I am supported by some remarks of Mr. R. P. Hardy, who from his ripe experience pointed out the danger of debiting a society with the assumed continuance of an undue past experience "which better management might improve."* In recent years the adjustment of table values by more or less "rule-of-thumb" methods has entered largely into unprofessional valuations, and I have shown that in one prominent case the form and extent of adjustment is directed by the rules of the organization concerned. It is worth consideration whether such adjustments can be reduced to a scientific system.

My own opinion is that excess of claims does not necessarily demand alteration of the standard values, that the probable causes of the excess must be searched out, and variation admitted only for such as are judged to be permanently operative. Given a valid standard, the chief factors productive of permanent excess are occupation risks (using the term in a wide sense) and ineffective rules, either as to the circumstances in which benefit may be drawn, e.g., the allowance of reduced pay with liberty to follow some light employment, or the imposition of an inadequate "off" period after prolonged claims. Where none of these factors operate, it is reasonable to assign an unfavourable experience to either (1) the relative efficiency of supervision, whether managerial or medical, or (2) accidental fluctuations. Both factors may be in operation together, and possibly in conjunction with one or more of the causes of permanent excess, but I submit that variations resulting from neither the quality of the supervision nor ephemeral conditions should be treated as permanent features of the experience.

* *J.I.A.*, vol. xxxv, p. 328.

The primary difficulty of resolving a past experience into its elements arises on the attempt to find a really valid standard. The aggregation of a large number of experiences involving much variety in claim conditions, occupations &c., e.g., the Friendly Societies Experience, 1876-80, would evidently not give such a standard, nor would the Manchester Unity Experience, 1893-97, Whole Society, although, from the greater degree of homogeneity in the regulations under which the claims arose, this experience might approach nearer to the ideal than that first named. A further step toward the essential uniformity is intended to be gained by the A H J Group of the Manchester Unity Experience, from the compilation of which the cases of marked excess believed to be due to occupation causes have been excluded, but even this group is in a measure heterogeneous, because division and subdivision must have their limits in practical operations. On p. 206 are shown the actual and expected sickness claims in a large number of branches, divided with reference to environment, and to the principal occupations followed. It will be seen that in No. 1 Group, composed of rural branches non-hazardous occupations, the claims were 104 per-cent of the expectation, and in No. 2 Group—the corresponding urban section—109 per-cent. Here, therefore, is one element of discordance in the data, and one apparently of a permanent character, though not of sufficient weight to have invalidated the combination of the two groups for practical purposes. But it is possible to go further, and to split up the respective groups according to the "size of lodge", a factor in the experience to which I have previously alluded. On this being done, the 1,697 branches in Group No. 1 and the 1,057 in Group No. 2 being divided respectively into three sections, the following variations appear :

Number of Members	PERCENTAGE OF SICKNESS COST TO EXPLOITATION BASED ON 1866-70 STANDARD	
	Rural	Urban
Under 150	97	104
150-249	103	108
250 and over	103	110

The average over the whole was 106 per-cent, and this may be taken to correspond nearly with 100 per-cent on the 1893-97

standard. If therefore this, or any subsequent *modern* standard be taken as the basis (a point of importance), it would seem that in the case of a friendly society of normal character a moderate variation, more or less calculable, may be ascribed to the permanent factor of locality, and the less certain but possibly permanent factor of relative facility of supervision.

In the case of a rural society of, say 100 members, the specific liability might be taken with some degree of confidence as, say

95
106 of that shown by the A H J Group of the Manchester Unity Experience, thus representing an abatement from the full standard values of 10 per-cent, whilst in an urban society of, say 300 members, a surcharge of 5 per-cent on that standard might be assumed to give the specific liability. In default of material for more extensive differentiation, these may be regarded as the limits of what I will designate "standard variability", as to sickness. Whether such quantity should find expression in the valuation estimates is a separate question. It is to be remembered that the rural mortality is somewhat lower than the average of rural and urban combined, and that, if the tables based on the latter are used, a moderate gain on sickness rates is required as the corrective of the effect of longevity. Similarly, in an urban society, a slight excess in the sickness rate may be deemed to be counterbalanced by shortened duration of life.

An estimate of the effect of this counter-play of forces may be obtained in the following way. Taking the Manchester Unity Experience, 1893-97, Areas 1, 3 and 2, Sickness Group A H J throughout, we obtain a series involving constant sickness with variable mortality. The reserves for certain benefits on these bases, and assuming a membership of 291 distributed as to age in accordance with the table on p. 204 are as follows :

Mortality Area	Sick pay.			Sick pay		
	1 per week. First 12 months	.5 per week. After 12 months	Death Benefits and Contributions corresponding	1 per week. First 6 months	.5 per week. Second 6 months	.25 per week. After 12 months
	AGE OF SOCIETY (YEARS)			AGE OF SOCIETY (YEARS)		
	20	40	70	20	40	70
1	6,760	10,529	12,798	4,808	7,392	8,877
3	5,809	9,298	11,387	4,312	6,741	8,134
2	4,858	8,070	10,006	3,816	6,691	7,398

The differences *down* the columns, it may be repeated, are due to variability of mortality alone.

In order to produce the same reserves on each basis, it is necessary that the variation of mortality be corrected by a variation of sickness, light mortality being combined with light sickness, and heavy mortality with heavy sickness. The uniform percentage variations in sickness rates necessary to produce this equality of reserves are as follows :

Area	HALF-PAY SCALE			QUARTER-PAY SCALE		
	20	40	70	20	40	70
1	- 6·1	- 6·9	- 7·4	- 4·2	- 5·0	- 5·5
3
2	+ 7·5	+ 8·6	+ 9·2	+ 5·0	+ 6·1	+ 6·7

It will be seen that the age of the lodge is of less significance in this connection than the scale of benefits.

Now, the differences in mortality which produce the above effects produce also the following differences in the values of \bar{e}_x :

	Age 35	Age 45
1	32·86	25·01
	$\Delta_a = 1·71$	$\Delta_a = 1·63$
3	31·15	23·38
	$\Delta_b = 2·08$	$\Delta_b = 1·82$
2	29·07	21·56

But the difference we are desirous of investigating is not that between areas, but between rural and urban sections of the same area, say for example Area 3, and the values of \bar{e}_x in this case are :

	Age 35	Age 45
Rural	31·79	23·89
	$\delta_a = .61$	$\delta_a = .51$
Combined	31·15	23·38
	$\delta_b = .51$	$\delta_b = .40$
Urban	30·64	22·98

The respective values of δ are much less than the corresponding values of Δ , and it may be assumed that the difference in reserves at each age corresponding to δ will be $\frac{\delta}{\Delta}$ of those corresponding to Δ . Taking the average age of the three assumed societies as 32, 38 and 42, the several values of the ratio become, approximately

$$\frac{\delta_a}{\Delta_a} = .39; .36; .33.$$

$$\frac{\delta_b}{\Delta_b} = .25; .23; .22.$$

Hence, to produce the same reserves on rural and urban mortality experiences as are yielded by the combined data, the adjustments of sickness rates, negative and positive, shown by the table of percentages given above, should be multiplied by these proportions. Such percentages then become :

Section of Area	HALF-PAY SCALE			QUARTER-PAY SCALE		
	20	40	70	20	40	70
Rural . . .	-2·4	-2·5	-2·4	-1·6	-1·8	-1·8
Combined
Urban . . .	+1·9	+2·0	+2·0	+1·3	+1·4	+1·5

Thus it would appear that, to counteract the effect of using a combined "area" mortality table for a rural society, the sickness rates experienced should be about $2\frac{1}{2}$ per-cent *below* the standard if continuous half pay is allowed, and from $1\frac{1}{2}$ to 2 per-cent *below* the standard if the continuous benefit is quarter pay. On the other hand, to counteract the effect of using a combined "area" table for an urban society, the sickness rates experienced should be about 2 per-cent *above* the standard in the one case, and about $1\frac{1}{2}$ per-cent *above* the standard in the other. Only such part of the quantity representing standard variability in sickness, as remains after these adjustments have been made, should be allowed effect in valuation ; it will generally be considered, I think, that (with some exceptions in the rural class) such remainder is too small

to require consideration, and that the standard values may be used with confidence as giving the nearest approach to accuracy that is attainable.

Excess arising from occupation risks in any definable group should be recognized as a matter of course, and, since the rules are the basis of the contract, any excess attributable to the operation of imperfect rules, if at all measurable, should also be allowed for. Such variations as may then remain should be ascribed to either fluctuations or other fortuitous elements, and should be excluded from the valuation estimates.

Whether a particular residuum of excess is definitely due to fluctuations, or slackness of management, or the proportions in which it may be due to both causes in conjunction, are problems which appear insoluble. Mr. G. F. Hardy has suggested means whereby the probable error (or average annual fluctuation) may be reached, but the probability that a particular residuum is due to fluctuation would appear to be another matter. I should not like to suggest (for the facts are admittedly otherwise) that the whole of the cases of excess contained in the Manchester Unity data are due to fluctuations, and that probabilities relating to this factor can be measured with confidence by reference to the "spread" of the data. It may be of interest, however, to exhibit the composition of a body of experience from which, chiefly, a standard sickness table has been evolved, and this is done in the following summary. It should be explained as the reason for using the table of 1866-70 as the standard of measurement that this table was employed as the basis of the valuation in connection with which the experiences were analyzed. The figures relate in all cases to a quinquennium within the years 1900-07. The number of lodges is reduced from the 2,754 previously mentioned to 2,729 by excluding a few established just before the valuation date, in which the experience, owing to extreme paucity of numbers, was obviously erratic.

The abnormal number of cases at the end of the table is traceable to the inclusion of "border line" lodges which might as fairly have been included in other occupation groups. Some of these are situate in districts where hazards of occupation are pronounced, and whilst the descriptions of employments given on the returns by the secretaries have led to their inclusion in the normal group, their experience suggests that they might more appropriately have been placed elsewhere.

Manchester Unity Lodges. Sickness Experience, 1900-07
(Normal Occupation cases only).

Percentage Sickness of 1866-70 Standard	RURAL			URBAN			Total
	Under 150 members	150 to 249 members	250 members and over	Under 150 members	150 to 249 members	250 members and over	
	Sickness Cost : Average Percentage of the Group to 1866-70 Expected						
	97	103	106	104	108	110	
Number of Lodges							
Under 30	2	1	3
30-39	3	4	7
40-49	20	9	29
50-59	21	3	1	19	2	...	46
60-69	66	14	6	31	6	1	124
70-79	108	45	24	40	18	12	247
80-89	105	69	54	42	31	33	334
90-99	129	85	109	71	46	53	493
100-109	95	81	110	62	51	73	472
110-119	76	63	86	60	36	67	388
120-129	44	41	51	28	33	55	252
130-139	29	22	27	33	27	22	160
140-149	21	11	15	22	11	10	90
150-159	11	9	3	10	3	2	38
160-169	8	3	...	8	5	...	24
170-179	5	2	1	...	8
180 and over	3	10	1	...	14
Total	746	446	486	452	271	328	2,729

The general tendency of the figures in each of the above columns is such as to suggest that a value can be assigned to the probability that the experience of a society of the normal class will be found divergent from the standard to any given extent. If this were so, it might be advantageous, in dealing with a particular case of high sickness, to contrast the realized fact with its antecedent probability. But bearing on this point other important factors must be considered.

One of these, the personal element, is referred to above. In the Manchester Unity, there are two definite types of lodge in which the rate of sickness is deflected by this element from the general average. In lodges of the first type, the recorded sickness is below the average, because a large proportion of the members, professional men, tradesmen, supervisors of labour, &c.,

habitually decline to claim. In lodges of the second type, composed (especially in recent years) of a different class, the accumulation of large amounts of funds has engendered a mistaken feeling of ease, leading to indulgent supervision, or ineffective rules, or inordinately large insurances, have brought about the same result, namely, excessive claims. These two sections tend to counterbalance each other, but their existence leads to a greater variation from the mean on either side (regarding the experience of the lodge as the unit) than would arise if all incapacity and nothing beyond it were represented by the claims.

The other factor in this question is the scale of benefits. The relative total cost of sickness depends partly upon the scale of reductions in protracted sickness, and it is quite possible that two societies meeting with identical experience might be found in different groups, because one society allows continuous half pay, and the other diminishes the protracted benefit to quarter pay. I have looked up some cases of quarter pay experience, and cite the following examples of this variation:

- (a) The relation of total cost to expectation was 99 per-cent, but the sick pay was so distributed, as to period, that if the continuous benefit had been half pay the relation to the standard would have been 113 per-cent.
- (b) The relation of the cost to expectation, with quarter pay included, was 104 per-cent, but in the alternative event would have been 109 per-cent.

The scales of benefit are too numerous to permit of discrimination in the table with respect to this element.

The probability that a given variation will arise in a particular case is thus a somewhat elusive quantity, and analytical methods, such, for instance, as the use of the normal curve of error, to which apparently each series should approximate if fluctuation were the only disturbing element, afford little, if any, assistance in reaching it. Perhaps the only use to which the several series can be put is to illustrate the "standard variability" of sickness as above defined, and this is exhibited best by a statement of the following form. In preparing this statement, the numbers previously given have been re-grouped round the general average of 106 per-cent (1866-70), which is called 100 per-cent of the later standard. The 14 abnormal cases of 180 per-cent and over have been eliminated for the reason given above.

Manchester Unity Experience.
Proportionate distribution of the Lodges with reference to the general average (100 per cent) of sickness cost.

Percentage of Sickness to Standard	RURAL			URBAN		
	Under 150 Members	150 to 249 Members	250 Members and over	Under 150 Members	150 to 249 Members	250 Members and over
	£000	£000	£000	£000	£000	£000
30 per-cent or more						
40	£000	£000	£000	£000	£000	£000
50992	1.000	1.000	.991	1.000	1.000
60958	.996	1.000	.955	.996	1.000
70943	.992	.996	.907	.981	.997
80786	.913	.965	.830	.937	.988
90615	.774	.873	.726	.811	.915
100457	.595	.700	.572	.719	.775
110286	.393	.449	.397	.471	.549
120201	.231	.245	.290	.338	.345
130107	.117	.111	.177	.190	.150
140062	.054	.044	.104	.079	.037
150034	.029	.008	.050	.038	.012
160018	.007	.000	.023	.023	.000
170007	.000	.000	.005	.004	.000

Other factors being assumed to operate with the same force in all classes, and so being *relatively* negligible, it will be seen from this table that it is about an equal chance that a rural society with under 150 members will experience (for example) 60 per-cent and upwards of the expected sickness cost, and that an urban society with over 250 members will experience 80 per-cent and upwards; or that a rural society with over 250 members will experience 100 per-cent and upwards, and an urban society of similar size 105 per-cent and upwards. It must not be overlooked that (as I have already shown) differences in the respective death rates go some way to nullify the financial effect of these variations.

Finally, with reference to this subject, I should like, at the risk of apparent repetition, to emphasize the point that the "*Whole Society*" Experience of the Manchester Unity is not a proper standard of sickness, and ought not to be employed, as a matter of course, in cases where a standard table has been hitherto used. The real standard of sickness is to be found in the A H J set of Tables; the "*Whole Society*" rates represent the result of incorporating with these a mass of extra risk cases, in the particular proportions in which these cases happened to exist in the Manchester Unity in the five years under observation. Societies with the experience of which the "*Whole Society*"

data will be homogeneous will be extremely rare, and probably will be found only in the centralized group. Extra risks of occupation are not diffused with equal pressure over the whole industrial population, but are concentrated with varying pressure in particular districts. The proper use of the Tables, as I conceive it, is to regard the A H J data as the standard, and to value the extra risk classes by the special tables appropriate thereto.

LEGISLATION AS TO FINANCE.

The many legislative experiments in which Parliament indulged with reference to friendly societies prior to 1875 are enumerated in the Annual Report of the Registrar for 1904, quoted in full in *J.I.A.*, vol. xl, p. 212. The principles which ought to govern such legislation are discussed in the Report of the Royal Commission of 1870, the majority of the signatories to which proposed, in furtherance of their adopted policy of making registration "more easy for the societies", that every condition affecting finance should be of a voluntary character, with the important exception of the quinquennial valuation. Even the compulsion recommended on the latter point was mitigated by the absence of any conditions directed to securing adequate valuation, the only proposal bearing on this point being that the valuation results should be submitted in prescribed form. A proposal was made that tables of premiums should be published from time to time by the Government for general information, but that the use thereof should not be compulsory.

Certain of the Commissioners, agreeing apparently with the view, expressed in an influentially signed memorial submitted to the Commission, "that the certificate of the Registrar of Friendly Societies should be so amended as to make it of weight in determining the trustworthiness of Friendly Societies", were of the opinion that the proposals of the majority were insufficient to meet the necessities of the case, and these Commissioners proposed that a discriminating system of registry should be set up and "Class A" certificates issued to societies which had adopted such tables of contributions and benefits as, taken with the rules, were in the opinion of the Registrar such as to offer a reasonable prospect that, apart from mismanagement and fraud, the societies concerned would be able to meet their engagements. It was proposed that this certificate when issued should stand good for six months beyond the date of the next quinquennial valuation.

This proposal was not incorporated in the Act which followed the Commissioners' Report. "From first to last," as Sir Edward Brabrook says, "the essential principle of the Friendly Societies Act is the voluntary principle. The act compels no society to be registered; it compels no registered society to adopt any fixed scale of contributions and benefits; it compels no society to carry on its business in any particular way; it compels no society to adopt reforms, however apparent may be the necessity."

There is a growing disinclination amongst the societies to acquiesce in these principles as the permanent basis of their relation to the law. For instance, it has been proposed by influential societies that tables of contributions, certified by a Government actuary, shall be adopted under penalty of cancellation of registry, and that the provision of the Act permitting exemption from valuation in certain cases shall be repealed.

On the first of these proposals the Chief Registrar has reported in the following terms:

"In considering the question of amending the Friendly Societies Act in the manner proposed, a fact of great importance must not be lost sight of, namely, that although a society's tables of contributions and benefits may have been certified by an actuary as sufficient, that circumstance in itself cannot be taken as a guarantee that a society will in the future be in a position to meet all its liabilities in full. There are, as is well known, many circumstances which may operate, so as to undermine the constitution of a society founded on a thoroughly sound financial basis.

"There is no doubt that the personal equation enters very forcibly into the administration and operation of a friendly society. This circumstance may be borne out by the fact that cases have come to light where it has been found that the experiences of two societies, practically within a stone's throw of one another, and whose members are engaged in similar employment, have varied considerably; and that, whereas the one society will be found to be solvent, the position of the other is anything but satisfactory; and this circumstance can only be attributed to the spirit in which the members treat their society, and the manner in which its administration is conducted."

The opinion here expressed will, I do not doubt, be very generally endorsed by actuaries, regretful as we may be to

observe the continuance of financial conditions that can only be successful in exceptional circumstances. A certificate once granted must be taken as permanently applicable—it would certainly be so taken by the public, however guarded might be its terms—but that condition would throw upon the actuary a responsibility which he cannot take and ought not to be expected to take. To quote one common case only, namely, that of radical change of occupations, due, perhaps, to the sinking of a coal mine in a locality hitherto agricultural, it is obvious that different conditions from those prevailing when the certificate was granted would be at once set up, and that continued dependence on the certificate could be nothing but delusive.

I have reason to think that the larger affiliated societies contemplate no difficulty in obtaining actuarial certificates for the tables of contributions on which they now are acting. In this I fear they are in error. It would be necessary for the certifying actuary to have regard to the circumstances of each branch, for each is responsible for its own sickness liabilities. It is true that there are provisions for assistance, but this is nearly always voluntary, and more or less sporadic in its incidence. Such provisions fall very far short of a definite guarantee. Recognizing that each branch is chargeable with its own liabilities, the question of occupations would at once arise, and would have to be dealt with in a much more searching manner than has been attempted at present. Even in the cases wherein a general table might be deemed suitable, the scales computed on old experiences, such as the Manchester Unity, 1856-60 or the Ancient Order of Foresters, 1871-75, at present employed by these two Orders as their respective contribution bases, would be ruled out by the universal increase of sickness disclosed by recent investigations. Possibly also the question of interest might be raised. Some societies assume 3 per-cent in their contribution tables, but others depend on $3\frac{1}{2}$ per-cent, a rate to which a Government actuary might well take exception, without questioning the arithmetical accuracy of the tables placed before him. From whatever cause it might come, it is safe to assert that an increase of rates, unexpected by some of its proposers, would follow upon the adoption of this regulation.

Development in this direction could hardly stop, moreover, at the enforced adoption of certified tables. Such other steps must be taken as may be necessary to ensure the security implied by the certificate. State control of the valuation and

compulsory revision of contracts upon the issue of an unfavourable report would follow, and the chain would be incomplete without State audit and State supervision of the investments. None of these may be inherently objectionable, but they are fatal to the educative self-government, which to many is more than an incidental advantage of the friendly society system.

The object of the proposal is avowedly that of discouraging the formation of dividing and deposit societies. It might not, however, have that effect. In the State of Victoria, where the system of certified tables has been adopted, the Government actuary is issuing certificates of sufficiency to dividing societies available for one year only! A more effective way of stultifying the actuary's authority could hardly be discovered.

The most recent proposal by the friendly societies is to the effect that the Act should be so amended as (1) to prevent the registration of societies which offer continuous membership and neglect to submit their affairs to actuarial valuation, and (2) to provide for the cancellation of registration of societies "which "fail to take steps for the removal of actuarial deficiencies when "such are found to exist." That registration should be a criterion of soundness is still the basis of the proposition, but the ground has been shifted from adequate tables to satisfactory valuation results. Whether the adoption of the later proposal, and the enforced application of financial remedies would be to the advantage of the friendly societies as a whole, is a matter upon which opinions may differ, but at least the proposal is free from some of the objections to which the earlier plan lay open. Whatever view may be taken of its practicability, the proposal, coming as it does from the National Conference of Friendly Societies, embodies an admission of the value of actuarial services which, in view of the former attitude of some of the societies, is significant. If the proposal should hereafter become law, the hope may be expressed that it will be accompanied by a further reform, directed to securing adequate valuations. Freedom in the case of the friendly societies is unaccompanied, in great measure, by publicity, as must inevitably be the case when the matter concerns 27,000 more or less separate organizations, many of them obscure and remote from the centre of authority. All who are qualified to speak upon the subject will agree that freedom without the restraint of publicity has a baneful effect upon the efficiency of the system of valuation as applied to the friendly societies.

INDUSTRIAL ASSURANCE (COLLECTING SOCIETIES) EXPERIENCE.

In 1904, Messrs. Ackland and Bacon contributed to the transactions of the Institute (*J.I.A.*, vol. xxxviii, p. 539) a valuable paper embodying the results of an investigation into the effect of the lapse element. In the course of the debate I referred to certain points, and now submit some interesting data bearing thereon.

It was made clear by Messrs. Ackland and Bacon that their assumption that mortality entered into the total decremental rate to the extent indicated by the English Life Table No. 3, and that the balance of the decrement, after allowing for mortality at this assumed rate, was attributable to lapses, was conditional upon the general suitability of the life table named as a measure of the emerging claims rate, and it is not to be presumed that an actuary of Mr. Ackland's wide experience would sanction the use of the method under any other conditions. The precision with which the method was stated, and the facilities which it presents for curtailing the heavy work of valuation involving the secession element might lead, however, to its use in less assured circumstances, and consequently I think it well to show that in organizations of this class the rate of mortality cannot safely be assumed as a matter of course to be in accordance with any standard table. In the first place, the distribution of the business geographically is a matter of much importance, the death rates at the higher ages in the manufacturing towns of the north being much above the general average. On this point I subjoin values of \bar{e}_x , derived from certain industrial life experiences in Lancashire and London respectively. The years at risk comprised within the observations yielding these results are respectively 452,000 and 299,000, numbers which, though comparatively small, are sufficient to show the general tendency.

Age x	COMPLETE EXPECTATION OF LIFE (\bar{e}_x)	
	Lancashire Experience	London Experience
10	45·04	49·11
25	32·52	36·83
40	21·26	25·27
55	12·47	15·95
70	6·70	8·65

I have had under examination at various times the experiences of several local societies in manufacturing districts, but have

never found a case wherein the mortality curve approximated to that of the English Life Table No. 3. The English Life Table No. 6 is now available for use, but, whilst this table approaches more nearly to the actual experience, especially at the younger and middle ages, it still affords an inadequate measure of the mortality met with in the more advanced period of life. The following are examples of actual cases illustrating these features ; it is reasonable to suppose that experiences which are so very generally discovered in local societies are repeated frequently in such of the larger societies or industrial companies as draw a great proportion of their business from these particular districts.

Ratio (per-cent) of Actual Mortality to the English Life Standards in certain local Burial Societies.

Groups of Ages	Lancashire		Lancashire		Lancashire		Cheshire		Staffordshire		Staffordshire		North of Scotland	
	65,000 Members		5,500 Members		25,000 Members		29,500 Members		38,000 Members		44,300 Members		52,500 Members	
	E. L. No. 3	E. L. No. 6	E. L. No. 3	E. L. No. 6										
11-25	52	90	47	82	46	81	44	79	40	73	51	92	60	109
26-40	67	90	58	78	54	71	63	85	61	85	65	87	67	100
41-55	114	107	75	70	90	84	106	100	95	92	80	76	88	95
56-70	134	117	133	116	127	112	139	121	121	106	93	81	113	108
Over 70	139	123	212	200	119	105	117	103	163	146	114	108

The second point for consideration is the incidence of the mortality with regard to duration of membership. There is reason to believe that in some cases (and it may be that the feature is a general one) the mortality rate, examined with regard to a constant age attained, diminishes as the duration of the assurance increases. For so long a period after entry is this feature traceable as to suggest that, as duration increases, the lives withdrawn by lapse or surrender constitute an increasingly inferior class, the exit of which exerts a material influence on the claim-rates of the assuring bodies.

This point is illustrated by the table following. Such table is obtained from the experience of a "collecting society" which has existed for over 50 years, and the records of which, in respect of mortality under weekly premiums, whole life tables, for the five years 1903-1907, yield about $7\frac{3}{4}$ million years of life, and nearly 171,000 deaths.

Age attained	COMPLETE EXPECTATION OF LIFE (\bar{e}_x)						English Life No. 6	Age attained
	Aggregate Experience	Experience after first 5 years	Experience after first 10 years	Experience after first 20 years	Experience after first 30 years	Σd_x		
	$\Sigma E_x = 7,715,275$	$\Sigma E_x = 4,317,697$	$\Sigma E_x = 3,068,630$	$\Sigma E_x = 1,678,980$	$\Sigma E_x = 732,591$			
x	$\Sigma d_x = 170,815$	$\Sigma d_x = 94,793$	$\Sigma d_x = 66,812$	$\Sigma d_x = 39,080$	$\Sigma d_x = 20,534$			Males
11	47·46	48·35	49·24	48·77	11	
21	39·06	39·80	40·63	41·85	...	40·22	21	
31	31·20	31·79	32·55	33·64	34·31	32·29	31	
41	23·83	24·15	24·82	25·81	26·42	24·95	41	
51	17·12	17·12	17·63	18·43	18·96	18·26	51	
61	11·46	11·27	11·58	12·03	12·36	12·36	61	
71	7·13	6·97	7·03	7·26	7·31	7·60	71	
81	4·03	4·01	4·01	4·11	4·12	4·11	81	

It is evident from this table that there is a considerable element of doubt in the assumption that mortality at all ages and durations follows a standard table.

The rates of lapse coincident with the mortality experience just exhibited are shown by the statement on the following page. These rates compare with those marked (β) in Messrs. Ackland and Bacon's Table I, subject to the slight difference that in this case

$wq_x = \frac{w_x}{l_x}$, whilst by Messrs. Ackland and Bacon's method

$wq_x = \frac{w_x}{l_x - \frac{1}{2}d_x}$. In constructing the previous table I have, of course, used the ordinary value of q_x , obtained by assuming the lapses of each year to be at risk for 6 months. In the work of valuation involving two or three decremental forces, it is more convenient to obtain the respective rates by the method referred to by Mr. King in *J.I.A.*, vol. xxxix, p. 134.

The institution experiencing these rates is accustomed to free the assured from premiums so soon after the attainment of 70 years of age as the period of 30 years of membership is reached. I do not think that this condition, which is a comparatively recent innovation, has much effect upon the lapse rates until the period of freedom is close at hand, and the general tendency of the table appears to me to support the opinion which I have previously submitted that the lapse element is not a permanent factor, either depending ultimately upon age, as suggested by Messrs. Ackland and Bacon's Table I, or assuming a constant form as exhibited by Dr. Sprague's Table (*J.I.A.*, vol. xxvii, p. 317). As regards the latter table, I may be again permitted, perhaps, to refer to a

*Lapse Rates (*Hazard Life*; *Weekly Premiums*) unadjusted.*

Curtate Duration	Central Age at Entry*										Curtate Duration								
	1*	2*	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	79	
0	.3017	.5823	.6151	.5655	.5413	.5076	.4832	.5045	.5110	.5090	.4589	.4252	.3839	.3517	.2904	.2534	.2425	0	
1	.1407	.2386	.2548	.1982	.1846	.1836	.1748	.1790	.1741	.1619	.1437	.1306	.1124	.0983	.0830	.0673	.0616	.0569	
2	.1036	.1628	.1683	.1229	.1131	.1169	.1131	.1108	.1054	.0964	.0853	.0821	.0719	.0632	.0536	.0365	.0382	.0508	
3	.0833	.1201	.1232	.0893	.0838	.0932	.0892	.0860	.0743	.0724	.0599	.0587	.0499	.0459	.0380	.0326	.0344	.0320	
4	.0714	.1017	.1053	.0709	.0647	.0708	.0741	.0712	.0619	.0567	.0480	.0435	.0362	.0310	.0259	.0286	.0337	+	
5	.0620	.0845	.0866	.0617	.0624	.0618	.0622	.0606	.0513	.0179	.0157	.0393	.0336	.0318	.0242	.0259	.0239	.0170	
6	.0550	.0765	.0812	.0585	.0547	.0559	.0544	.0504	.0432	.0395	.0388	.0330	.0235	.0219	.0183	.0188	.0128	.0386	
7	.0494	.0619	.0670	.0511	.0542	.0555	.0515	.0473	.0418	.0311	.0308	.0281	.0235	.0171	.0154	.0132	.0192	.0215	
8	.0437	.0475	.0552	.0501	.0503	.0512	.0147	.0382	.0371	.0305	.0274	.0247	.0199	.0191	.0147	.0147	.0138	.0431	
9	.0396	.0433	.0566	.0463	.0176	.0481	.0129	.0360	.0373	.0287	.0255	.0224	.0204	.0129	.0093	.0129	.0031	.0000	
10-14	.0294	.0373	.0391	.0119	.0112	.0365	.0319	.0310	.0281	.0213	.0170	.0155	.0116	.0096	.0088	.0119	.0085	.10-14	
15-19	.0203	.0245	.0303	.0118	.0126	.0233	.0194	.0162	.0147	.0130	.0108	.0087	.0081	.0065	.0058	.0119	..	.15-19	
20-24	.0196	.0229	.0235	.0243	.0220	.0168	.0118	.0103	.0090	.0068	.0067	.0066	.0049	.0030	.0041	.0114	..	.20-24	
25-29	.0190	.0191	.0197	.0184	.0142	.0091	.0078	.0070	.0039	.0026	.0030	.0027	.0023	.005225-29	
30-34	.0190	.0190	.0186	.0153	.0095	.0066	.0055	.0054	.003030-34	
35-39	.0154	.0128	.0139	.0105	.0075	.0052	.0041	.002235-39	
40-44	.0146	.0097	.0102	.0060	.0049	.0038	.002240-44	
45-49	.0074	.0117	.0057	.0016	.0034	.002045-49	
50-54	No valid experience									50-54
55-59) beyond duration 50,									55-59

* See Author's remarks on p. 260 (Ed. J.J.A.).

point touched by me in the discussion upon Messrs. Ackland and Bacon's paper. From durations 30 to 70, the table may fairly be described as inconclusive, the society from the experience of which it was deduced having practically begun operations in 1850, or 28 years only from the latest date comprised within the observations upon which the table was based.

It is worthy of note that the low rates of lapse for age 1 at entry exhibited by Messrs. Ackland and Bacon, in comparison with the rates for higher entry-ages, are also disclosed by my observations. Even in comparison with entry-age 2, the experience of entry-age 1 exhibits a remarkably low lapse rate. It would be interesting to have some explanation of this peculiarity.

The following are the corresponding experiences under monthly premium (whole life) policies up to duration 24, which is the limit of this particular experience. It should be said in respect of mortality, and will be seen in regard to lapse, that the rates are much below those exhibited by the weekly premium assurances, so bringing out the further point that if the experience of a particular organization is to be used, regard must be had separately to the working under each important class of assurances. It is worthy of observation that in this group there is no clear evidence to connect the rate of mortality with the duration of assurance. The aggregate number of exposures to risk in this class is about 250,000.

Monthly Premium Assurances (Whole Life).

Comparative number of Deaths experienced to 100 "Expected" in the following curtailed durations of assurance.

(Expected Deaths based on English Life No. 6).

Central Age attained	CURTAILED DURATION					Central Age attained
	0-4	5-9	10-14	15-19	20-24	
22	92	22
27	102	88	27
32	81	74	74	32
37	73	70	68	74	...	37
42	83	123	78	67	56	42
47	77	83	97	69	83	47
52	79	88	113	83	74	52
57	94	107	98	106	124	57
62	92	102	106	114	106	62

Lapse Rates (Whole Life: monthly premiums) unadjusted.

Curtate Duration	CENTRAL AGE AT ENTRY									Curtate Duration
	20	25	30	35	40	45	50	55	59	
0	.2058	.2487	.2235	.2486	.2246	.2138	.2141	.1794	.1468	0
1	.1227	.1206	.1043	.1039	.1367	.1080	.0852	.0940	.0883	1
2	.0834	.0892	.0900	.0859	.0831	.0583	.0587	.0739	.0456	2
3	.0614	.0674	.0523	.0501	.0535	.0534	.0301	.0212	.0238	3
4	.0447	.0411	.0499	.0519	.0314	.0268	.0276	.0204	.0237	4
5	.0248	.0500	.0523	.0306	.0298	.0200	.0299	.0113	.0578	5
6	.0412	.0424	.0281	.0305	.0381	.0269	.0221	.0276	.0141	6
7	.0341	.0421	.0306	.0212	.0169	.0160	.0124	.0174	.0166	7
8	.0344	.0364	.0368	.0244	.0316	.0311	.0160	.0069	.0162	8
9	.0264	.0244	.0245	.0186	.0116	.0114	.0254	.0096	.0000	9
10-14	.0267	.0199	.0151	.0158	.0128	.0110	.0077	.0068	.0069	10 14
15-19	.0150	.0093	.0073	.0072	.0056	.0059	.0041	.0045	.0063	15-19
20-24	.0161	.0108	.0083	.0025	.0049	.0000	.0048	.0000	.0000	20-24

An interesting piece of further evidence, as to the effect of the selection exercised in the choice of assurance, is afforded by a comparison of adjusted mortality and lapse rates under life assurance and endowment business respectively. The age at admission is 1 in each case, and in both classes premiums are payable weekly. The endowments carry return of premiums in the event of death before age 14, so that there is no doubt as to the notification of all the deaths that occurred in this class.

Entry Age 1.

Age attained	MORTALITY RATE		LAPESE RATE	
	Life Assurance	Endowment	Life Assurance	Endowment
1	.0757	.0269	.2209	.1700
2	.0307	.0123	.1137	.0540
3	.0159	.0094	.0878	.0320
4	.0114	.0077	.0728	.0190
5	.0079	.0063	.0617	.0130
6	.0062	.0050	.0548	.0078
7	.0050	.0038	.0500	.0039
8	.0041	.0028	.0450	.0030
9	.0034	.0023	.0400	.0019
10	.0029	.0020	.0350	.0015

The chief impression left by further study of Messrs. Ackland and Bacon's investigation is that, circumstances being

favourable, it is advisable to rely upon a strict net premium valuation with lapses excluded, or upon a gross premium valuation similarly based with a suitable reserve for expenses. The lapse element may, nevertheless, be more than a merely academic study. Examination of a particular experience may show, as in the case now submitted, that the mortality differs sensibly from any standard, and that if the valuation is to have due relation to the facts, the Actuary must rely upon the actual experience. In such a case, the use of the lapse element may be unavoidable, and in the absence of a standard experience, the data here given may sometimes be found of service for comparative purposes. I have refrained from exhibiting specimen valuation results, partly because much of the ground is covered by Messrs. Ackland and Bacon's valuable researches, and partly because the circumstances vary too much—not only as regards the observed rates of lapse and mortality (the latter chiefly), but also as regards the distribution of the premium income with reference to duration of assurance—to satisfy me that results based on a "model office" can have any general application. On this latter point I may be allowed, perhaps, to suggest that Messrs. Ackland and Bacon's Table VI falls somewhat short of a representative body of facts, by reason of the paucity of data relating to assurances of more than 20 years duration.

I am constrained to emphasize the necessity of recognizing the incidence of the experience when using the lapse element, by reason of the latitude assumed by some operators when handling this very difficult factor in our work. I have recently seen a valuation of a large collecting society, wherein, without examination of the experience, either of mortality or lapse, a constant lapse rate at all ages and durations of 1 per-cent per annum was assumed and incorporated with the mortality rates of the English Life Table No. 3, a standard which I have reason to believe the valuer would have found to be largely exceeded in the actual mortality experience at the higher ages. The effect of employing the empirical lapse rate was to reduce the English Life No. 3 reserves, on a lapse excluded basis, by about 25 per-cent, such reduction representing about four times the surplus declared. It is obvious that whilst a valuation so conducted proves nothing as to the position of a Society, it may from mere show of authority be the means of doing serious harm.

The system of valuation "with secessions" is likely to

be found, as heretofore, of greatest use in the case of the large collecting societies, and it may be well to point out that these societies, although paying surrender-values to only a limited extent, are accustomed to allow a much greater latitude in the matter of "arrears" than is the custom of the more strictly administered industrial companies. Consequently, in using a lapse rate, it is necessary to take into consideration the period, sometimes 13 weeks, for which the risk continues after the payment of the last premium. This may be conveniently done by valuing say $\cdot25P'_x$ as a benefit payable on lapse, taking out \bar{A}_x (the full reversion) by conversion tables, and obtaining the lapse reversion by the formula $A''_x = A_x - \bar{A}'_x$ where \bar{A}'_x is equal to $\frac{\Sigma r^2 d q_{x+n} D_{x+n}}{D_x}$.

In concluding my references to this class of societies, it may be useful to detail the adjustments which should be made upon the tabular results in drawing up the valuation balance sheet. As regards expenditure for management, it is usual to set apart a fixed proportion of the premiums after the first twelve months, say $37\frac{1}{2}$ per-cent. A much larger proportion, say 80 per-cent, is applied to this purpose from the first year's premiums. Against this the full benefit does not enure, as a rule, until after 12 months from the date of entry. It should not be deemed sufficient to oppose the two latter factors, and assume the one to counterbalance the other. On a close agreement between the aggregates of assets and liabilities, the method of treating these items might determine the side of the account on which the final balance should appear. It is advisable, therefore, to make the best estimate of the value of each that circumstances will permit. Similarly, an investigation should be made of the probable relief to the liabilities, resulting from the adjustment of sums assured in correction of errors in the stated entry ages, which adjustment will be found in the case of a large concern to be so regular on average as to be susceptible of estimation. Assuming that the full benefits are valued in the case of juvenile assurances, and the allowance for the payments of lower amounts in the event of death under age 10 made subsequently, which at times is a convenient course, and assuming also that negative values are excluded (an interesting point dealt with fully by Messrs. Ackland and Bacon, and in the discussion following their paper), the adjustments relative to a gross premium income of £100,000

arising out of a well-established business will be somewhat as follows :

Value of Gross Premiums	£1,000,000
Deduction for Arrears lost by Lapses	8,000
	<hr/>
	992,000

Deduction for Management and Collection:

37½ per cent of gross	£372,000
Extra 42½ per cent for remainder of first year	4,500
	<hr/>
	376,500
	<hr/>
615,500	
Negative Values	40,000
	<hr/>
Net Value of Premiums	£575,500
	<hr/>

(It is assumed as a matter of course, but the point is perhaps worth mention, that the annual premiums are valued as 52·18 times, and not merely 52 times the weekly premiums).

Value of Maximum Sums assured	£880,000
Deductions for Deaths at ages under 10	£10,000
Deductions for Deaths within 12 months from entry	2,000
Deductions for Errors in age at entry	3,500
	<hr/>
	15,500
	<hr/>
	£864,500

The reserve thus determined is £289,000, whereas, if the minor adjustments be disregarded, the net premiums being valued at £623,500 less negative values £40,000, and the sums assured at £880,000 less £10,000 for the contractual abatements in respect of juveniles, a reserve of £285,000 results. Whilst the difference is not large, it cannot be regarded as unimportant. Even were it smaller it would justify investigation.

FRIENDLY SOCIETIES ACCOUNTS AND VALUATION DATA.

Much interesting matter bearing on this subject is to be found in the first Report of the Actuarial Commission of 1875-6. I cannot learn that the models submitted in this report have been

adopted to any great extent, but the origin of the various forms of return issued by the Registry Office is clearly traceable in the proposals of the Commissioners. Important contributions to the subject have been made by Mr. G. F. Hardy (*J.I.A.*, xxvii, pp. 272-5) and by Mr. Vyvyan Marr, who submitted a paper embodying a complete system of accounts to the Glasgow Chartered Accountants' Students' Society in 1907. My judgment, after careful consideration of all that emanates from these three sources, is that the ideal system is to be traced in the recommendations of the Actuarial Commission, but that there is still scope for study of the subject, and particularly for illustration of the links connecting the ordinary items of account with the data required by the Actuary.

Much praise has been bestowed upon the card system, and I suppose that all actuaries undertaking friendly society practice have employed it. Mr. Marr, in the paper above referred to, goes so far as to advocate its use for practically all the purposes of a society's accounts. My own opinion is that in this particular connection the card system is somewhat over-rated, although I grant that circumstances must frequently arise in which it is obviously the proper method to adopt. Looking at the subject generally, it is clear that great advantage is gained, if the preparation of experience data and other actuarial requirements proceeds concurrently with the ordinary work of a society throughout the quinquennium, but I have never yet met a case where a card system was employed in which the writing up of the cards was not deferred till the end of the quinquennium was at hand. This, no doubt, is a fault of management rather than of the card system, but at least it may be concluded that the system offers no inducement to officials to treat the work as part of their ordinary routine. The results, moreover, as they leave the hands of the secretaries, are decidedly unsatisfying. Instead of applying himself at once to such actuarial problems as the case may present, the Actuary has to spend much valuable time in supervising the reduction to order of the mass of disconnected facts which the cards contain—a task which could be avoided with really no more trouble to the officials of the societies, if a well-devised system of tabulation were laid down in the first instance.

In this belief, I submit the three following forms, each of which, independently, is in actual use, but the linking up of which with the others will conduce in my belief to the preparation of a set of valuation data with as much ease and as little delay as can

CONTRIBUTION BOOK

SICK PAY REGISTER.
Six Months ending 31 December, 19...

VALIDATION AND EXPERIENCE BOOK.

Year of Birth.....

Register Number	Name	Year of Entry	Weekly Full Sick Pay	Member's Death Benefit	Contributions for Benefits			Wife's Death Benefit drawn		
					Sickness (or combined)	Death	Year	Amount	Year	Cause
					£	s.	d.	£	s.	d.

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SICKNESS EXPERIENCE.				190.....				190.....				190.....			
Time	Cost	Time	Cost	Time	Cost	Time	Cost	Time	Cost	Time	Cost	Time	Cost	Time	Cost
Full Pay	Reduced Pay	Full Pay	Reduced Pay	Full Pay	Reduced Pay	Full Pay	Reduced Pay	Full Pay	Reduced Pay	Full Pay	Reduced Pay	Full Pay	Reduced Pay	Full Pay	Reduced Pay

W D W D £ s. d.	£ s. d.	W D £ s. d.	£ s. d.	W D £ s. d.	£ s. d.	W D £ s. d.	£ s. d.	W D £ s. d.	£ s. d.	W D £ s. d.	£ s. d.	W D £ s. d.	£ s. d.	W D £ s. d.	£ s. d.
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reasonably be looked for. The forms provide respectively for the contribution book, sick pay register, and valuation and experience book ; other books will necessarily be required in the conduct of a friendly society, *e.g.*, the register, the general ledger, the latter providing a ledger account for each *fund*—I do not consider a ledger account for each *member* necessary or advisable—and one or two minor books, but it is no part of my purpose to discuss these, which must be in more or less the same form in all well-conducted societies. The object of the three forms given is to show the connection between the items of account and the actuarial statistics, and to indicate how the latter may be so scheduled as to give at the end of *any* year not only the experience of the quinquennium up to such year but the essential facts for valuation summarized in the shape in which the Actuary has occasion to use them. The form is very similar to that proposed by the Actuarial Commission, and its advantages are precisely those claimed for it in the report of that body.

In the Contribution Book, the names should be inserted in alphabetical order, thus facilitating reference back to this book from the sick pay register, in order to insert the year of birth and register number in the latter. The names in the sick pay register will necessarily appear in order of date of claim, and the year of birth and register number are required in each case, in order to transfer the claim items to the name of the member in the valuation and experience book. Deaths and lapses may be transferred directly to this book from the contribution book, wherein they will of necessity be posted when “contributions due” cease to be carried forward.

Everything paid by the members on each meeting night should be entered in the contribution book, regular monthly payments being posted on the line opposite the name, and sundries over the line. The total of the columns for each date will then give the full receipts from the members on that date, and the treasurer should sign for this exact amount. I digress to mention this, because the practical advantage which results from the enforced concentration of all the receipts of each meeting is real, but is not so fully appreciated as it should be.

If the members are engaged in occupations bringing frequent accident claims, the letter *a* should be written against each claim of this kind in the weekly columns of the Sick Pay Register. In the “total” columns, accident claims should be inserted in red ink. This applies also to the valuation and experience book.

In the case of members who claim in the first half year, but not in the second, the weeks of claim, as inserted in the first half year's summary, will be carried direct to the valuation and experience book.

If there are more than two rates of sick pay, further "reduced pay" columns may be added to the summary columns of the sick pay register as may be necessary. This applies also to the valuation and experience book.

The Valuation and Experience Book is in the simplest form, assuming that there is more than one scale of benefit. If all the members are assured on one scale, further simplification is possible by omitting the columns of benefits and the "cost" items in the experience. If, on the other hand, classes of sick pay ceasing at different ages, with or without pensions, have to be provided for, further columns will be necessary, both in the "existing" and "exit" columns. Separate tabulations, however, may be preferred for different classes, if these are numerous.

On this book being written up at the beginning of the quinquennium, and each age totalled, it will give (*a*) particulars of assurances then existing, (*b*) numbers and amounts assured brought forward into risk for the first year of the quinquennium. To save space, the total *number* of assurances thus brought forward may be inserted in the "year of entry" column in the line of totals. If the society is large, a separate set of sheets should be kept for members beginning the quinquennium, and for members entering in each of the five years following—six sets in all. If the numbers are not large—say under 2,000—the particulars of entrants in the first year may be inserted immediately following those relating to entrants by survivance, and a combined total made at the end of the year. The exit columns will be filled up for the cases going off during the year, and the total of these for the year should be carried under the corresponding totals in the "existing" columns and deducted. The balances will represent the several amounts going forward to the beginning of the second year, and the same processes will be repeated year by year during the quinquennium, the "year of entry" column being used in the case of all totals for numbers of members. The "existing" columns will by this process appear somewhat as follows :

Register No.	Name	Year of Entry	Weekly Full Sick Pay	Member's Death Benefit	Sickness (or com- bined)	CONTRIBUTIONS FOR BENEFITS		WIFE'S DEATH BENEFIT DRAWN	
						—	—	—	—

A = Entries in detail existing at beginning, say 1 January 1905.

B = Totals of foregoing.

C = Details of New Entrants, 1905.

D = B + C.

E = Totals of corresponding "Exit" Columns, 1905.

F = Difference, D - E = Quantities, beginning 1906.

G = Details of New Entrants, 1906.

H = F + G.

J = Total of corresponding "Exit" Columns, 1906.

K = H - J = Quantities, beginning 1907.

&c.

It should be noted that, by the use of the new entrants and exits columns, due effect may be given to all additions to or reductions of assurances.

The number of claims at each age for full and reduced pay respectively is a very useful function, and will be obtained by counting the items in the claims columns, on the "experience" half of the page.

The object of scheduling the cost of sick pay in addition to the weeks of claim is to facilitate comparison of the expected and actual cost, as well as expected and actual time sick. As I have shown previously in this paper, investigation on this point may be well worth undertaking.

Should it be necessary to dispel doubts as to the feasibility of this system of classification, I am able to point to one case, at least, in which it is actually in use, namely, the London Friendly Institution, a society of the centralized type, with about 2,000 members assured on very varied scales of benefits. The Secretary of this society adopted the system many years ago, and

has maintained it so successfully as to be able to publish with each annual balance sheet (issued in February) a complete schedule giving the experience of the society at each age during the preceding year, with all the data required for valuation as at 31 December.

No spaces are provided in the experience record for dates of sickness attacks. For the ordinary purpose of examining the experience of a single society, these are not required, all the lives under observation being subject to the same condition as to duration of pay at each rate (full, half, &c.) and as to the control of the claims at the higher rates by the "off" period. If a standard table is to be compiled from the experience of many societies, the circumstances are different, for here the periods at the various rates of pay will differ, 3 or 6 months obtaining in some cases, and 12 in others, and the mere record of weeks paid for in a society with a long run of full benefit, say 12 months, will afford no true criterion as to the proper number of weeks falling within the shorter periods of 3 or 6 months. Moreover, in such circumstances the "off" periods will vary, and some societies will pay at the full benefit rate, for weeks, which in other societies would only qualify for reduced pay. It is accordingly necessary, when constructing a standard table, to disregard the divisions made by the societies, and to re-distribute every member's experience on a basis to be laid down at the beginning of the work. For this purpose, the actual dates of sickness are required, together with the position, relative to duration of attack, of each member who was sick on the commencing date. All this information will be yielded by the sick pay register, but as the collection of data for standard purposes is only required at long intervals, it is not thought necessary to demand that such troublesome details should be entered up in the valuation records.

In a brief but pregnant communication to a recent meeting of the Faculty of Actuaries (in connection with the discussion on a very interesting little paper by Mr. Vyvyan Marr) Mr. R. P. Hardy, pointing out the complexities of examining the working of a friendly society, says,* "the grand test that I endeavour to "apply (not always successfully) is to make up a final statement "ear-marking the profit and loss. This, I have found, brings "matters home to the administration who cannot follow the "technicalities of a balance sheet or even tabular percentages."

* (*See T.F.A.*, vol. iv, p. 170.)

No further weight can be added by the endorsement of any member of our body to a statement of this character made by Mr. Hardy, but it may be of interest, nevertheless, to some to know that this test is one which I habitually apply, not only as affording beneficial information to the members, but also as supplying an independent check upon the work of the clerical staff. The preparation of a profit and loss statement gives rise to some very interesting points, particularly in the case of branches of a "district" where the death benefits are re-assured, sometimes by direct transfer of the contributions received for these allowances, and sometimes by assessment for current liabilities, either on a proper mortality basis, or by some more or less convenient and certainly more or less inaccurate plan of levying. Features of this kind sometimes prevent a close balance, as do circumstances of which from his remarks Mr. Hardy has also had experience, nevertheless the attempt to reconcile the present and past results is often very fruitful. Occasionally, with other advantages, it puts the Actuary on the track of a financial irregularity which the periodical audit should have but has not revealed. Thus is established in the minds of the members a connection between the labours of the Actuary (which are still to many in the nature of mysteries) and the realities of the common round, with results that, from the Actuary's point of view, are eminently satisfactory.

These remarks, with many previous passages, are intended less for the consideration of the experienced (whom I cannot presume to instruct) than for the information of our junior Fellows and Students, for the entry of some of whom into the field of friendly society work I earnestly look. By what means, short of legislation, that result (so eminently desirable in the interests of the societies) may be attained, I do not know. Present conditions stand condemned by the consideration that probably 20,000 friendly societies and branches, with funds amounting to many millions of pounds, are conducting, without proper actuarial guidance, a class of operations in respect of which such guidance is essential, not merely to success, but often to the avoidance of disaster. I submit this paper in the hope that it may prompt discussion such as will direct public attention to this unseemly state of things.

ABSTRACT OF THE DISCUSSION.

MR. VYVYAN MARR referred to the summary, published by the Registrar of Friendly Societies, as to the numerical strength of the societies, and the author's reference to a quotation made by him

(Mr. Marr) a year ago on the subject. He thought that he was right in including the whole of the societies, his motive being an attempt to ascertain the present position of thrift institutions. Certainly the growth of the Holloway Societies had been remarkable in recent years. He thought it right to say, however, that he did not approve of the illusory estimates of future dividends, which were put forward in some cases to attract new entrants. They were, of course, based upon recent selections, and whether the estimates would be realised in the future he should not like to say.

Much had been written about the improvement of friendly societies as a whole, and he thought the recent valuation report of the Manchester Unity of Oddfellows, the largest of the affiliated Orders, was of very great interest. Mr. Watson, in his recent report, pointed out that in 1870 there was a net deficiency of £1,343,447, and in the last valuation but one that had been changed into a net surplus of £33,346; whereas in the last valuation there was a net surplus of £407,625, with a reduction in the gross deficiencies.

The President, in his prize essay, gave an interesting summary of the valuation returns published in 1880, prepared by Sir Edward Brabrook, and contained in a paper read to the Royal Statistical Society. The result for the five years ending 31 December 1880 showed that in 5,030 cases there were deficiencies of over £5,155,816, and in 1,537 cases there were surpluses of £885,382, or a net deficiency of £4,270,434. Looking at the returns for the five years to 31 December 1908, it would be found that 2,340 societies showed deficiencies of £5,210,366, and 1,232 societies showed surpluses of £2,195,323, a net deficiency of just over £3,000,000. Therefore there was a decrease in the net deficiency in those 28 years of about a million and a quarter. Of course, those figures could only be considered as extremely approximate, because they included ordinary collecting societies, and it had to be remembered that under the head of "Friendly Societies" the funds were some £55,000,000 in all, whereas in the summary he had just mentioned the amount involved was only £22,000,000.

Mr. Watson, in referring to the statutory valuation abstract, said that he would like to see it enlarged, to include specimen values of the tables used, when valuations purported to be made at rates of interest other than those on which tables had been published. He certainly endorsed that opinion of Mr. Watson's, and at the same time should like to see rules guarding against fictitious rates of interest, and providing for periodical valuations by every society that offered continuous membership.

In discussing centralised societies, Mr. Watson compared the Hearts of Oak with the Manchester Unity, and mentioned that the comparison told in favour of the former, because it excluded miners. Mr. R. P. Hardy in his paper in the *Journal* (vol. xxxi, p. 97) pointed out that up to 1887 the Hearts of Oak had a schedule of interdicted occupations, but it appeared that the interdict was not operative, saving as to miners, and this would surely have some effect upon the claims. Mr. Watson mentioned that the

system of the Rational Association was open to criticism, in assigning the important work of supervision to those who were not directly responsible for the monies disbursed, and it had been considered that a large excess of sickness claims had arisen from that cause, especially in prolonged cases. That was most important, in view of any proposals for the State to undertake the functions of invalidity or sickness insurance, about which so much had been heard of late, and he thought it was a point which should be kept prominently in view. There was no more difficult question in regard to friendly society work than a heavy rate of invalidity claims, coupled with a very low rate of mortality, and in that connection he thought the Workmen's Compensation Act of 1906 would have a very injurious effect. The amount of claim for permanent incapacity was fixed at 50 per-cent of the average weekly earnings during twelve months, with a maximum of 20s. That practically had the effect of reducing a man's earning power by 50 per-cent. In the same way, an optional pension reduced the value of the work to the difference between the amount of salary and the amount of pension, and no doubt had a potent influence on the secession rates. For instance, Mr. Watson gave a table which contrasted the secession rates before and after the establishment of a pension scheme. In Germany, he believed, the definition of incapacity was the inability to earn one-third of normal wages.

He should certainly like to see Mr. Watson's analysis of sickness and accident claims carried further, and, in connection with the various examples given, he should like Mr. Watson to give a note of the number of sickness and accident claims respectively. In the annual reports of the chief inspector of factories and workshops, tables were given, showing the number of reported accidents, tabulated according to industry, age and sex, and every three years a census of the persons employed in the factories was given. Those, he thought, would give some indication as to the proportion of claims in respect of accidents that should be looked for in any one society, or society confined to one class of workmen. In that connection, also, the fact should not be lost sight of that the rate of accident claims, if he might use that expression, was to some extent influenced by a liability of the accident proving fatal. He thought it pointed to the question of sickness being completely differentiated from accident, as was done in Germany. He should like to see more attention paid to preventive treatment, by the establishment of Convalescent Homes, nominations to Hospitals, and the like, which was one of the great features of the German insurance scheme.

An interesting application of the method advocated by the author for the valuation of fixed annual subsidies was to be found in the case of Mr. Hewat's recent valuation of the widows' fund of ministers of the church of Scotland, in which the stipends from vacant charges were always credited to that fund, and were practically a uniform amount. On the question of the orphan funds, and the case of posthumous children, he had to deal with a case where quite an appreciable allowance had to be made, in tabulating

the ages of children as at the age of the death of the members, for the purpose of ascertaining the amount required to be set aside for annuities to any orphan children which might then become payable. With regard to the tables, he wished to ask Mr. Watson one or two questions. On page 196, there was a curious rise in the Adjusted Lapse Rates at age 17, and he should like to know whether that was accounted for in any way by entrants coming in at just about 16? Then, on page 198, Mr. Watson contrasted the rate of mortality between the active members and superannuated members of the Railway Pension Fund, the rates for superannuated members being extremely high. He should like to know whether the excess was evenly distributed throughout the remainder of life, after age 64, or whether it was more or less confined to the first few years? He had always had the impression that the effect of superannuation on the man who had no hobbies told greatly upon his constitution when he ceased from active work. With regard to the Lapse Rates on page 235, there was certainly a most curious fall in the rates referring to central age at entry 1, and he was wondering whether the entrants were equally distributed from ages $\frac{1}{2}$ to $1\frac{1}{2}$, or whether there were more entrants over age 1, and if 1 could be taken as the mean age in question?

In connection with the forms given by Mr. Watson on pages 242, 243, and 246, he could instance the case of a society, founded in 1838, that had had its experience regularly summarised on similar forms since the outset, and, in connection with the valuation experience book, it carried the totals for each year to a register arranged according to ages, and therefore, from the summing of the last 5 or 10 columns of that register, one could obtain at once the exposed to risk and the number of sickness claims for a 5 or 10 year investigation without any difficulty.

Mr. Watson, in conclusion, had referred to two small contributions of his (Mr. Marr's) on the subject of Friendly Societies. The card system advocated might appear very elaborate, but that was due to an endeavour to distinguish between the actual claims due in a year and the actual amount paid. Whilst a revenue account should be strictly prepared in terms of the Friendly Societies Act, it was usual only to give the accounts in the form of a cash account, giving the actual receipts and payments in the year.

MR. H. J. P. OAKLEY said that he would like to raise the question of the legal position of unregistered friendly societies. Registration under the Friendly Societies Act was voluntary, and it would therefore seem that many of the societies, being unregistered, came under the category of companies referred to in the first Section of the Assurance Companies Act of 1909. That Act, however, in Section 35, exempted unregistered friendly societies, on application being made to the Board of Trade. The inference was clear, namely, that the unregistered societies came within the scope of the new Act. Many of those societies had never heard of the Assurance Companies Act, and it would be interesting to know what steps, if any, were likely to be taken, either by the Board of Trade or by the Registrar

of Friendly Societies, to bring those unregistered societies either under the one Act or the other?

MR. H. W. MANLY said that it always appeared to him that one of the greatest difficulties friendly societies had to contend with was that of supervision. Small societies, if the members were properly instructed as to their common interests, might keep within the bounds of reasonable sick-pay, but when they become large they grew out of hand, and Mr. Watson had given some most instructive tables, showing how, the larger the society became, the more difficult was the supervision, and therefore the greater the sickness. If that was so in affiliated societies, how much more difficult must it be with centralised societies, where they had not that local supervision, and the personal interests of members were not brought into play in keeping down malingering. Many years ago, he was concerned with an association where he found the sickness was three times that of the Manchester Unity tables. It was a centralised society, with no supervision, taking in anybody and everybody, and paying all the claims that arose. By dividing the members up into districts and sub-districts, and giving them a table by which they could find out what the sick rate ought to have been in the previous year, an attempt was made to reduce it, where the sick rate was highest, by sending some one down to enquire into all the cases, and it was found that the rules had been systematically broken in the great majority of instances, and most of the bad cases were knocked off the funds. In time, the sickness was reduced very considerably.

There was another point, with regard to the perfunctory way in which many of the medical officers did their duty. Medical officers were appointed in the districts, and were paid generally by a fee of so much per member, and it was not to the doctor's interest to keep a man off a fund, or to put him on. He was probably a medical officer for several funds or clubs, and took very little interest in club patients, and therefore it seemed that perhaps the principle of paying a man so much per head was not altogether the right method. In the old days of the slave trade, when slaves were shipped from Africa to America, there was always a medical officer on board. He was paid so much per head, and he did not care whether the slaves were landed alive or dead, and consequently they died off like rotten sheep, and very few were landed in anything like a sound condition. Afterwards, the system was changed, and the doctor was paid so much per head per sound man landed, and consequently, the mortality went down markedly, and, the doctor took great care to keep them alive, and land them in good health. Therefore, he thought it would be better to pay the doctor so much per member kept off the fund than so much per member in the district, if it could be done. The difficulty in connection with medical examination, malingering, and so on, was certainly one which would become very serious, if the State itself undertook to deal with sickness business. There would then be a very large society, with a supervision that was not of the best, and a fund which was unlimited to draw upon: and everyone would feel that it was almost his duty to get some

thing out of the State, and the State would probably have very great difficulty in preventing men from applying.

MR. C. W. KENCHINGTON said that, in advising a small friendly society some short time ago, he came across a form of benefit of which he did not recollect to have seen any mention in the pages of the *Journal*. It was to the effect that members who were able to keep off the funds were entitled at seven year intervals to an increase in their assurance benefit, which benefit went by the term of "equitable assurance." Any sick-pay received, diminished to the same extent the sum payable as "equitable assurance." He should be glad to know if this was a usual form of benefit in societies? It was a matter of some difficulty to ascertain the best way to value such a benefit. The sum payable depended upon the duration of membership, and on the amount of sick-pay received, and he finally found it was best to value the equitable assurance, by setting out the benefit in the form of an assurance according to the year of age passed through. Owing to the fact that the sickness benefit was practically unlimited in duration that "equitable assurance" started at a minimum, rose to a maximum, and then vanished completely. In the particular society he referred to, the supervision of claims was most excellent, the claim rate being only about one-third of that of the Manchester Unity.

MR. T. G. ACKLAND said that the subject of Friendly Societies had not been largely treated, and it was extremely valuable for a member with such special knowledge of the matter as Mr. Watson to prepare and submit to the Institute so many practical thoughts on different aspects of the subject. Dealing first with general points of principle, there were the interesting facts Mr. Watson brought forward, as to the very large extent of friendly society membership, and the interesting indications of the recent tendency to a fall in numbers, owing to the reduction of young entrants. Those who had had much to do with the valuation of friendly societies, and had noticed the numbers of new entrants and existing members, compared with those of previous valuation periods, would have remarked the tendency that Mr. Watson referred to. He was afraid it was very wide-spread, and arose from the present generation not having that desire for provident benefits which their predecessors more strongly evinced. It was very encouraging, however, to see the progressive financial improvement of friendly societies generally. Most of those who had to do with friendly society valuations had noticed that the general tendency was for the societies to improve with regard to their financial position, and he quite agreed with Mr. Watson's remarks as to the increasing respect that was being paid to the views of the actuary on the subject.

The advantages of grouping friendly societies in affiliated orders must be very great, as enabling a greater control to be exercised over the financial position of branches, and generally a better system of supervision in respect of their sickness claims. With regard to the Friendly Societies Acts and their working, Mr. Watson had

quoted an interesting remark by Sir Edward Brabrook as to the purely voluntary nature of the Friendly Societies Acts and their provisions. It occurred to him that perhaps in some respects those measures were too voluntary in their nature, and in particular that, where there was a provision under those Acts that a registered friendly society was to make a valuation quinquennially, it seemed to be unfortunate that there was no adequate machinery, and no sufficient financial aid, to enable the Registry to bring pressure upon the societies to have those valuations made. There were probably many thousands of societies, duly registered, that escaped valuation altogether, and, as far as one could observe the practice of the Registry, it appeared to be that it did not intervene in the matter, unless the society came along and wished to alter its rules, or in some other way brought itself under the notice of the department. That did not seem to be a satisfactory state of things, but he did not think any blame was to be attributed to the officials in the matter. All present would also agree that where a valuation was made, it should be made by a competent valuer. It seemed a great pity that, under the Act, it was left to the societies to have absolute freedom to select whom they liked to undertake their valuations. It was to be hoped that, as time progressed, and knowledge of actuarial matters became more extended, some suitable definition of the term "Actuary" would be adopted, so that the valuer should be a competent man, within the actuarial meaning of the term.

Another question dealt with by Mr. Watson was the availability of sickness tables for purposes of invalidity. That was a point of growing interest, because there were indications that the State might be taking up, at some future date, the question of invalidity insurance. So far as he was aware, there were no suitable data available to supply their needs in that respect, but it was interesting to learn from Mr. Watson that the Manchester Unity experience included approximately all sickness, and therefore must give some reasonable approximation, perhaps as near as could be obtained at present, to invalidity. He supposed that, under any State scheme, there would be included the collection and tabulation of experience in respect of such invalidity, and it would probably be necessary to take cautious steps, somewhat in the dark at the outset, before the matter could be fully or adequately dealt with.

With regard to the proposals for legislation amending the Acts, such as a qualifying certificate, authorizing a society to proceed on a presumably sound financial basis, or State supervision and control, or cancellation of registry where the condition continued unsatisfactory, those suggestions appeared to him to teem with difficulties. They had been discussed in one form or another for a great many years, and apparently no substantial progress had been made up to the present. Everyone would agree as to the desirability, if it could be secured, of a qualifying certificate being given to a society, but the difficulties were very great, because, as had been pointed out, a society might have suitable contributions,

actuarily certified, and yet might be so managed and administered as very speedily to become insolvent, and it would be very unfortunate if such a society were to be officially certified.

He thought that Mr. Watson had not done entire justice to Deposit Societies. Whilst he agreed with a good deal of what the author had said on the subject, it was a fact, he thought, that the leading societies of this class were working on well-considered lines, and with successful results. One notable and outstanding point was, that the principle of charging some stated proportion of the sick pay against the deposit of the member had a very important effect in keeping down the sickness experience, and the fact that the member had to contribute, say, fourpence in the shilling out of his own pocket towards his own sick pay, had a very material effect in reducing sickness. Although the system of charging so much per head in respect of sickness, without discrimination of age, was not altogether a satisfactory one, he thought that in the past it had been found to work fairly well. Certainly Mr. Watson's remarks should be qualified in respect of the old-age allowances of such societies, which had been investigated scientifically, and funds set up in respect of their liabilities.

With regard to the experience of abstainers, as shown by the Rechabites Society, he quite agreed that, in view of the way in which the facts were got out, and the incidence of recent entrants, those results could not be considered as at all conclusive, as to the greater longevity of abstainers, nor, he supposed, as to their excessive sickness under age 60—for they appeared to point both ways.

With reference to Railway Societies, he wished to make a remark as to the peculiarly select experience of some classes of railway employees that had come under his notice. In valuing a society of Enginemen and Firemen in one of the largest English railway companies, he had occasion to investigate their death rate before age 65 (at which age there was a definite retirement), as compared with the H^m Table, and he found that the expected deaths, for the number of members under observation up to age 65, was 79, while the actual number of deaths was only 28 from natural causes, and two from accident, making 30: and he was able to include the cases of early incapacitation before 65, which were 31 in number, as presumed deaths, making in all a figure of 61, still well below the expectation of 79. That was a remarkable illustration of the select nature of this class, doubtless arising from the fact that the engineman on the footplate was at once taken off, if there was the least sign of anything approaching illness or inability, and only reinstated when the company were fully satisfied that he was fit to resume his duties.

With regard to aggregate lapse rates, personally he demurred, as Mr. Watson appeared also to do, to lapse rates computed at ages passed through, but he thought the matter was materially improved by the ingenious suggestion, first made, he thought, by Mr. Watson, of eliminating from the experience the lapses in respect of the new entrants during the quinquennium; and he had adopted that course

himself in more than one case, because the computation of lapse rates for select tables was a very laborious undertaking, and sometimes an impracticable one.

The varying mortality of active and retired members over age 65, referred to also by Mr. Marr, was very curious. Whether it arose from the high scale of pay adopted on retirement or not, it seemed to be peculiarly significant. He thought that the suggestion made by Mr. Watson, where there were no data adequate to form a table, to value the combined facts by the total cost at each age, was a particularly valuable and useful one. As regards allowances on retirement through lapse, he had recently had a case of a society of tramway employees, where a return was made, at age 65, of a stated proportion of their individual contributions, after deducting their individual sick pay, and it was found, on investigating the whole experience, that the proportion in question was exceeded by the sickness experience of the members as a whole, so that a special provision had to be made for that liability to the individual member who was fortunate enough not to have reached the average proportion. It was a question whether the actuary could properly recommend such a provision for insertion or retention in the rules of such societies.

With regard to orphan benefits, Mr. Watson referred to some interesting complex conditions of benefit. He did not know whether Mr. Watson had seen the data, in respect of orphans, for the five years 1902 to 1906, published a few years ago by the authorities in New Zealand. It appeared to be the practice there, when a death certificate of a father was obtained, to have compulsory particulars as to the number of children living, and their ages, and they were all tabulated as at the death of the father, and formed a useful body of facts. He thought Mr. King had dealt with similar data, but in respect of an earlier period (*see J.I.A.*, vol. xxx, p. 299). The later figures had been found useful, in deducing certain results for the information of the Royal Commission on the Poor Laws.

With regard to occupation risks, Mr. Watson put forward his specimens and resulting rates with a good deal of modesty, and expressed some doubts as to their being sufficient or adequate for the purpose. Their value seemed to be qualified by the fact that they would only apply, where the age distribution in question was similar to that which Mr. Watson adopted as a basis; and the same remarks, as to the specimen values being rather suggestive than final, applied to the accident rates given by Mr. Watson. It was extremely desirable, especially in these days of Workmen's Compensation Acts, to be able to distinguish accident risks from sickness risks, but at present there did not seem to be very much data available for the purpose.

In reference to valuations with allowance for lapses, Mr. Watson referred to the paper that Mr. Bacon and he had contributed some years ago, but he wished to say at once that that paper had nothing to do with the experience or valuation of collecting societies. They were, in that paper, investigating the experience of an industrial

assurance company, which appeared to him to be quite another thing. It might be suitable in such a case to adopt a population mortality table, whereas, in the case of a collecting society, there would be at least a tendency for the classes assured to be in geographical areas, or in occupational groups; and, in particular, the management of a collecting society was, he thought, conducted on somewhat different lines from that of an industrial assurance company. He did not, moreover, think that it was intended in that paper to advocate the adoption of the English Life Table No. 3 as a suitable mortality table, even in the case of an industrial assurance company; but in that particular case they satisfied themselves that the combined rate of exit did not differ very materially from the combined rates of death and lapse assumed, and he thought that was good enough for their purpose, which was mainly academic, and to submit specimens of methods, rather than practical results which could be relied upon in particular cases. He agreed with Mr. Watson that, in dealing with the most delicate question of valuations with allowance for lapses, great skill and tact were required. Thanks partly to Mr. King's labours, and also to Mr. Marr's excellent tabulations, the actuarial world was now in possession of the English Life Table No. 6, and no doubt that table would be adopted in similar cases, although Mr. Watson had shown, even there, curious departures from the experience of a particular society. The diminution, with increased duration, of the mortality rate experienced in certain societies, appeared to him to be a very singular phenomenon, and the suggestion thrown out by the author, that it might arise from the lives lapsing being of an inferior class, appeared to him to require a good deal of confirmation, before it could be accepted. The varying rates of lapse, in respect of whole-life and endowment assurances, was what one would expect from the facts. In an endowment assurance, the policyholder put a larger stake into the concern, and wagered, as it were, that the life would survive; in the case of whole-life assurance, the premium was smaller, and the sum was payable at death only, and it was quite natural there should be a better staying power in one class than in the other.

Towards the close of the paper, Mr. Watson dealt with various allowances and adjustments which had to be made, some of which were very interesting, and new to him. An allowance for errors in age was an important point, and also the question of "arrears," which presented considerable difficulty when investigating the mortality experience of an industrial assurance company, because it appeared to be extremely difficult to determine when the risk actually ceased by lapse. He entirely agreed with the remarks as to the analysis of profit and loss, and he thought it should be part of the duty of every actuary who valued a friendly society to try and reconcile his results with those of the previous valuation, allowing for differences in the rate of interest and in the mortality table adopted, and for all the facts that were under observation. That was not only a useful check on the accounts, but also on the accuracy of the valuation.

THE PRESIDENT said that Mr. Watson was recognized as being a master, not only of the scientific principles which underlay the operations of friendly societies, but also of the practical details of their working. His paper should be thoroughly studied by all the members of the Institute, and much good would be done, if it reached the large body of persons outside the Institute who were practically concerned in the management of friendly societies.

There were a great many points of interest in the paper. The first that struck him was the stationary character of the membership of the large societies during the last few years. Mr. Watson reduced the supposed number of about six million members to something like four-and-a-half millions, which represented not much more than about 40 per-cent of the workers in this country who might be expected to avail themselves of the advantages of such societies. That ratio seemed to be a very unsatisfactory one, and, when one saw how large a number of the workers neglected their opportunities, one could hardly wonder that schemes of compulsory insurance were in the air.

It was satisfactory to find that the societies were gradually reaching a better financial position than formerly. At first sight, it seemed curious that that should be so, because it was found that, generally speaking, the average sickness experience of the societies was somewhat in excess of the standard by which they valued, and yet as time went on the position of the societies as a whole improved. That no doubt was due to two causes, first, because the worst societies gradually dropped out and became dissolved, or special efforts were made locally to put them in a sound position; and, secondly, because no doubt the valuations made, by taking no account in the majority of cases of the element of lapse, somewhat underestimated the actual financial strength of the societies. It was quite right that the element of lapse should not be taken account of in small individual cases; but in dealing with thousands of societies its effect was in course of time to materially assist in improving their financial position as a whole.

The moral element, in relation to sickness claims, was a very important one. Mr. Watson showed that the rate of sickness was a function influenced by considerations other than those purely physical or physiological. The relation borne by the amount of sick-pay to the average wage of the person assured, was a most important question, and it was somewhat disturbing to find that even half-pay during prolonged sickness was an inducement to numbers of claimants to remain unduly on the sick list, and that the societies were finding that they could not afford to give more than quarter-pay in cases of prolonged illness. Various suggestions had been made from time to time as to how this moral problem was to be met. Mr. Kenchington had alluded to a society in which the death benefits varied, from time to time, with the amount of sickness claimed by the assured during the preceding period. He had himself met with a society of that kind, in which the effect on the sickness-rate was very marked. The death benefit in that case was a function

of the total amount that the deceased member had drawn for sick pay. But none of those suggestions really met the case, and he was afraid that the problem of providing in prolonged sickness a benefit which really met the requirements of the sick man, and at the same time prevented malingering, was, for the present at all events, unsolved.

The effect of legislation on friendly societies was an important matter, and he thought that effect would be found, so far as the Workmen's Compensation and the Old Age Pensions Acts were concerned, eventually to be a very advantageous one. The financial difficulty of the societies was to keep down the sickness rates to the standard required by their valuations, and also to provide for the prolonged sickness in old age, for which most of them had charged inadequate contributions. The Workmen's Compensation Act, if properly utilised, ought to assist the societies who were suffering from excessive sickness to reduce the claims, by not giving allowances to members who were already benefiting under that Act. Mr. Watson had given a very interesting series of tables, in which he showed that the excess sickness, due to certain occupations, and particularly in connection with certain societies, was mainly due to sickness resulting from accident, which was met by the Act in question. The Old Age Pensions Act certainly ought to assist societies in reducing the provision for old age sickness, which they had nominally, but not really, made in the contributions actually charged.

The position of the societies with respect to their periodical valuations was one of the greatest importance. It seemed to him that some change in the law was certainly required, and that appeared to be the trend of the discussion. The powers of the Registry ought to be increased, and the powers which they had already should certainly be exercised. It was absurd that a society should be compelled to have a valuation of its assets and liabilities made, and then that a valuation should be accepted by the Registrar which was based on a rate of interest of $5\frac{3}{4}$ per-cent. The only way of meeting that difficulty seemed to be, to require that those who undertook the by no means simple problem of valuing friendly societies should pass some test as to their knowledge, which would ensure that they should understand the principles and the technical methods upon which that work depended.

A cordial vote of thanks was then unanimously passed to Mr. Watson.

MR. A. W. WATSON, in reply, said that the Holloway Societies had not been excluded from the membership of permanent societies, but were included under Item No. 7 of his summary statement, "Deposit Societies." That statement, however, excluded the members of the merely burial societies (some of which were local and some called collecting societies), and also the merely juvenile members; whilst, in the Registrar's summaries, the figures of all kinds of societies were lumped together, whether burial societies, or juvenile membership, or ordinary sickness societies.

With regard to the Workmen's Compensation Act, 1906, and the Old Age Pensions Act, 1908, either the societies might continue under their present conditions, in which case they would certainly have an increase of sickness claims, both in respect of members invalidated by accident, and in respect of members in receipt of old age pensions; or they could follow what he thought was the sensible course, by providing that their members should not receive benefits from the society when in receipt of accident allowances, and so be paid twice over, and should not receive sick pay when in receipt of pensions. Within the last month, he had laid it down to a very large society that its first concern should be to ensure that its branches would be solvent, on the condition that they paid the full benefits to members under seventy years of age, who were incapacitated by sickness and not by accident, and that any assets they might have over, after providing for that particular liability, should be applied to the provision of a supplementary pension after seventy years of age. That he took to be the logical outcome of recent legislation, and he considered that all friendly societies ought to be advised to re-arrange their constitution, and place it on that basis. If they did not do so, he was quite satisfied that the recent social legislation would eventually have the effect of making it quite impossible for large numbers of them to carry on their business. Mr. Marr referred to some small points in the tables. The reply to his enquiry as to those on pages 196 and 198, was that, both with regard to the young members in one case, and the members over 79 years of age in the other, the data were too small to enable him to give any conclusive information. With regard to ages 1 and 2 on page 235, the "central age at entry" referred to in the heading of course applied to ages 5, 10, 15, &c. At ages 1 and 2, it was the age next birthday at entry. He had no doubt that the preponderance of the lives at age 1 were entered very shortly after birth.

Mr. Oakley had raised a point of the greatest interest with regard to unregistered friendly societies. The legal position of those societies under the new Act certainly required definition. The difficulty would be to discover the unregistered friendly societies, and to make them, either submit themselves to registration, or make formal application to the Board of Trade for exemption. With reference, however, to these unregistered societies, great as was the task of bringing them into line, he thought the duty should be undertaken. He had long held the view that, if they would not take advantage of the Friendly Societies Act, and obtain the privileges of that Act by registration, that was no reason why they should continue to conduct business without meeting the obligations of the Act. They invited the public to join them, in precisely the same way as did the registered friendly societies, and he saw no reason why they should be exempt from rendering returns to the Registrar, or making valuations, so that the public might see their position. Certainly the powers of the Registrar, and the staff of the office, would have to be greatly extended, if it were deliberately decided to explore the country, and to bring the unregistered friendly societies

into some kind of relation with a Government Department.

Possibly Mr. Manly's illustrations as to medical certificates were a little extreme, but certainly there were some evils with regard to medical service. He believed, however, that, taken as a whole, medical men rendered to the societies much greater service than was recompensed by the comparatively small emolument that as a rule was paid to them.

Mr. Ackland questioned whether he (the author) had done deposit societies justice. He was aware that it was claimed for their system that it tended to keep down sickness claims, although that contention had not been established by any definite investigation; but, however that might be, there was something more in insurance than keeping down claims. Friendly societies, like other insurance bodies, existed for the payment of claims, as much as they did for the receipt of premiums; and he was strongly of opinion that any system that tempted a man to put the present needs of himself and his family in the background, in order that he might amass a substantial bank account for a future day, was not a proper system of assurance, and ought not to be actuarially praised, or even condoned, because it had the effect of keeping down sickness claims. There was much in the management of deposit societies no doubt worthy of commendation, but there was also much in their constitution which should be condemned with much greater force than he had attempted in the paper; and he was bound to say that a profound consideration of the system of the deposit societies had convinced him that they were not sufficiently reliable media through which the working classes could effect sickness assurance.

Formulae for the Valuation of Premiums payable more frequently than once a year. By GEORGE J. LIDSTONE, F.I.A., Actuary and Secretary of The Equitable Life Assurance Society.

The following Paper, based on notes made for the most part several years ago, has now been prepared with the aid of Messrs. H. E. MELVILLE, F.I.A., and L. A. BULLWINKLE, without whose kind assistance in checking the work, calculating tables and preparing for press, the writer would have been unable to put his notes in form for publication.

PRELIMINARY NOTE.

I. THE object of this paper is to investigate, and collect in a compendious and complete form, the principal formulae (both theoretical and practical) for the valuation of half-yearly, quarterly, monthly, and *mthly* premiums, whether payable for the whole of life or for a limited term only. The subject is touched upon in the Institute of Actuaries' *Text-Book*, Part II, and is more completely dealt with in a very useful paper by Mr. J. J.

McLauchlan, entitled, "On some Formulas for use in Life Office Valuations", printed in the *Transactions of the Actuarial Society of Edinburgh*, vol. ii, pp. 347 *et seq.* The present writer makes no claim to any essential novelty either of method or result, his principal object being to complete and arrange existing materials; but the reference in the title to the "Valuation of Premiums", instead of the more usual form, namely, the "Valuation of Policies", indicates a difference in the point of view. For practical valuation work, it is not usual (at any rate in Great Britain) to operate with policy-values, but rather to strike a balance between the total value of the sums assured (including bonus additions) and the total value of the premiums. Since the former values are not affected by differences in the mode of payment of the premiums, it is thought both clearer and more convenient to omit any reference to them, and to deal solely with the valuation of the premiums. This is the way in which the problem presents itself in practice, and this way of treating it has the advantage that the resulting formulæ in most cases apply to office premiums as well as pure premiums (which is not the case with formulæ based on policy-values), and, moreover, in the case of premiums payable for a term only, the formulæ apply equally to endowment assurances, temporary assurances, and whole-life assurances by limited payments.

2. It will first be desirable to find and record certain special annuity-values, and for completeness of record the formulæ will be taken as far as the terms involving the first differential coefficients of D_x and M_x (*i.e.*, terms involving μ and δ), though these are usually omitted in practice. These theoretical formulæ will be given in the earlier part of this paper (pars. 3-19); while in the later part (pars. 20-31) the terms involving μ and δ will be omitted, and practical working rules deduced.

FORMULÆ FOR WHOLE-TERM ANNUITY-VALUES.

3. From the general formula (*Text-Book*, Part II, p. 477, form. 22)—

$$\begin{aligned} \sum_m u &= \frac{1}{m} (u_0 + u_1 + \dots ad inf.) \\ &= \int_0^\infty u_x dx + \frac{1}{2m} u_0 - \frac{1}{12m^2} \frac{du_0}{dx} + \frac{1}{720m^4} \frac{d^3u_0}{dx^3} \dots \quad (1) \end{aligned}$$

(in which formula it is assumed that u and its differential coefficients all vanish at the higher limit), we find at once, on

replacing u_0 by u_t , or $u_0 + t \frac{du_0}{dx} + \frac{t^2}{2} \frac{d^2 u_0}{dx^2} + \frac{t^3}{6} \frac{d^3 u_0}{dx^3} + \dots$, and making corresponding substitutions in $\frac{du_0}{dx}$, etc. :—

$$\begin{aligned}\Sigma^m u_t &= \frac{1}{m} (u_t + u_{t+\frac{1}{m}} + u_{t+\frac{2}{m}} + \dots) \\ &= \int_0^{\infty} u_x dx + \left(\frac{1}{2m} - t \right) u_0 - \frac{1 - 6mt + 6m^2t^2}{12m^2} \frac{du_0}{dx} + \dots \quad (1A)\end{aligned}$$

(See Woolhouse, *J.I.A.*, xv, p. 102).

When $m=1$, we have

$$\Sigma u_t = \int_0^{\infty} u_x dx + \left(\frac{1}{2} - t \right) u_0 - \frac{1}{12} (1 - 6t + 6t^2) \frac{du_0}{dx}, \dots \quad (1B)$$

and hence,

$$\Sigma u_t - \Sigma^m u_t = \frac{m-1}{2m} u_0 - \frac{1}{12m^2} (m^2 - 1 - 6mt, m-1) \frac{du_0}{dx}. \dots \quad (1C)$$

Putting $u_t = D_{x+t}$, and hence $\frac{du_0}{dx} = \frac{dD_x}{dx} = -D_x(\mu_x + \delta)$,

formula (1A) gives (omitting subscripts for brevity and dividing out by D_x)—

$${}^t \mathbf{a}^m = a + \left(\frac{1}{2m} - t \right) + \frac{\mu + \delta}{12m^2} (1 - 6mt + 6m^2t^2). \dots \quad (2)$$

$$= a + \left(\frac{m+1}{2m} - t \right) + \frac{\mu + \delta}{12m^2} (1 - 6mt + m^2\overline{6t^2-1}) \dots \quad (2A)$$

4. If the values of t be equally spread over the interval 0 to $\frac{1}{m}$, the mean annuity-value will be

$$\frac{\int_0^{\frac{1}{m}} {}^t \mathbf{a}^m dt}{\frac{1}{m}} = m \int_0^{\frac{1}{m}} {}^t \mathbf{a}^m dt, \text{ or say } \frac{1}{m} \mathbf{a}^m.$$

It is easily shown that between these limits, the average value (*i.e.*, m times the integral) of t is $\frac{1}{2m}$, that of $1 - 6mt + 6m^2t^2$ is 0, and that of $1 - 6mt + m^2(6t^2 - 1)$ is $-m^2$. Hence, substituting in (2) and (2A)—

$${}_0 \dots {}_1 \frac{1}{m} \mathbf{a}^m = \bar{a} \quad \text{for all values of } m \quad . . . \quad (3)$$

$$= a + \frac{1}{2} - \frac{\mu + \delta}{12} \quad , , , , , \quad . . . \quad (3A)$$

(*cf.* Woolhouse, *J.I.A.*, xv, p. 109, and Errata, *J.I.A.*, xviii, pp. 151-2).

5. Putting $m=1$ in (2), (2A), (3), we have for the special case of a yearly annuity—

$${}_t \mathbf{a} = \bar{a} + (\frac{1}{2} - t) + \frac{\mu + \delta}{12} (1 - 6t + 6t^2) \quad \quad (4)$$

$$= a + (1 - t) + \frac{\mu + \delta}{2} (t \cdot t - 1) \quad \quad (4A)$$

(*cf.* *Text-Book*, Part II, p. 175, formula (27), putting $\frac{1}{t}$ for t and $\frac{dD_x}{dx}$ for Δ_x); and

$${}_0 \dots {}_1 \mathbf{a} = \bar{a} \quad \quad (5)$$

6. The whole-term annuity-values hitherto deduced are collected in Table A (page 265), which gives also the numerical values of the coefficients for half-yearly, quarterly, and monthly cases.

VALUE OF THE UNPAID INSTALMENTS OF PREMIUM FOR THE YEAR OF DEATH.*

7. If the m thly payments of premium are treated as *instalments* of the annual premium (and not as true m thly *premiums*), any instalments which may remain outstanding, in respect of the policy-year current at the date of death, will be paid by deduction,

* The work in this section is believed to be new.

W. note Term Annuities. Values of $i \alpha^{(m)}$ for different values of t and m .By adjustment of \bar{a} By adjustment of \bar{a} General form for any value of t

m	$a + \left(\frac{1}{2m} - t\right) + (\mu + \delta)\left(\frac{1}{12m^2} + \frac{t(12t-1)}{2m}\right)$	$a + \left(\frac{m+1}{2m} - t\right) + (\mu + \delta)\left(\frac{1}{12m^2} + \frac{t(12t-1)}{2m} - \frac{1}{12}\right)$
1	$a + \left(\frac{1}{2} - t\right) + (\mu + \delta)\left(\frac{1}{12} + \frac{t(t-1)}{2}\right)$	$a + (1-t) + (\mu + \delta)\left(\frac{t(t-1)}{2}\right)$
2	$a + \left(\frac{1}{4} - t\right) + (\mu + \delta)\left(\frac{1}{48} + \frac{t(2t-1)}{4}\right)$	$a + \left(\frac{3}{4} - t\right) + (\mu + \delta)\left(\frac{t(2t-1)}{4} - \frac{1}{16}\right)$
4	$a + \left(\frac{1}{8} - t\right) + (\mu + \delta)\left(\frac{1}{192} + \frac{t(4t-1)}{8}\right)$	$a + \left(\frac{5}{8} - t\right) + (\mu + \delta)\left(\frac{t(4t-1)}{8} - \frac{5}{64}\right)$
12	$a + \left(\frac{1}{24} - t\right) + (\mu + \delta)\left(\frac{1}{1728} + \frac{t(12t-1)}{24}\right)$	$a + \left(\frac{13}{24} - t\right) + (\mu + \delta)\left(\frac{t(12t-1)}{24} - \frac{143}{1728}\right)$

Value when $t = \frac{1}{m}$, i.e., ordinary curtate annuity

m	$a - \frac{1}{2m} + (\mu + \delta)\frac{1}{12m^2}$	$a + \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\mu + \delta)$
1	$a - \frac{1}{2} + (\mu + \delta)\frac{1}{12}$	a
2	$a - \frac{1}{4} + (\mu + \delta)\frac{1}{48}$	$a + \frac{1}{4} - \frac{1}{16}(\mu + \delta)$
4	$a - \frac{1}{8} + (\mu + \delta)\frac{1}{192}$	$a + \frac{3}{8} - \frac{5}{64}(\mu + \delta)$
12	$a - \frac{1}{24} + (\mu + \delta)\frac{1}{1728}$	$a + \frac{11}{24} - \frac{143}{1728}(\mu + \delta)$

Mean Value (whatever the value of m) when t ranges equally over the interval 0 to $\frac{1}{m}$

All values

 a $a + \frac{1}{2} - \frac{1}{12}(\mu + \delta)$

from the sum assured. The value of this payment in respect of outstanding instalments may be found as follows:

8. Let the annual premium be 1, payable in m thly instalments of $\frac{1}{m}$ each; and let the age of the life be x . Further, let

$$\bar{M}_x = \int_0^{\infty} \mu_{x+t} \cdot D_{x+t} \cdot dt = D_x \bar{A}_x = D_x (1 - \delta \bar{u}_x) = D_x - \delta \bar{N}_x,$$

so that

$$\frac{d\bar{M}_x}{dx} = \bar{M}'_x = -\mu_x D_x.$$

Then if death occur in the first $\frac{1}{m}$ th of the first policy-year, the outstanding instalments will amount to $\frac{m-1}{m}$, and the corresponding present value of the payment (omitting the denominator D_x) is

$$\frac{m-1}{m} (\bar{M}_x - M_{x+\frac{1}{m}})$$

If death occur in the second $\frac{1}{m}$ th of the first policy-year, the outstanding instalments will amount to $\frac{m-2}{m}$, and the value is

$$\frac{m-2}{m} (\bar{M}_{x+\frac{1}{m}} - \bar{M}_{x+\frac{2}{m}})$$

and so on. Thus the total value of the payment in respect of the first policy-year is

$$\begin{aligned} & \frac{m-1}{m} (M_x - \bar{M}_{x+\frac{1}{m}}) + \frac{m-2}{m} (\bar{M}_{x+\frac{1}{m}} - M_{x+\frac{2}{m}}) + \dots \\ & \quad + \frac{1}{m} (\bar{M}_{x+\frac{m-2}{m}} - \bar{M}_{x+\frac{m-1}{m}}) \\ &= M_x - \frac{1}{m} (\bar{M}_x + \bar{M}_{x+\frac{1}{m}} + \bar{M}_{x+\frac{2}{m}} + \dots + \bar{M}_{x+\frac{m-1}{m}}) \end{aligned}$$

Similarly, for the second policy-year, the value is

$$\bar{M}_{x+1} - \frac{1}{m} (\bar{M}_{x+1} + \bar{M}_{x+1+\frac{1}{m}} + \bar{M}_{x+1+\frac{2}{m}} + \dots + \bar{M}_{x+1+\frac{m-1}{m}})$$

and so on for succeeding years. Thus the total value over the whole of life is

$$\bar{M}_x + \bar{M}_{x+1} + \dots - \frac{1}{m} (\bar{M}_x + \bar{M}_{x+\frac{1}{m}} + \dots)$$

which is equal to*

$$\frac{m-1}{2m} \bar{M}_x - \frac{m^2-1}{12m^2} \frac{d\bar{M}_x}{dx} \dots = \frac{m-1}{2m} \bar{M}_x + \frac{m^2-1}{12m^2} \mu_x D_x \dots \quad (6)$$

Dividing by D_x , we have the required value, namely,

$$\frac{m-1}{2m} \bar{A}_x + \frac{m^2-1}{12m^2} \mu_x \dots \dots \dots \quad (7)$$

9. The last expression gives the value, at the commencement of a policy-year, of the payment to be made at death in respect of outstanding instalments. Adding this to the value of the ordinary m thly annuity-due at the same epoch, namely,

$$\mathbf{a}_m^m = \mathbf{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2} (\mu_x + \delta),$$

the following expression (in which subscripts may be omitted) is obtained for the value at the commencement of a policy-year of an annual premium of 1 payable by m thly instalments of $\frac{1}{m}$, with the stipulation that any instalments that remain outstanding at the date of death in respect of the policy-year then current shall be paid or deducted from the sum assured, namely,

$$\mathbf{a}^m + \left[\frac{m-1}{2m} \bar{A} + \frac{m^2-1}{12m^2} \mu \right] \dots \dots \dots \dots \dots \quad (8)$$

$$= \mathbf{a} - \frac{m-1}{2m} - \frac{m^2-1}{12m^2} (\mu + \delta) + \frac{m-1}{2m} \bar{A} + \frac{m^2-1}{12m^2} \mu$$

$$= \mathbf{a} - \frac{m-1}{2m} (1 - \bar{A}) - \frac{m^2-1}{12m^2} \delta$$

$$= \mathbf{a} - \frac{m-1}{2m} \delta \bar{a} - \frac{m^2-1}{12m^2} \delta$$

$$= \mathbf{a} - \frac{m-1}{2m} \delta \bar{a} + \delta \left(\frac{m-1}{4m} - \frac{m^2-1}{12m^2} \right)$$

$$= \mathbf{a} - \frac{m-1}{2m} \delta \bar{a} + \delta \left(\frac{2m^2 - 3m + 1}{12m^2} \right)$$

$$= \mathbf{a} - \frac{m-1}{2m} \delta \bar{a} + \delta \frac{(2m-1)(m-1)}{12m^2} \dots \dots \dots \quad (9)$$

* This result follows at once by putting \bar{M}_x for u_0 in the general formula $\Sigma^{(1)} u - \Sigma^{(m)} u = \frac{m-1}{2m} u_0 - \frac{m^2-1}{12m^2} \frac{du_0}{dx} \dots$, which is a simple transformation of formula (25), *Text-Book*, Part II, p. 477, or follows at once from formula (1c) of this paper, putting $t=0$.

10. From another point of view, the same result may be obtained by an alternative process which will be found instructive. Since the outstanding instalments are deducted at death, the total amount paid under the given conditions will be identical, whether the annual premium be paid in one sum in advance or in m thly instalments ; and the difference in value will be due entirely to the loss of interest arising from the deferred payment of the second and subsequent instalments in each year. If the outstanding instalments, instead of being payable in one sum at death, were respectively payable on their ordinary due dates in the year of death, the value of the m thly payments would obviously be—

$$\begin{aligned} \mathbf{a}_x \times \frac{\mathbf{a}_{\overline{1}}^{(m)}}{\mathbf{a}_{\overline{1}}} &= \mathbf{a}_x \times \frac{d}{f^{(m)}} = \mathbf{a}_x \times \frac{\delta - \frac{\delta^2}{2} + \frac{\delta^3}{6} - \frac{\delta^4}{24} \dots}{\delta - \frac{\delta^2}{2m} + \frac{\delta^3}{6m^2} - \frac{\delta^4}{24m^3} \dots} \\ &= \mathbf{a}_x \left(1 - \frac{m-1}{2m} \delta + \frac{(m-1)(2m-1)}{12m^2} \delta^2 \dots \right) \end{aligned}$$

since an annual payment of 1 at the beginning of each year would be replaced by the m thly annuity-certain, of which the value is $\mathbf{a}_{\overline{1}}^{(m)}$. Let n be the number of instalments outstanding at the date of death ; thus, the amount then to be paid or deducted will be $\frac{n}{m}$, while at the same date the average *value* of the outstanding instalments (considered as payable on their ordinary due dates) will be—

$$\frac{1}{\frac{1}{2m} + \frac{m}{m}} \mathbf{a}_{\overline{n}}^{(m)} = (1+i)^{\frac{1}{2m}} \cdot \mathbf{a}_{\overline{n}}^{(m)} = \frac{e^{\frac{\delta}{2m}} - e^{-\delta(\frac{2n-1}{2m})}}{j_{\cdot m}} = m(e^{\frac{\delta}{m}} - 1)$$

$$= \frac{\frac{n}{m} \delta - \frac{n(n-1)}{2m^2} \delta^2 + \dots}{\delta + \frac{\delta^2}{2m} + \dots} = \frac{n}{m} - \frac{n^2}{2m^2} \delta \dots$$

The difference between this value and the actual amount of the outstanding instalments will be $\frac{n^2}{2m^2} \delta$, and, assuming the

deaths to be equally distributed over the year, the average magnitude of this quantity will be—

$$\frac{1}{m} \sum_{n=0}^{n=m-1} \left(\frac{n^2}{2m^2} \cdot \delta \right) = \frac{1}{m} \cdot \frac{\overline{m-1} \cdot m \cdot 2m-1}{6} \frac{\delta}{2m^2} = \frac{(m-1)(2m-1)}{12m^2} \delta.$$

This being the value as at the date of death, the present value will be—

$$\frac{(m-1)(2m-1)}{12m^2} \delta \bar{A}_x = \frac{(m-1)(2m-1)}{12m^2} (\delta - \delta^2 a_x),$$

which may be replaced by

$$\frac{(m-1)(2m-1)}{12m^2} (\delta - \delta^2 a_x),$$

within the limits of accuracy of formula (9) in which terms of the order δ^2 are omitted. The last expression, namely—

$$\frac{(m-1)(2m-1)}{12m^2} (\delta - \delta^2 a_x),$$

thus gives approximately the amount by which $a_x \cdot \frac{a_1^{(m)}}{a_1}$ is to be increased in order to arrive at the full value, and the latter is therefore equal to

$$\begin{aligned} & a \cdot \frac{a_1^{(m)}}{a_1} + \frac{(m-1)(2m-1)}{12m^2} (\delta - \delta^2 a) \\ &= a \left[1 - \frac{m-1}{2m} \delta + \frac{(m-1)(2m-1)}{12m^2} \delta^2 \right] + \frac{(m-1)(2m-1)}{12m^2} (\delta - \delta^2 a) \\ &= a - \frac{m-1}{2m} \delta a + \delta \cdot \frac{(m-1)(2m-1)}{12m^2}, \end{aligned}$$

agreeing with formula (9).

11. The maximum value of the correction $\frac{(m-1)(2m-1)}{12m^2} \delta \bar{A}$ is $\frac{\delta}{6}$ (which is reached only when $m=\infty$ and $\bar{A}=1$), and it will usually be much smaller; this term will not, therefore, sensibly affect the third decimal place, and for practical purposes it may be neglected, giving the convenient formula $a_x \times \frac{d}{f_m}$ for the

value of the m thly premiums at the commencement of any policy-year. The following table shows how the values by this short formula compare with those based on the more complex formula No. (9).

TABLE B.

RATE OF INTEREST, 3 PER-CENT

a_x	$m=2$		$m=4$		$m=12$		$m=\infty$	
	$\frac{d}{f_{(2)}}$	Form. (9)	$\frac{d}{f_{(4)}}$	Form. (9)	$\frac{d}{f_{(12)}}$	Form. (9)	$\frac{d}{f_{(\infty)}} = \frac{d}{\delta}$	Form. (9)
25	25.809	25.810	25.714	25.715	25.651	25.652	25.619	25.621
20	20.846	20.847	20.769	20.770	20.718	20.720	20.693	20.695
15	15.883	15.884	15.824	15.826	15.785	15.788	15.766	15.768
10	10.919	10.920	10.879	10.881	10.852	10.855	10.839	10.842
5	5.956	5.958	5.934	5.937	5.919	5.923	5.912	5.916

RATE OF INTEREST, 4 PER-CENT

	$m=2$		$m=4$		$m=12$		$m=\infty$	
	$\frac{d}{f_{(2)}}$	Form. (9)	$\frac{d}{f_{(4)}}$	Form. (9)	$\frac{d}{f_{(12)}}$	Form. (9)	$\frac{d}{f_{(\infty)}} = \frac{d}{\delta}$	Form. (9)
20	20.796	20.796	20.695	20.695	20.627	20.628	20.594	20.595
15	15.845	15.846	15.767	15.769	15.716	15.718	15.690	15.693
10	10.893	10.894	10.849	10.842	10.805	10.808	10.787	10.791
5	5.942	5.944	5.913	5.916	5.893	5.898	5.884	5.889

12. In paragraph (8), an expression was found for the value of the deduction for outstanding instalments reckoning from the beginning of a policy-year : the value may now be found reckoning from any other point of time in the year. Let there be k mthly instalments outstanding, and let the interval to the next mthly renewal date be t . Then, omitting the denominator D_x , the value up to the next anniversary is

$$\begin{aligned} & \frac{k}{m} (M_x - M_{x+t}) + \frac{k-1}{m} (M_{x+t} - M_{x+t+\frac{1}{m}}) + \dots \\ & \quad + \frac{1}{m} (M_{x+t+\frac{k-2}{m}} - M_{x+t+\frac{k-1}{m}}) \\ & = \frac{k}{m} M_x - \frac{1}{m} [M_{x+t} + M_{x+t+\frac{1}{m}} + \dots + M_{x+t+\frac{k-1}{m}}] \quad \dots \quad (A) \end{aligned}$$

From the next anniversary, the value for the remainder of life will be found—as in paragraph (8)—to be

$$M_{x+t+\frac{k}{m}} + M_{x+t+\frac{k}{m}+1} + \dots - \frac{1}{m} (\bar{M}_{x+t+\frac{k}{m}} + \bar{M}_{x+t+\frac{k+1}{m}} + \dots) . \quad (\text{B})$$

13. Adding expressions (A) and (B), the total value is found as follows :

$$\begin{aligned} \frac{k}{m} M_x &= \frac{k}{m} M_x \\ M_{x+t+\frac{k}{m}} + M_{x+t+\frac{k}{m}+1} + \dots &= \int_0^\infty \bar{M}_x dx + \left(\frac{1}{2} - t - \frac{k}{m} \right) M_x \\ &\quad - \frac{1}{12} \left(1 - 6t + \frac{k}{m} + 6t + \frac{k^2}{m^2} \right) \frac{d\bar{M}_x}{dx} * \\ \frac{1}{m} (M_{x+t} + M_{x+t+\frac{1}{m}} + \dots) &= - \int_0^\infty \bar{M}_x dx - \left(\frac{1}{2m} - t \right) M_x \\ &\quad + \frac{1}{12m^2} \cdot (1 - 6mt + 6m^2t^2) \frac{dM_x}{dx} \dagger \\ \text{Total} &= m - 1 \cdot M_x - \frac{m^2 - 1}{12m^2} \cdot \frac{d\bar{M}_x}{dx} \\ &\quad + \frac{dM_x}{dx} \left[t \left(\frac{1}{2} - \frac{1}{2m} \right) + \frac{k}{2m} - \frac{kt}{m} - \frac{k^2}{2m^2} \right] \end{aligned}$$

14. Dividing by D_x , and replacing $\frac{1}{D_x} \frac{dM_x}{dx}$ by its equivalent $-\mu_x$ [see par. (8)], we have as the final result—

$$\left[\frac{m-1}{2m} A_x + \frac{m^2-1}{12m^2} \mu_x \right] - \mu_x \left[t \cdot \frac{m-(2k+1)}{2m} + \frac{k(m-k)}{2m^2} \right] \dots (10)$$

which may be put into the form—

$$\left[\frac{m-1}{2m} A_x + \frac{m^2-1}{12m^2} \mu_x \right] - \frac{\mu_x}{2} \left[\left(\frac{k}{m} + t \right) \left(1 - \frac{k}{m} + t \right) - t \left(\frac{1}{m} - t \right) \right] \dots (10\alpha)$$

where the first term represents the value previously found as at the beginning of a policy-year, and the second is the additional

* From formula (1B), putting $u = \bar{M}_x$, and $t + \frac{k}{m}$ for t .

† (1A),

term introduced at any other point of time. It is not difficult to show that if all possible values of t and k are equally likely—*i.e.*, if the business is equally spread over the year reckoning from the date of valuation—the average value of this second term is

$-\frac{m^2-1}{12m^2}\mu_x$, so that the average value of the whole expression is $\frac{m-1}{2m}\bar{A}_x$. This result is also evident from general considerations.

Under the given conditions, the average amount of the outstanding instalments will always be the same, being the average of all possible values of $\frac{k}{m}$, *i.e.*,

$$\frac{1}{m} \sum_{k=0}^{k=m-1} \frac{k}{m} = \frac{1}{m} \cdot \frac{m(m-1)}{2m} = \frac{m-1}{2m}.$$

This being the amount—taking one ease with another—receivable at death whenever it happens, the value must be accurately $\frac{m-1}{2m}\bar{A}_x$, as found by direct algebraical work.

15. From formula (10A) it is not difficult to find the maximum numerical value of the corrective term $[\quad]\mu_x$. It will be found that when m is even, which of course it usually is in practice, the maximum value is $\frac{1}{8}\mu_x$, which occurs in the middle of the year, when $k = \frac{m}{2}$ and $t = 0$. When m is odd, the maximum value is $\frac{1}{8} \left[1 - \frac{1}{m^2} \right] \mu_x$, which occurs at every point within the middle $\frac{1}{m}$ th of the year, *i.e.*, when $k = \frac{m-1}{2}$, and t takes any value from 0 to $\frac{1}{m}$.

16. Adding formula (2A) to (10A), and transforming as in Par. 9, we shall find the following expression for the total value of a premium of 1 payable by m thly instalments, with the stipulation that any instalments that remain outstanding at the date of death in respect of the policy year then current, shall be paid or deducted from the sum assured, there being k m thly instalments outstanding at the present time, and the interval to the next renewal date being t .

$$\left[\mathbf{a} \left(1 - \frac{m-1}{2m} \delta \right) + \delta \frac{(2m-1)(m-1)}{12m^2} \right] \\ - \left[t + \frac{\delta t}{2} \left(\frac{1}{m} - t \right) + \frac{\mu}{2} \left(\frac{k}{m} + t \right) \left(1 - \frac{k}{m} + t \right) \right] . . . (11)$$

where the first line is the corresponding value at an anniversary (formula 9) and the second line is the correction at any other point. This correction vanishes as a whole, and in respect of t , δ and μ separately, at each anniversary; the terms involving t and δ vanish also at each renewal date.

TEMPORARY ANNUITY-VALUES.

17. The necessary temporary annuity-values can easily be found from the whole-life annuity-values already determined, the general formula of relation being—

$${}_{t+n} \mathbf{a}_x^m = {}_{t+1} \mathbf{a}_x^m - \frac{D_{x+n}}{D_x} {}_t \mathbf{a}_{x+n}^m$$

or
$${}_{t+n} \mathbf{a}^m = {}_{t+1} \mathbf{a}^m - {}_n E \cdot {}_t \mathbf{a}^m.$$

Substituting the values of ${}_{t+1} \mathbf{a}^m$ and ${}_n E$ from formula (2), this gives—

$$\begin{aligned} {}_{t+n} \mathbf{a}^m &= \bar{a} + \left(\frac{1}{2m} - t \right) + \frac{\mu + \delta}{12m^2} (1 - 6mt + 6m^2t^2) \\ &\quad - {}_n E \cdot \bar{a} - \left(\frac{1}{2m} - t \right) {}_n E + \frac{{}_n E (\mu + \delta)}{12m^2} (1 - 6mt + 6m^2t^2) \\ &= {}_n \bar{a} + \left(\frac{1}{2m} - t \right) (1 - {}_n E) + \frac{1 - 6mt + 6m^2t^2}{12m^2} \times \\ &\quad [\mu + \delta - {}_n E (\mu + \delta)] . . . (12) \end{aligned}$$

Similarly, from formula (2A)—

$$\begin{aligned} {}_{t+n} \mathbf{a}^m &= {}_n a + \left(\frac{m+1}{2m} - t \right) (1 - {}_n E) + \frac{1 - 6mt + m^2(6t^2 - 1)}{12m^2} \times \\ &\quad [\mu + \delta - {}_n E (\mu + \delta)] . . . (12A) \end{aligned}$$

Also, by equating the last two results, or directly from formula (3A)—

$${}_n \bar{a} = {}_n a + \frac{1}{2} (1 - {}_n E) - \frac{1}{12} [\mu + \delta - {}_n E (\mu + \delta)] . . . (13)$$

18. To find $t \ n + \frac{k}{m} \mathbf{a}^m$, the value of 1 per annum payable by m thly instalments for n complete years and k months of a year, the first payment being due after the interval t , we may proceed in two alternative ways, each leading to the same result.

(i) From formula (12) we have—retaining only terms involving $_n E$ and its first differential coefficient, $_n E'$, and remembering that $[\mu + \delta - {}_n E({}^n \mu + \delta)] = - {}_n E' + {}_n E' -$

$$\frac{d}{dn} (t \ n \mathbf{a}^m) = {}_n E - \left(\frac{1}{2m} - t \right) {}_n E'$$

$$\frac{d^2}{dn^2} (\quad, \quad) = \quad \quad \quad {}_n E'$$

Therefore, by Taylor's theorem,

$$\begin{aligned} t \ n + \frac{k}{m} \mathbf{a}^m &= t \ n \mathbf{a}^m + \frac{k}{m} \left[{}_n E - \left(\frac{1}{2m} - t \right) {}_n E' \right] + \frac{k^2}{2m^2} {}_n E' \\ &= t \ n \mathbf{a}^m + \frac{k}{m} {}_n E + {}_n E' \left(\frac{kt}{m} + \frac{k \cdot k - 1}{2m^2} \right) \dots \dots \dots \quad (14) \end{aligned}$$

(ii) From first principles,

$$\begin{aligned} t \ n + \frac{k}{m} \mathbf{a}^m &= t \ n \mathbf{a}^m + \frac{1}{m} \cdot {}_{n+t} E + \frac{1}{m} \cdot {}_{n+t+\frac{1}{m}} E + \dots + \frac{1}{m} \cdot {}_{n+t+\frac{k-1}{m}} E \\ &= t \ n \mathbf{a}^m + \frac{k}{m} \cdot {}_n E + \frac{1}{m} \cdot {}_n E' \left[t + \left(t + \frac{1}{m} \right) + \dots \right. \\ &\quad \left. + \left(t + \frac{k-1}{m} \right) \right] \\ &= t \ n \mathbf{a}^m + \frac{k}{m} \cdot {}_n E + {}_n E' \left(\frac{kt}{m} + \frac{k \cdot k - 1}{2m^2} \right), \text{ as before.} \end{aligned}$$

Now—

$${}_n E' = \frac{d}{dn} \left(\frac{D_{x+n}}{D_x} \right) = \frac{dn}{D_x} \frac{D_{x+n}}{D_x} = \frac{-D_{x+n}({}^n \mu + \delta)}{D_x} = - {}_n E ({}^n \mu + \delta).$$

Substituting this value in (14), and replacing $t \ n \mathbf{a}^m$ by its value according to (12) and (12A), we have, finally—

$$\begin{aligned} t \ n + \frac{k}{m} \mathbf{a}^m &= {}_n u + \left(\frac{1}{2m} - t \right) + {}_n E \left(\frac{k}{m} + t - \frac{1}{2m} \right) + (\mu + \delta) \frac{1 - 6mt + 6m^2t^2}{12m^2} \\ &\quad - {}_n E ({}^n \mu + \delta) \frac{1 - 6mt + 6m^2t^2 + 12mkt + 6k \cdot k - 1}{12m^2} \dots \dots \quad (15) \end{aligned}$$

$$= {}_n a + \left(\frac{m+1}{2m} - t \right) + {}_n E \left(\frac{k}{m} + t - \frac{m+1}{2m} \right) + (\mu + \delta) \frac{1 - 6mt + m^2(6t^2 - 1)}{12m^2} \\ - {}_n E({}^n \mu + \delta) \frac{1 - 6mt + m^2(6t^2 - 1) + 12mkt + 6k \cdot k - 1}{12m^2} \dots (16)$$

since ${}_n \mathbf{a} = {}_n a + 1 - {}_n E$,

$${}_{t \cdot n + \frac{k}{m}} \mathbf{a}^m = {}_n \mathbf{a} - \left(\frac{m-1}{2n} + t \right) + {}_n E \left(\frac{k}{m} + \frac{m-1}{2m} + t \right) + \text{small terms.} \dots (16A)$$

19. If t range equally over the interval 0 to $\frac{1}{m}$, the mean annuity-value will be by reference to \bar{a}_{x^n} ,

$$\dots \frac{1}{m} \cdot {}_{n+\frac{k}{m}} \mathbf{a}^m = {}_n \bar{a} + \frac{k}{m} {}_n E - {}_n E({}^n \mu + \delta) \frac{k^2}{2m^2} \dots \dots \dots (17)$$

$$= {}_n \bar{a} + \frac{k}{m} \cdot {}_{n+\frac{k}{m}} E \dots \dots \dots \dots \dots \dots (18)$$

or by reference to a_{x^n} ,

$$= {}_n a + \frac{1}{2} + {}_n E \left(\frac{k}{m} - \frac{1}{2} \right) - \frac{1}{12} (\mu + \delta) \\ - {}_n E({}^n \mu + \delta) \left(\frac{k^2}{2m^2} - \frac{1}{12} \right) \dots (19)$$

PRACTICAL WORKING FORMULE.

20. In practice it is neither convenient nor necessary to use the complicated formulæ which have been deduced and recorded in the earlier part of this paper. The terms involving μ and δ can be safely discarded, and other approximations made in order to arrive at practical working formulæ. These we may now proceed to discuss. It will be desirable to consider separately the case of (1) true m thly premiums, and (2) instalment premiums where the outstanding instalments are deducted from the sum assured. In each case, alternative formulæ will be required according as the equivalent annual premium or the actual premium payable per annum be valued; the first plan being usually the more convenient for pure and the second for gross premiums. It will be necessary also to consider the case of premiums payable for a limited term, and premiums payable for the whole of life; though the former case leads to the latter, if the age up to which the premiums are payable exceed the limit of life.

TRUE MONTHLY PREMIUMS.

21. If there are k monthly instalments outstanding, in respect of the current year's premium, and the next fall due after the interval t , then assuming that the actual monthly premiums (and not the equivalent annual premiums) are to be valued, the annuity-values required will be ${}_t \mathbf{a}^{(m)}$ for whole life payments and ${}_{t+n+\frac{k}{m}} \mathbf{a}^{(m)}$ for temporary payments. It will be convenient to put these into the form [corresponding annuity for yearly case] + [correction for monthly case], so that in the first instance all the premiums may be valued together, leaving a small correction to be made for monthly cases. Discarding terms involving μ and δ , we shall have the following results:

Whole-Life Payments :

$$\begin{aligned} {}_t \mathbf{a}^{(m)} &= \mathbf{a} - \left(\frac{m-1}{2m} + t \right) \\ {}_{t+\frac{k}{m}} \mathbf{a} &= \mathbf{a} - \left(\frac{k}{m} + t \right) \\ \therefore {}_t \mathbf{a}^{(m)} &= {}_{t+\frac{k}{m}} \mathbf{a} + \left(\frac{k}{m} - \frac{m-1}{2m} \right) \quad \end{aligned} \quad (20)$$

Temporary Payments :

$$\begin{aligned} {}_{\frac{k}{m}+t} {}_n \mathbf{a} &= {}_n \mathbf{a} - \left(\frac{k}{m} + t \right) + {}_n E \left(\frac{k}{m} + t \right) \\ {}_{t+n+\frac{k}{m}} \mathbf{a}^{(m)} &= {}_n \mathbf{a} - \left(\frac{m-1}{2m} + t \right) + {}_n E \left(\frac{k}{m} + \frac{m-1}{2m} + t \right) \\ &= \frac{k}{m} + t {}_n \mathbf{a} + \frac{k}{m} {}_n E + \left(\frac{k}{m} - \frac{m-1}{2m} \right) (1 - {}_n E) \quad . . . \end{aligned} \quad (21)$$

22. If the business be equally spread over the year the average value of $\frac{k}{m}$ will be $\frac{m-1}{2m}$ and the last term in (20) and (21) will vanish; while, if the business be proportionately heavier at the end of the year $\left(\frac{k}{m} - \frac{m-1}{2m} \right)$ will have a small positive average value, and if this term be omitted the value of the premiums will be slightly understated, which will be on

the safe side.* Thus the practical rule when true m thly premiums are valued is to use the same annuity-value as would be used for an annual case, so that the premiums may be valued with the yearly premiums; and (if the premiums are payable for a limited term) add the outstanding instalments multiplied by the pure endowment payable at the end of the term, a correction which vanishes when premiums are payable throughout life. In practice this correction would often be omitted, giving a small error on the safe side; but if the policies are arranged in groups according to valuation ages or unexpired terms, an average value of nE_y can, with sufficient accuracy, be found and used for each group; or, if in any special case greater accuracy be required, the corrections can be valued individually by means of Crelle's or Cotsworth's tables. The outstanding instalments for each policy being the same at each valuation can be conveniently recorded on the valuation cards. In most cases temporary annuity-values are tabulated and nE_y can be found to three places—which will be sufficient for practical work—from the difference $a_{y\bar{n}} - a_{y\bar{n}-1}$.

23. If it is desired to value the equivalent annual premiums, a course which is specially convenient for the pure premiums, we may make use of the following lemma.

24. *Lemma.*—Let P be the annual premium and $\mathbf{a}_{x\bar{r}}$ the corresponding annuity-value. Let π be the corresponding premium and $\mathbf{a}_{x\bar{r}} - e(1-rE_x)$ the corresponding annuity-value when the mode of payment is changed, the present value remaining unaltered. Then

$$\begin{aligned}\pi &= \frac{P\mathbf{a}_{x\bar{r}}}{\mathbf{a}_{x\bar{r}} - e(1-rE_x)} = P \frac{1}{1-e \frac{1-rE_x}{\mathbf{a}_{x\bar{r}}}} = P \left[1 + \frac{\frac{e}{1-r} E_x}{1 - \frac{e}{1-r} \frac{1-rE_x}{\mathbf{a}_{x\bar{r}}}} \right] \\ &= P + e\pi(P_{x\bar{r}}^1 + d) \text{ since } 1-rE_x \div \mathbf{a}_{x\bar{r}} = P_{x\bar{r}}^1 + d.\end{aligned}$$

* Taking the distribution of business shewn in Mr. Fraser's Table 2, *J.I.A.*, xxxviii, p. 389, we shall find the following values:

m	$\frac{k}{m}$ (Average)	$\frac{m-1}{2m}$	$\left(\frac{k}{m} - \frac{m-1}{2m}\right)$
2	.313	.250	.063
4	.471	.375	.096
12	.572	.458	.114

In each case $\left(\frac{k}{m} - \frac{m-1}{2m}\right)$ is almost exactly equal to $\frac{1}{5} \cdot \frac{k}{m}$, so that if the business is similarly distributed an adjustment for the neglected term can easily be made if thought necessary.

Let the annuity-value for the valuation of π be

$$\mathbf{a}_{yn} - e(1 - {}_nE_y) + g_n E_y - f$$

where $y+n=x+r$. Then the value of π or $P + e\pi(P_{yr}^1 + d)$ is

$$[P + e\pi(P_{yr}^1 + d)] [\mathbf{a}_{yn} - e(1 - {}_nE_y) + g_n E_y - f]$$

or remembering that $1 - {}_nE_y = \mathbf{a}_{yn}^{-} (P_{yn}^1 + d)$

$$= P \mathbf{a}_{yn}^{-} + e\pi \mathbf{a}_{yn}^{-} (P_{yr}^1 + d) - e\pi \mathbf{a}_{yn}^{-} (P_{yn}^1 + d) + g_n E_y \pi - f \pi$$

$$= P \mathbf{a}_{yn}^{-} - e\pi \mathbf{a}_{yn}^{-} (P_{yn}^1 - P_{yr}^1) + g_n E_y \pi - f \pi$$

$$= P \mathbf{a}_{yn}^{-} - e\pi \cdot r \cdot {}_nV_{xr}^1 + g_n E_y \pi - f \pi \quad \quad (22)$$

Making r and n such that $x+r$ and $y+n$ exceed the limiting age, we have the case of whole-life payments, and ${}_nE_y$ disappears while ${}_r \cdot {}_nV_{xr}^1$ becomes ${}_{y-x}V_x$. Thus the value of π for the whole of life becomes

$$P \mathbf{a}_y - f \pi - e\pi \cdot {}_{y-x}V_x \quad \quad (23)$$

25. In the case of premiums payable for r years from the commencement, the initial annuity-value corresponding to the m thly premiums, will be

$$\mathbf{a}_{xr}^{(m)} = \mathbf{a}_{xr} - \frac{m-1}{2m} (1 - {}_rE_x)$$

so that in using the lemma we must put $\frac{m-1}{2m}$ for e ; and the annuity at the date of valuation will be

$$t \mathbf{a}_{xr}^{(m)} = \mathbf{a}_{xr} - \left(\frac{m-1}{2m} + t \right) (1 - {}_nE_y) + \frac{k}{m} {}_nE_y$$

$$\text{or } \mathbf{a}_{yn}^{-} = \frac{m-1}{2m} (1 - {}_nE_y) - t + \left(\frac{k}{m} + t \right) {}_nE_y$$

so that $f=t$ and $g=\frac{k}{m}+t$. Hence, if P be the equivalent annual premium, the value of the m thly premiums is equal, by the lemma, to

$$P \mathbf{a}_{yn}^{-} = \frac{m-1}{2m} P \cdot {}_{r-n}V_{xr}^1 + \left(\frac{k}{m} + t \right) {}_nE_y P^m - t P^m$$

Since the coefficients of $P^{(m)}$ are small and $P^{(m)}$ is nearly equal to P , we may substitute the latter, giving, with some rearrangement of terms,

$$P\mathbf{a}_{y-n} - tP + \left(\frac{k}{m} + t\right)P_n E_y - \frac{m-1}{2m} P_{.r-n} V_{x_i}^1$$

The corresponding value in a yearly case would be $P_{.t+\frac{k}{m}} \mathbf{a}_y$.

or $P\mathbf{a}_{y-n} - \left(t + \frac{k}{m}\right)P + \left(t + \frac{k}{m}\right)P_n E_y$

The difference between the two last expressions is

$$\frac{k}{m} P - \frac{m-1}{2m} P_{.r-n} V_{x_i}^1 \quad \quad (24)$$

which, when premiums are payable for the whole of life, becomes

$$\frac{k}{m} P - \frac{m-1}{2m} P_{.y-x} V_x \quad \quad (25)$$

26. Since policy-values for temporary assurances grow slowly to a small maximum and then decrease to zero, the average value of the deductive term in formula (24) will be very small, as will be seen from Table C on the next page; so that in cases of premiums payable for a limited period (including whole-life limited payment policies, endowment assurances and temporary assurances where these are accurately valued) this term may be safely neglected (unless the premiums are payable up to a very advanced age) and the value of the m thly premiums found by adding the actual outstanding instalments to the value of the annual premiums.

27. In the case of premiums payable for the whole of life the deductive term $\frac{m-1}{2m} P_{.y-x} V_x$ grows steadily with the duration of the policies, and with an old-established business it may be too large to be neglected, particularly as it represents an addition to the reserves. The value may, however, be estimated with sufficient accuracy without detailed calculation. As pointed out in par. (22), if the business is equally spread over the year, $\frac{m-1}{2m} P$ will in the aggregate be equal to $\frac{k}{m} P$, while if the business be concentrated towards the end of the year $\frac{k}{m} P$, will be greater. Thus $\frac{k}{m} P$, the actual outstanding instalments, may safely be substituted for $\frac{m-1}{2m} P$, with the advantage that different

values of m need not be treated separately. An average value of V , sufficiently accurate for the purpose, can easily be determined from the class as a whole. Calculations based on Mr. King's Model Office (No. 1) with O^M mortality and 3 per-cent interest, shew that the approximate results are respectively, 90 per-cent, 94 per-cent, and 98 per-cent of the true results for the Model Office of age 15 years, 30 years, and 45 years respectively; giving a small error in the opposite direction to that of substituting $\frac{k}{m} P$ for $\frac{m-1}{2m} P$.

TABLE C. (*Basis O^M 3 per-cent*).

$$100_n V_{x:r} \times \frac{1}{2} P_{xr}.$$

r = Original number of Years, at the end of which the Sum Assured is payable

Age at Entry x	10 years		20 years			30 years			Age at Entry x	
	5	5	10	15	5	10	15	20		
20	.010	.015	.023	.020	.017	.032	.041	.042	.031	20
30	.017	.026	.040	.036	.033	.061	.082	.090	.070	30
40	.030	.054	.089	.083	.075	.146	.203	.229	.187	40
50	.074	.139	.232	.225	.192	.375	.528	.613	.536	50

The above table is based on the maximum value of the second factor, namely, $\frac{1}{2} P_{xr}$, which occurs only in the case of an endowment assurance policy subject to premiums payable continuously when $\frac{m-1}{2m} = \frac{1}{2}$. In monthly cases $\frac{1}{2}$ will be replaced by .458, in quarterly cases by .375, and in half-yearly cases by .25; and in limited premium whole-life cases P_{xr} will also be replaced by the smaller quantity a_x .

INSTALMENT PREMIUMS.

28. When the m thly premiums are instalment premiums, so that the outstanding instalments at the date of death are deducted from the sum assured, we have seen (par. 16) that if the next instalment is due after the interval t the value of a premium of 1 per annum is $a_x \left(1 - \frac{m-1}{2m} \delta\right) - t = (a_x - t) \left(1 - \frac{m-1}{2m} \delta\right)$

neglecting small terms in μ and δ . Similarly, after an interval of $n + \frac{k}{m} + t$, the value of the remaining premiums (the first instalment due and unpaid) will be $\mathbf{a}_{x+n+\frac{k}{m}+t} \left(1 - \frac{m-1}{2m} \delta\right)$ (par. 9). Hence the present value of premiums payable for $n + \frac{k}{m}$ years, the next instalment due after the interval t , will be

$$\begin{aligned} & \left(1 - \frac{m-1}{2m} \delta\right) (\mathbf{a}_x - t - n + \frac{k}{m} + t) \mathbf{E}_x \cdot \mathbf{a}_{x+n+\frac{k}{m}+t} \\ &= \left(1 - \frac{m-1}{2m} \delta\right) (\mathbf{a}_x - t - n) \mathbf{E}_x \cdot \mathbf{a}_{x+n} \\ &= \left(1 - \frac{m-1}{2m} \delta\right) [\mathbf{a}_x - t - n \mathbf{E}_x (\mathbf{a}_{x+n} - \frac{k}{m} + t)] \\ &= \left(1 - \frac{m-1}{2m} \delta\right) [\mathbf{a}_{x_n} - t + (\frac{k}{m} + t) n \mathbf{E}_x] \quad \quad (A) \end{aligned}$$

Now the corresponding annuity in a yearly case will be

$$t + \frac{k}{m} n \mathbf{a}_x = \mathbf{a}_{x_n} - \left(\frac{k}{m} + t\right) + \left(\frac{k}{m} + t\right) n \mathbf{E}_x.$$

Hence expression (A) = $\left(1 - \frac{m-1}{2m} \delta\right) \left[t + \frac{k}{m} n \mathbf{a} + \frac{k}{m}\right]$. As already pointed out, we may safely substitute $\frac{k}{m}$ for $\frac{m-1}{2m}$ in a negative term, giving

$$\left(1 - \frac{k}{m} \delta\right) \left(t + \frac{k}{m} n \mathbf{a} + \frac{k}{m}\right) \quad \quad (26)$$

Thus, when the m thly instalment premiums are valued we may use the same annuity-value as in the equivalent yearly case, adding the actual outstanding instalments of premiums and multiplying the result by $\left(1 - \frac{k}{m} \delta\right)$, where $\frac{k}{m}$ will have the average value $\frac{k}{m} P \div \Sigma P$. This correction can be applied to the total of the whole class, remembering that $\frac{k}{m} P$ will be the actual total of outstanding instalments in the m thly cases and ΣP the total premiums of the whole class, including yearly as well as m thly premiums.

29. If it be desired to value the equivalent annual premiums, we shall have, if $P^{[m]}$ denote the actual premium payable per annum, and P the equivalent annual premium,

$$P^{[m]} = P \mathbf{a} \div \left[\mathbf{a} \left(1 - \frac{m-1}{2m} \delta \right) \right] = P \div \left(1 - \frac{m-1}{2m} \delta \right)$$

Value of $P^{[m]}$

$$\begin{aligned} &= \left[P \div \left(1 - \frac{m-1}{2m} \delta \right) \right] \left[\left(1 - \frac{m-1}{2m} \delta \right) \left(t + \frac{k}{m} n \mathbf{a} + \frac{k}{m} \right) \right] \\ &= P \left[t + \frac{k}{m} n \mathbf{a} + \frac{k}{m} \right] = P \cdot t + \frac{k}{m} n \mathbf{a} + \frac{k}{m} P^m \text{ nearly . . . } (27) \end{aligned}$$

So that we have only to use the equivalent annual premiums and annuity and add the actual amount of the outstanding instalments.

30. In practice it is generally convenient to use the equivalent annual premiums where pure premiums are concerned, but the actual *mthly* gross premiums in order that these may be agreed in total with the office books. It is desirable to point out, what may easily be overlooked, that whereas for pure premiums the formulæ involving the annual premiums are approximately equivalent to those involving the actual *mthly* premiums, there is a systematic difference as regards gross premiums. In both cases, the use of the annual premiums gives the present value of the *mthly* premiums theoretically equivalent to the annual premiums; and the balance of the customary percentage addition to the gross premiums for *mthly* payments, over and above the small theoretical addition, is thrown off and remains unvalued, which is not the case with the formulæ involving the actual *mthly* premiums. The point is immaterial in the case of a pure premium valuation, since the values of the gross premiums do not affect the result. In the case of a valuation based on modified *gross* premiums, different views may be held, and reference may be made to some remarks by Messrs. King and Sprague (*J.I.A.*, xxiii, p. 256), where one side of the case is presented.

31. The practical working formulæ based on the results obtained in pars. 21–30, are summarized in the appended table. It will be observed that they are all reduced to such a form as to be independent of m so that yearly, half-yearly, quarterly, and monthly cases can all be valued together.

TABLE D.—SUMMARY OF WORKING FORMULAE.

Showing the correction to be made when the premiums, whether true annual premiums or instalment premiums, are valued by the same annuity as would be used for yearly premiums.

Premiums Payable	monthly Premiums Valued	Equivalent Annual Premiums Valued
For whole of life. (No correction).	Add (outstanding instalments) $\times \frac{1}{n} E_g$ *	Add (outstanding instalments) $\times (1 - V)$ †
For a limited term	Add (outstanding instalments) $\times n E_g$ *	Add (outstanding instalments)*‡
	May be valued individually by Circle, using $n E_g \cdot a_m - a_n$ if; or an average value of $n E_g$, corre- sponding to the average value of a_m , can be found for each age or term.	† V represents the mean value found by dividing the total reserves (excluding value of bonus or reduction of premium) by total sums assured. ‡ Assuming that the premiums are not payable up to a very advanced age.
For whole of life.	Add (outstanding instalments) and multiply the total by $\left[1 - \frac{\sum p}{m} \cdot \delta \right] ^{\frac{k}{m}}$	Add (outstanding instalments).
For a limited term	Do. do. do. do. do.	Do. do. do. do. do.
	$\frac{k}{m} \cdot P$ —total outstanding instalments of the class or group.	
	ΣP —total premiums (yearly and monthly) of the class or group.	

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., Barrister-at-Law.

Right to retain
income tax
deducted from
annuity pay-
ments. THE case of *The Edinburgh Life Assurance Company* v. *The Lord Advocate* [1910] A.C. 143, recently decided

by the House of Lords, is one of great importance to all companies granting annuities, dealing as it does with the liability of such companies to account to the Crown for income tax deducted from annuity payments. The facts have already been given in connection with the report of the proceedings in the Scottish Courts (*J.I.A.* vol. xliii, p. 222), but it will, perhaps, be a convenience if they are recapitulated briefly. The company grants annuities which are charged, indiscriminately with its other obligations, on its whole funds. It is assessed to income tax on its interest and not on its profits, and during the quinquennium immediately preceding the commencement of the action, the interest and other income from invested funds amounted to £804,731, as against £218,463 paid to annuitants. Had the Inland Revenue authorities elected to assess the company upon its profits and gains in the trading sense, tax would have been paid upon £334,450 of net profits, together with tax on £218,463 of annuity payments, or upon £552,913 in all. In these circumstances the assurance company contended that the annuities in question must be deemed to have been paid out of profits and gains already brought into charge, and that they were therefore entitled to retain the whole of the tax deducted by them from the annuity payments.

On the case coming before the Lord Ordinary, he found in favour of the company, but on appeal to the First Division of the Court of Session, this judgment was varied in an important particular, it being laid down by that Court that enquiry must be made as to what proportion of the company's total income over the period in question was derived from untaxed sources and what from taxed sources; that the income tax deducted from annuity payments must be divided in the same proportion; and that only such portion of the tax as, on this basis, represented the proportion derived from taxed interest, could be retained by the company, the remainder having to be accounted for to the Crown.

The assurance company appealed against this decision, and on the case coming before the House of Lords, their appeal was allowed and judgment given in their favour, it being held that in the circumstances they were entitled to retain, as against the Crown, the whole of the income tax deducted by them from the annuity payments. Lord Atkinson, in the course of his judgment, said : " From the accounts furnished it is clear that " had the company been assessed upon their trade profits for " the five years ending 31 December 1907, the Inland Revenue " would have received from them income tax on £334.450, and " have received from the annuitants income tax upon £218.463, " making together the sum of £552.913. The Inland Revenue " did not take that course: they collected the tax upon the " company's investments . . . with the result that they have " already received income tax on a much larger sum (a sum " amounting, after deducting the tax, to £804.731) than they " would have received had they taxed it on its profit. And if " the contention on which their present claim is based be sound, " they would be entitled to receive income tax on the amount " of the annuities paid, namely : £218.463, besides ; that is, " income tax on over £1,023.194, instead of on £552.913. The " respondents . . . contend that as the annuities are not made " a special charge on any particular fund belonging to the " company, and are in fact paid out of a mixed fund, the " payments must be ascribed to the whole sources of income " in the proportions which each source of income bears to the " whole income."

" It cannot be disputed that the annuities, though not " exclusively charged upon this taxed income, are payable out " of it, in the sense that they are charged upon it, may legitimately " and properly be paid out of it, and can be paid out of it in fact " as it is ample to meet them."

He then referred to the case of London County Council *v.* Attorney General ([1901] A.C. 26, *J.L.A.* vol. xlvi, p. 223) and quoted from the judgment of Lord Davey, who said, *inter alia*, " The general principle of payment in due course of adminis- " tration is to pay annual charges in the first place out of annual " income. It is not required by the Income Tax Acts, in order " to raise the right of deduction and retention, that the interest " or annual payment should be exclusively charged or payable " out of profits or gains brought into charge. It is enough if " the interest is charged upon or payable out of the taxable

" income, though there may be other subjects of charge."

Lord Atkinson went on to say: "The feature which, in addition to the insufficiency of the tax-bearing income to pay all the claims upon it, distinguishes that (the London County Council) case from the present is that there the taxed income had been set apart as a separate fund, and the interest and dividends on which the income tax required to be returned had been paid were, in fact, paid out of the fund so set apart. The question is whether a manipulation such as that by the company of its funds, a setting apart of less than one-third of their taxed income, to pay these annuities, which they can any day readily accomplish, and which if done could not have any effect on their balance-sheet or financial position, is a condition precedent which must be performed in order to entitle them to retain, under the provisions of section 24 of the Income Tax Act of 1888, the sum deducted and now sued for."

" In my opinion, where annuities such as these are charged upon a tax-bearing fund amply sufficient to pay them in full, though not set apart for that purpose, they cannot be held to be 'not payable' or 'not wholly payable' out of gains and profits brought into charge within the meaning of the 24th section. For the purposes of that section I think that the interest or annuities charged upon the tax-bearing fund must under such circumstances be treated as payable out of that fund, so far as it will reach. If the taxed fund be insufficient to pay all the interest and annuities, then the income tax deducted on the interest or annuities not satisfied out of it must be accounted for. In short, I attach no special virtue to the manipulation of the funds of a corporation in the manner above mentioned, as a means of escape from a liability to income tax. To do so would in effect be, I think, to lose sight of what appears to me to be one of the main objects, if not the main object, of the section, namely, to avoid obliging a subject to pay income tax twice over on the same sum. That object would, in the result, be defeated if the subject were obliged first to pay income tax on a given fund, and then to pay income tax on sums properly payable out of it, simply because he had omitted formally to dedicate the funds specially to that use, and formally to pay those sums out of it. On this ground I think the appeal in this case should be allowed."

Lord Gorell said : " The question depends upon the meaning " and effect of section 102 of the Property Tax Act, 1842, section " 40 of the Property Tax Act, 1853, and section 24, subsection 3 " of the Customs and Inland Revenue Act, 1888. . . . Lord " Macnaghten has examined these sections very fully (in the " London County Council case 1901). He points out how they " authorize a person who has paid income tax on what is not " really available income, because it includes money which he " has to pay over to someone else, to deduct and retain the tax " upon that payment."

" The annuities are in my opinion payable, within the " meaning of the Acts, out of the interest, dividends, and rents " received by the company, from which income tax is deducted " before the moneys are received by the company—they are " payable out of profits or gains brought into charge by virtue " of the Acts. But they are not payable out of these profits " and gains exclusively, and the question appears to be whether " that prevents the company from having the right of deduction " and retention to the extent which they claim. . . ."

" There does not appear to be any authority applicable to " this case for the proposition on which the learned judges of " the First Division acted, that because there are two funds " practically charged with the payment of the annuities, the " liabilities must be apportioned rateably between them. The " appellants may pay the annuities out of which funds they " please, and the question of their rights and liabilities with " regard to the Crown must depend on the sections of the Acts " above referred to, according to which the tax may be deducted " and retained, although the annuities are not payable ex- "clusively out of the taxable income of the company. . . . It " does not appear to me . . . that where the annuities are " payable out of profits or gains brought into charge it is necessary " to use in paying the annuities the actual moneys received in " respect of the profits or gains in order to obtain the benefit " of the deduction and retention. In the case of a business " like the appellants', and taking into account the language " and object of the three Acts, it seems to me that if the annuities " are made payable out of the interest, dividends and rents " charged with the tax, it is immaterial whether the money " to pay them is taken out of the general till of the company or " not, provided that it does not exceed the amount of income " on which tax is charged."

The decision appealed against was reversed accordingly, and judgment given in favour of the assurance company with costs both in this Court and in the Courts below.

Validity of charge by wife on policy issued for her benefit under the Scottish Act of 1880.

I am indebted to Dr. A. E. Sprague, F.I.A., F.F.A., for particulars of a case dealing with a mortgage of a wife's interest in a settlement policy issued under The Married Women's Policies of Assurance (Scotland) Act, 1880, which came recently before the Scottish Courts. The case was one of Multiplepoinding, which is the Scottish equivalent of the English process of Interpleader, and was brought by The Edinburgh Life Assurance Company against James Balderston and others to determine the rights to the

proceeds of a policy of assurance effected with that company by one James Shearar for the benefit of his wife under the Scottish Act of 1880. The policy was dated 30 April 1891, and on 29 March 1894, James Shearar and his wife, together, assigned the policy to the claimants, James Balderston and James Robertson Walker, who were creditors of James Shearar to the extent of £1,000, which sum he was not then in a position to repay. The premiums on the policy were, pursuant to a covenant in the deed to that effect, paid by James Shearar up to, and including 30 March 1895. In June 1895, his estates were sequestrated, the surrender-value of the policy at that time being £41 7s. This amount was deducted from the amount of the claim of Balderston and Walker in the sequestration as the value of the security they held in respect of the assignation. Under the deed the assignees were not bound to keep the policy in force, but as a matter of fact they did pay the premiums from the date of the sequestration to the death of James Shearar, which took place on 25 January 1909. The amount so paid by them was £290 16s. 3d. On the policy becoming a claim, the widow claimed the policy moneys, less the premiums paid by the assignees with interest. The assignees, on the other hand, claimed the whole amount due under the policy, or, alternatively, that amount less £41 7s.—the surrender value at the date of the sequestration. In reply to this claim, the widow contended that as the policy was of the nature of a post-nuptial provision to her, the assignation to creditors of her husband was bad, and that, therefore, they were not entitled to claim more than the premiums and interest.

Lord Mackenzie gave judgment in favour of the widow, and in doing so, said : " In my opinion it was incompetent for the " the husband here, who occupied the position of trustee—his " wife being a beneficiary—to become a party to an assignation " of the policy in security of his own debt. The policy might " no doubt be surrendered as was done in the case of Schumann. " 13 R. 678. That, as Lord Kyllachy pointed out in the case " of Barras, 2 F. 1094, would not necessarily be inconsistent " with the trust, and indeed might be quite necessary for its " protection. Nor would it be incompetent to grant an assigna- " tion of the policy for purposes ancillary to the trust. To grant " an assignation, however, to creditors of the husband for an " antecedent debt was in effect to revoke the trust. . . . The " case of the Scottish Life Assurance Company Limited *v.* John " Donald Limited, 1901, 9 S.L.T. 200 (*J.L.A.* vol. xli, p. 182) " decided by Lord Stormonth Darling seems like the present. " It was there held that an assignation to creditors of the " husband of a policy of insurance effected ' for the benefit and " separate use of his wife under the provisions of the Married " Women's Policies of Assurance (Scotland) Act, 1880', was " invalid. . . . In the later case of Hay's Trustees, 6 F. 978 " (*J.L.A.* vol. xli, p. 180) . . . both Lord Young and Lord " Moncrieff expressed the opinion that if the policy was regarded " as a provision, it was not revocable. There is therefore, " in my opinion, a sufficient body of authority to justify the " conclusion that the policy in the present case could not be " assigned to the husband's creditors."

. . . " The assignation is, therefore, in my opinion bad, but " the assignees . . . say that even if the assignation was invalid " as regards the property secured to the wife, the amount of " that was only £41 7s., the surrender value at the date of the " sequestration. They maintain that under the assignation " which declares that there is no obligation upon them to " keep the policy in force, that the present value of the " policy is due to the premiums voluntarily paid by them, " and that they are, therefore, entitled to the value of the " policy and bonus in so far as these exceed the sum of " £41 7s. The true view, however, appears to me to be that " the principle to be applied is that of recompense. The " assignees have, in *boni fide*, spent money upon property " which they believed was theirs, but which has been held to " belong to another, and by reason of their expenditure a benefit

" has been created for the true owner. This is just a case in which the person making the expenditure is entitled to be recouped the money expended with interest, but no more. . ."

" I am accordingly of opinion on the whole case that the claimants, Balderston and Walker are entitled to be ranked and preferred to the sum of £290 16s. 3d., with interest at the rate of five per centum per annum on each premium paid subsequent to 30 March 1895 from the date of payment thereof, and that the claimant, Mrs. Shearar, is entitled to be ranked as an individual to the balance."

There has been a marked tendency in recent years Conversion of friendly society into assurance company. Powers of new company. on the part of the large collecting friendly societies to seek conversion into assurance companies. The case of *Blythe v. Birtley* (1910) 26 T.L.R. 115, 215,

which is concerned with the conditions under which such a conversion can be effected, and the powers of the new company when converted, is therefore both of interest and importance. The facts of the case are as follows: The Royal Co-operative Collecting Friendly Society was a friendly society registered under the Act of 1896, and having for its objects " the insuring sums of money to be paid on the death of a member, or for the funeral expenses of the husband, wife or child of a member, or the widow of a deceased member, for rendering assistance to members when sick and thereby not able to follow their employment, and for the endowment of members or nominees of members." The society proposed, under the powers to that end conferred by the Friendly Societies Act, 1896, to convert into a company, to be called The Royal Co-operative Life and General Insurance Company Limited, with power to carry on the business of life assurance in all its branches, the granting of annuities of all kinds, sinking fund assurances, dealing in and lending on life and reversionary interests, fidelity insurance, guaranteeing titles, undertaking the office of trustee, executor or administrator, and other forms of business associated with a company transacting all branches of assurance. The resolutions required for such conversion by the Friendly Societies Act, 1896, were duly passed, whereupon the plaintiff, suing on behalf of himself and the other members of the society, applied for an injunction to restrain the defendants, who were the trustees and officers of the society, from registering

or acting upon these resolutions, on the ground that the society had no power under the Friendly Societies Act, 1896, to convert itself into a company having objects widely different from those of the society as defined by its rules, and outside the scope of friendly society business.

The case came before Joyce, J., in the first instance, and he granted the injunction asked for. In doing so, he said that "without going so far as to say that the objects of the company into which a friendly society might be converted under section 71, must be precisely limited to the purposes defined in the rules as the objects of the society, or must be strictly confined to the objects specified or referred to in section 8, he was of opinion that a friendly society could not, under the guise of conversion into a company in pursuance of section 71, by a mere special resolution as defined in section 74, and without any such assents as were required by section 70 (3), be transformed (to take an instance suggested in argument) into a brewery company. It could not be so converted into a company having a range of objects going so far beyond those specified in rule 2 of this society or referred to in section 8, or constituted so differently as the proposed company in this case was to be."

The society appealed, but on the case coming before the Court of Appeal, the decision of Mr. Justice Joyce was affirmed. In delivering judgment to this effect, Cozens-Hardy, M.R., said : "I have no doubt that what is proposed to be done here is not authorized by the 1896 Act, and is plainly contrary to the scheme and policy of that Act, and must therefore be restrained by injunction. All that a society can and ought to be able to obtain by special resolution under section 71 of this Act is a matter of machinery only. Here is a friendly society with certain limited objects; it is not a corporation, but it may convert itself into a company under the Companies Act, and when it is so converted it may then exercise the powers of enlarging and altering its objects with the sanction of the Court as required by the Companies Act."

Covenant by
surety to pay
interest and
premiums.
Bankruptcy of
principal
debtor.

These Notes refer, as a rule, only to recent cases, but Mr. G. J. Lidstone, F.I.A., has called my attention to the case of *In re Moss, ex parte Hallet* [1905] 2 K.B. 307 ; and he suggests that particulars of it are likely to be of interest to life assurance officials

in view of its bearing on the granting of loans with sureties. In this case, by an indenture dated 24 October 1901, Moss mortgaged to Hallet, *inter alia*, a policy of assurance on his own life to secure an advance with interest of £700 made by Hallet to Moss. By an indenture dated 11 December 1901, in consideration of £500 advanced by one Cooke to Moss, the latter covenanted to repay that amount with interest on the following 11 March; Moss and Hallet jointly and severally covenanted to pay to Cooke interest so long after that date as any principal money should remain due; Moss, as beneficial owner assigned to Cooke all his interest in the policy of assurance on his own life, subject to the previous mortgage to Hallet; and Moss and Hallet jointly and severally covenanted with Cooke to pay all premiums and other sums of money which should from time to time become payable for keeping the policy on foot.

On 28 November 1903 a receiving order in bankruptcy was made against Moss. At that date the principal sum of £900 remained due under the indenture of 11 December 1901, and the trustee admitted a proof by Cooke for that sum, less £100, which was the amount at which Cooke valued his equity of redemption in the life policy. Hallet lodged a proof against the debtor's estate, claiming in respect of (1) £687 due under the mortgage of 24 October 1901, (2) £813, being his estimated liability in respect of the premiums under his covenant in the indenture of 11 December 1901, and (3) £600, being his estimated liability for interest under the same indenture, less the value of his interest in the policy.

The trustee admitted the proof as to the first item, but rejected the last two items on the ground that the alleged liability to pay premiums and interest after the date of the receiving order was not provable within the meaning of section 37 of the Bankruptcy Act, 1883: that Cooke had proved against the estate for the principal debt and had valued the policy in respect of which interest and premiums were respectively payable; and that Hallet, as surety, was not entitled to prove also.

On the case coming before the County Court judge for Hertfordshire, he affirmed this decision. Hallet then appealed to the Divisional Court, which dismissed the appeal. In delivering judgment to this effect, Bigham, J., said: "The first question is whether the appellant can prove against the debtor's estate in bankruptcy in respect of the appellant's covenant to pay

" Cooke interest so long as any principal money remains due.
" In my opinion, as soon as the principal debtor became bankrupt,
" which has happened, it would be untrue to say that the principal
" money remains due from him to Cooke. The only liability
" then existing was the liability on the trustee in bankruptcy,
" in administering the bankrupt's estate, to pay to Cooke a
" dividend in respect of that debt. That being so, my view is
" that the liability of the appellant to pay interest to Cooke
" was also gone. It cannot be said that there was still a liability
" to pay interest if no part of the debt remained due, and if the
" appellant is not liable to pay interest to Cooke, he cannot
" prove in the bankruptcy."

" The deed also contained a covenant by which the appellant
" and the debtor jointly and severally covenanted with Cooke
" to pay the premiums necessary to keep up the policy of
" insurance on the debtor's life. What has happened is that
" Cooke, in proving in the bankruptcy, has put a value on his
" interest in the policy, and has included that value in his
" proof; and the position, therefore, is just the same as if he
" had sold the policy to a stranger. It does not make any differ-
" ence whether Cooke sold the policy or kept it, or whether he
" decides to pay the premiums himself or to surrender the policy.
" In any event the liability of the appellant under his covenant
" is gone."

ACTUARIAL NOTE.

*On some approximations to the values of Joint Life Annuities,
&c., where the mortality tables employed are not graduated
by Gompertz's or Makeham's law.* By W. PALIN ELDERTON,
F.I.A., and ALBERT E. KING, A.I.A.

THE object of the present note is to show how approximations such as those arising from the assumption of Gompertz's or Makeham's law can be made, even when the table has not been graduated by either of these laws. These approximate methods might be developed on the theoretical side to show how a table deviated from the assumed hypothesis, while they might be of some practical use in checking work which had already been done more exactly, or in showing how rough valuations can be made for more or less complicated benefits.

I.—JOINT LIFE ANNUITIES.

Taking the simplest case to start with, we have, if Gompertz's law holds,

$$a_{xy} = \sum v^n n p_{xy} = \sum v^n n p_w = a_w \quad (1)$$

and if Makeham's law holds, we have either

$$a_{xy} = \sum v^n n p_{xy} = \sum v^n n p_{uu} = a_{uu} \quad (2)$$

or $a_{xy} = \sum v^n n p_{xy} = \sum (v')^n n p_t = a'_t \quad (3)$

As indicated above, however, it seems that it should be possible to apply these approximations, even if the laws do not hold throughout the table on which the monetary values are based. Evidence in favour of this view has already been produced, so far as Makeham's hypothesis is concerned, by Messrs. Austin and Symmons, in their recent publication* of monetary tables based on the O^M Table, but it seems worth while to carry their work a little further, to see if we can approximate even when such an allied table as the O^{M,5} is not available.

Returning to formulæ (1), (2), and (3), it is well known that (1) holds if $\mu_{x+n} + \mu_{y+n} = \mu_{w+n}$ for all values of n , (2) holds if $\mu_{x+a} + \mu_{y+n} = 2\mu_{u+n}$ for all values of n , and (3) holds if $\mu_{x+n} + \mu_{y+n} = \mu_{t+n} + \text{constant}$, for all values of n .

Now it is clear that there must be some age w at which $a_{xy} = a_w$, and also an age u at which $a_{xy} = a_{uu}$, and so on, but the difficulty in approximating is to guess the equivalent age required without working out the whole of the joint life annuity table. To put this difficulty in another way, we may say, that if Gompertz's law holds, we can use the values of μ at the ages at which the annuity is calculated (*i.e.*, x and y); if it does not hold can we use them at those ages for approximating, or may we use them at some other ages which we have yet to find? As soon as the question is stated in this way, it occurs to one that a good result might be obtained by averaging the values of μ from the ages at entry onwards, or by taking them arbitrarily some years later. The former method is in some respects like regraduating a portion of the table by the law assumed, but while we have found that it can

* British Offices Life Tables, 1893; Tables of Two Joint Lives of equal Age; Aggregate and Select Tables. H. H. Austin and F. P. Symmons. 1907. (Introduction p. iv.)

give good results,* it entails more work, and we shall confine our attention to the second method indicated. The number of years to be chosen depends, so far as our investigations have gone, on the value of the joint life annuity which it is required to find, and we are inclined to think that it would not be very difficult to evolve comparatively simple expressions for the number of years for such a table as the O^M,† but as the method of approximation is wanted, if it is to be of practical use, in its simplest form, lengthy preliminaries must necessarily be avoided.

Let us take an example and find values for $a_{20:40}$ at O^M 3 per-cent on the assumption that the only tabular values available are a_x and μ_x .

The obvious way of approximating is by taking

$$\mu_{20} + \mu_{40} = .00399 + .00900 = .01299 = \mu_{47.7}$$

$$\therefore a_{20:40} = a_{47.7} = 14.861$$

* We have, in fact, made a considerable number of trials on these lines, and have succeeded in getting some close approximations, not only in simple cases, but even in that of the compound survivorship annuity.

It may occur to some readers that satisfactory results could be reached by finding approximation to $\log e$ in some such manner as that suggested (for purposes of graduation) by Mr. G. F. Hardy in Appendix G of his recent work, "The Theory of the Construction of Tables of Mortality, &c." Our reasons for not adopting any such method were :

- (1) That the values of $\log e$, ascertained from different parts of some tables, show wide variations, and, as an example, we find that, for the O^M Table, the value of $\log e$ about age 40 is .025, while at the end of the table it is .039.
- (2) The Gompertz constant would require a separate calculation, and would also show wide variations.
- (3) A value of e by Makeham's hypothesis is not very helpful for survivorship or complicated benefits.
- (4) Auxiliary tables would be necessary, which our method avoids.

On the other hand, if it happened that a mortality table were of such a nature that a table of uniform seniority, already calculated for another table, could be used, a considerable amount of labour would be saved in the calculation of joint-life annuities.

+ (e.g.) As a rough guide, the following could be used for joint life annuities, by the O^M Table—

NUMBER OF YEARS TO BE ADDED TO THE INITIAL AGES

the sum of initial ages = f	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140
makeham's Hypothesis f	24	21	18	16	14	12	10	9	8	7	6	5	4	3	2	1	0	0	0	0	0
gompertz's Hypothesis f	23	21	19	17	15	14	13	12	11	10	9	8	7	7	6	6	5	5	4	4	3

If, however, instead of taking the values of μ at the ages at entry, we had taken them, say, 15 years later, we should have had

$$\mu_{20+15} + \mu_{49+15} = .00725 + .02000 = .02725 = \mu_{44.5+15}$$

$$\therefore a_{20:40} = a_{44.5} = 15.948$$

which is a good approximation to the true value 15.958. Here we chose 15 years, as our first approximation led us to think it a good number, but the term can often be guessed, and in doing this it is helpful to bear in mind that the joint life annuity is less than the single life annuity on the elder life.

Taking as a second example, $a_{30:30}$, we have

$$\mu_{30} + \mu_{30} = .01168 = \mu_{45.63}$$

or

$$a_{30:30} = a_{45.63} = 15.570$$

while taking*

$$\mu_{30+16} + \mu_{30+16} = .02378 = \mu_{41.55+16}$$

we have

$$a_{30:30} = a_{41.55} = 16.905$$

which is close to the true value of 16.933.

These two examples show that good approximations can be obtained from Gompertz's law if the values of μ are not used at the ages at entry, but at later ages.

If we had based our approximation in our first example on Makeham's law, we should have had

$$\mu_{20+15} + \mu_{49+15} = .02725 = 2\mu_{48.59}$$

$$\therefore a_{20:40} = a_{33.59:33.59} = 15.905$$

which is not quite so close an approximation as that previously found.

II.—JOINT LIFE TERM ANNUITIES.

In making approximations to joint life term annuities, we can follow the same rules as those indicated above, but generally speaking, we have only single life term annuities on which to work, and must therefore restrict ourselves to approximations based on Gompertz's law, or on the second approximation (formula 3) based on Makeham's law.

* In the examples which follow we have not used the table given in the footnote to p. 295, but we have chosen the terms on the assumption that we have no auxiliary table available—the only guide being the annuity values found from the first approximation.

Taking the former method first, we can get an idea of the limits by calculating the age of the single life from the ages of the two lives at each end of the term, thus for the O^M 3 per-cent value of $a_{20:32:\overline{10}}$, we have

$$(1) \mu_{20} + \mu_{32} = .01035 = \mu_{43.12}$$

$$(2) \mu_{20+10} + \mu_{32+10} = .01567 = \mu_{41.06+10}$$

\therefore taking the mean of the equivalent ages, the age required = 42.09,

$$\therefore a_{20:32:\overline{10}} = a_{42.09:\overline{10}} = 8.025$$

The true value is 8.018, and $a_{43.12:\overline{10}}$ would have given 7.999. Taking $a_{41:37:\overline{22}}$ as a second example,

$$(1) \mu_{41} + \mu_{37} = .01731 = \mu_{52.72}$$

$$(2) \mu_{41+22} + \mu_{37+22} = .06160 = \mu_{48.02+22}$$

\therefore age required = 50.37,

$$\therefore a_{41:37:\overline{22}} = 12.483$$

The true value is 12.474.

We may mention that both these results are far closer approximations than would have resulted from finding the equivalent age w to form the equation $a_{xy} = a_w$.

The second method of approximating by means of a modified rate of interest appears a little difficult in application, owing to the difficulty of splitting up the force of mortality into its component parts, A and Bc^x , and the method is therefore of most service in a few rather special cases. One of these is in connection with the valuation of Joint Life Endowment Assurances. In many offices where there are only a few of these policies they are valued separately, and if the net premium has already been calculated, a 3 per-cent valuation can be obtained very easily by interpolation in a $3\frac{1}{2}$ per-cent table.* Taking as an example a Joint Life Endowment Assurance on the lives of two people aged (41) and (37) for 23 years, and assuming the net premium at O^M 3 per-cent to be given as .04509, we can obtain $a_{41:37:\overline{22}}$ by conversion tables, and find it

* We take $3\frac{1}{2}$ per-cent because the Text-Book Graduation showed that the single life equivalent with Makeham's hypothesis has to be used with an increase of about $\frac{1}{2}$ per-cent in the rate of interest. The same rule holds approximately for the $O^{M(5)}$ Table.

to be 12·474, which is the same as $a_{47\cdot2:\bar{22}}$ at 3½ per-cent. For the next few years' valuations, we take $a_{48\cdot2:\bar{21}}$; $a_{49\cdot2:\bar{20}}$, and so on, or which comes to the same thing, we require $nV_{47\cdot2:\bar{23}}$ at OM 3½ per-cent.

The table of policy-values for maturing at age 70 will be almost exact, and the following table gives the comparison :

Table showing the value of a joint life Endowment Assurance on two lives aged 41 and 37, for 23 years.

Number of years in force	True Value	Approximate Value
5	15·42	15·40
10	32·96	32·99
15	53·56	53·63
20	79·52	79·57

In incidently, this shows that Mr. Lidstone's method of approximation to endowment assurances could be employed by using his values of Z with single life equivalent net premiums and annuity-values.

A second special case is interesting and sometimes useful. It can be most easily stated in the form of an example.

Given $a_{44:39:\bar{20}} = 11\cdot586$ at OM 3 per-cent, find $a_{50:45:\bar{9}}$.

$$a_{44:39:\bar{20}} = 11\cdot586 = a_{49\cdot8:\bar{20}} \text{ at OM } 3\frac{1}{2} \text{ per-cent}$$

$\therefore a_{50:45:\bar{14}} = a_{55\cdot8:\bar{14}}$ by the rule underlying the valuation at 3½ per-cent shown above.

$$\therefore a_{50:45:\bar{9}} = a_{55\cdot8:\bar{9}} = 6\cdot724,$$

which fortunately agrees exactly with the true result.

III.—SURVIVORSHIP ASSURANCES.

In the case of survivorship assurances, we will confine our attention to Gompertz's law,* and we have

$$\bar{A}_{xy}^1 = \bar{A}_{xy} \div (1 + e^{y-x})$$

but since $\mu_x = Bc^x$, we have

$$\bar{A}_{xy}^1 = \bar{A}_{xy} \frac{\mu_x}{\mu_x + \mu_y}$$

* The term of years to add may be chosen from the table already given, but as explained in the footnote to p. 296, we did not adopt this course.

The easiest way to show the application is to take an example, and to find $A_{20:60}^1$ at $O^n 3$ per-cent, the following is the procedure :

$$\mu_{20+10} = .00584 \quad \mu_{60+10} = .06150$$

$$\mu_{20+10} + \mu_{60+10} = .06734 = \mu_{61.1+10}$$

$$\therefore A_{20:60}^1 = A_{61.1} \times \frac{.00584}{.06734} = .0590$$

as against the true value of .0574. The ten years ahead were taken from a_{60} .

As a second example, we may take $A_{40:70}^1$,

$$\mu_{40+6} = .01189 \quad \mu_{70+6} = .10140$$

$$\mu_{40+6} + \mu_{70+6} = .11329 = \mu_{71.3+6}$$

$$\therefore A_{40:70}^1 = \frac{.01189}{.11329} \times A_{71.3} = .0823,$$

while the true value is .0814.

It is, however, for annual premiums that this method would be simplest, as the formula becomes, for instance, $P_{71.3} \times \frac{.01189}{.11329}$ in the second case, and the result is .01117, while the true result is .01105.

IV.—COMPOUND SURVIVORSHIP ANNUITY.

As has been already mentioned, the method is useful for giving a rough check on complicated benefits, which have been calculated more accurately, and, as an example, we will take the compound survivorship annuity.

$$\begin{aligned} \tilde{\sigma}_{x,y,z}^1 &= \int_0^\infty v^t t p_{xyz} \mu_{y+t} \tilde{\sigma}_{z+t} dt \\ &= \int_0^\infty v^t t p_{xyz} (\mu_{x+t} + \mu_{y+t}) \frac{\mu_{y+t}}{\mu_{x+t} + \mu_{y+t}} \tilde{\sigma}_{z+t} dt \\ &= \frac{\mu_{y+n}}{\mu_{y+n} + \mu_{z+n}} \tilde{\sigma}_{x,y,z} \end{aligned}$$

where the fraction is taken as an average value.

An example will be of assistance, and we will take $\tilde{a}_{45:60:30}^1$ by the H^M 4 per-cent Table, Text-Book Graduation, on the assumption that we only know a_x and μ_x .

We require a value for the fraction, and values for a_{30} and $a_{45:60:30}$.

Taking eight years ahead for the joint life annuity, we have

$$\mu_{53} = .01833$$

$$\mu_{68} = .05396$$

$$\mu_{38} = .00928$$

$$\overline{.08157} = \mu_{73}$$

$$\therefore a_{45:60:30} = a_{65} = 7.850; \text{ the true value is } 7.764.$$

$$a_{30} = 17.155$$

$$\therefore a_{45:60:30} = 9.305$$

$$\text{Again, } \frac{\mu_{58}}{\mu_{53} + \mu_{68}} = \frac{.05396}{.07222} = .7464$$

\therefore our approximation to $\tilde{a}_{45:60:30}^1$ is $.7464 \times 9.305$, or 6.946, while if we had used the true value of $a_{45:60:30}$, we should have found 7.010.

On p. 266 of *Text-Book*, Part II, the value by a formula of approximate summation is given as 6.9957.

Taking nine years ahead, the O^M 3 per-cent value is 8.735, and the true value 8.781.

Another trial, taking the values of μ at seven years older, was made with $\tilde{a}_{40:60:50}^1$, for which we found 1.258 instead of 1.264, by O^M 3 per-cent, while, by *Text-Book* 4 per-cent, by which the true value is 1.116, we found 1.104 when using the true value for $a_{40:50:60}$, and 1.092 when approximating roughly to this annuity, in the way indicated above.

It should be borne in mind that the so-called true values are calculated by approximate summation, and may be slightly in error in the last figure.

V.—CONCLUSION.

From these notes it would appear that a large number of very simple approximations might easily be evolved, which could with

a little practice be made to give fairly accurate results without much arithmetical work, and even when a table has not been graduated by either Gompertz's or Makeham's hypothesis. In fact we almost think that some, at any rate, of the advantages of a Makeham graduation could be read into almost any table, and a certain freedom might be gained by using the approximation that a Makeham graduation implies at the final stage instead of at the initial stage (graduation). This is, however, a matter of opinion, and is rather apart from the object of these notes, which are put forward in case others may find them of practical use for checking or other purposes.

EDITORIAL NOTE.

JOSHUA MILNE.

AT the Annual Meeting of the Sun Life Assurance Society, held on 2 March 1910, the Chairman (Mr. Marlborough R. Pryor) referred to the approaching Centenary of the Society, which was established on 2 April 1810; and made some remarks as to its first Actuary, Mr. Joshua Milne, which will, we think, be of interest to readers of the *Journal*. After referring to the establishment of the Society, Mr. Pryor proceeded as follows (as reported in *The Times* of 3 March 1910):—“They selected Mr. Joshua Milne as Actuary. Mr. Joshua Milne was a capable mathematician, and had directed his attention to statistics of vitality. His attention had been called to those of the City of Carlisle. There was a good old Bishop of Carlisle in the middle of the 18th century, Bishop Littleton, who saw the very great importance of vital statistics, and he ran a census of Carlisle ‘off his own bat’ in 1768, I think it was, and from that time, apparently, not only the ordinary bills of mortality and the Church Registers were probably kept with rather special attention, but one or two people, and especially a Dr. Heysham, of Carlisle, paid very special attention to the vital statistics of Carlisle. His work in collecting them came to the knowledge of Milne, who entered into correspondence with him, and in 1815 (Waterloo year) he produced his great book, and in the words of his biographer, his treatise on the valuation of annuities and assurances of lives, &c., resulted in a revolution in actuarial science. Things were put on a really sound foundation, and he does deserve, as I have said, the gratitude of humanity. Shortly afterwards, he prepared the Life Tables

which appeared in our prospectus. They appear in it to this day unaltered, and their being unaltered is most fully justified. They were quite sound, and in the year before 1822 Joshua Milne evolved a contributive system of bonuses under which, supposing the tables did not exactly correspond to the changed experience of mortality which there always is after the lapse of time, his system for distributing bonuses made up adequately for any inequities which might arise from the tables not conforming to the more recent experience. In 1822, Mr. Milne made our first valuation, and it was very satisfactory. A very large profit was ascertained. That was at the end of 12 years, according to our constitution. The valuations then succeeded each other at the end of every seven years. In 1836 it became apparent that the profits of Life Assurance justified the sharing of them with the assured, and in an Act of 1837 this Society took power so to assure them. Joshua Milne made our last valuation in 1843, in which the profits were divided in equal moieties, and the assured shared under his contribution system in one-half. Then he retired, and spent the remaining seven or eight years of his life in the study of natural science, especially botany."

2, 3

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

THE DEATH OF KING EDWARD VII.

AT the SIXTY-THIRD ANNUAL MEETING of the INSTITUTE OF ACTUARIES, held on Monday, June 6th, 1910, at STAPLE INN HALL, HOLBORN, the President, Mr. G. F. HARDY, made the following remarks from the Chair, the members all rising and standing whilst he spoke :

Now that our meeting is duly constituted, there is one subject that I am sure you will all feel must take precedence of our ordinary business.

Since we last met in this Hall, the death of our King has plunged the nation and the Empire into the deepest grief, a grief felt by every one of his subjects from the highest to the humblest, a grief which has found an echo in every part of the world. Never has a King more closely identified himself with those aspirations and ideals which, in spite of minor differences, weld the many diverse types and classes of this realm into one loyal and patriotic whole. Short as was King Edward's reign, we all feel that it has been a reign of profound import and significance for the country and the Empire he loved, to whose welfare he so entirely devoted himself, and, it may well be, of deep significance also for the future history of the world. In the distinguishing feature of that reign, we as actuaries may find inspiration. In its noblest aspect, our work is a fight against some of the wasteful and destructive forces of the world, an attempt to parry the formidable blows of death, accident and disease. He of whom we have been so suddenly bereaved, in a wider arena and upon a

more illustrious field, devoted himself to the same great cause, and in the place of war, strife, bitterness and misunderstanding, how much has he not done to bring peace, friendship and co-operation. It is not given to any of us to play a great part in the world, but to the extent of our ability we may follow the noble example of devotion to duty which King Edward has left us.

As a touching proof of the universal sorrow felt in every part of the world for the King's death, I think you would like me to read to you a letter which I received only this morning from Japan. It is dated from the Nippon Life Assurance Company, Ltd., and is addressed to me, as President of the Institute:

“ DEAR SIR,—The whole land of Japan has been startled from end to end by the unexpected sudden news of the death of His Majesty the King Edward VII. I hereby most respectfully beg to present, on behalf of the Directors, Officers and Clerks of the Nippon Life Assurance Company, expression of the deepest sympathy and condolence with you and your countrymen at home and abroad in the irreparable loss the Great British Empire sustains in the untimely departure of the noblest and the ablest among the rulers of the day. I can not possibly find out proper words with which I can fairly represent the unspeakable sentiment of grief, which the hearts of the whole people of Japan are now fully possessed of.

“ Again assuring you of my profound sympathy in your national mourning,

“ I remain, dear Sir,

“ Yours truly,

“ N. KATAOKA (*President*).”

That is a letter which will, I am sure, touch the hearts of everyone present. The Council, on your behalf, will in due form pray His Majesty King George to accept an expression of their respectful sympathy and an assurance of their enthusiastic loyalty.

At the Council Meeting held on 14 June 1910, the following Memorial was ordered to be signed and sealed, and dutifully offered to the King :

To THE KING'S MOST EXCELLENT MAJESTY.

MOST GRACIOUS SOVEREIGN,

We, your Majesty's loyal and dutiful subjects, the President, Council and Members of the Institute of Actuaries, humbly approach your Majesty with the expression of our profound sorrow at the irreparable loss which your Majesty's subjects throughout the Empire have sustained by the deeply-lamented death of our late revered Sovereign, KING EDWARD VII., and our respectful condolence with your Majesty, her Majesty the Queen, her Majesty Alexandra the Queen Mother, and the other members of the Royal Family, in the bereavement which has befallen your Royal House.

We are mindful of the abiding interest which his late Majesty manifested in the promotion of thrift among his people, and the encouragement of those provident institutions with which the work of our Institute is so closely associated.

We also desire most respectfully to offer to your Majesty the assurance of our heartfelt loyalty and of our devotion to your Royal Person, with our humble congratulations upon your Majesty's accession to the Throne. We fervently pray that your Majesty may long be spared to rule in peace and happiness over a united and contented Empire.

Given under the Common Seal of the said Institute at Staple Inn Hall this fourteenth day of June, in the year of our Lord one thousand nine hundred and ten.

G. H. RYAN, *President.*

W. P. PHELPS } *Honorary*
L. F. HOVIL } *Secretaries.*

L.S.

*Some Notes on the Establishment of the Office of Public Trustee
in England.* By WILLIAM CHARLES SHARMAN, F.I.A.,
Barrister-at-Law, of the Prudential Assurance Company.

[Read before the Institute, 28 February 1910.]

Introduction. IN bringing these Notes under the consideration of the members I feel some apology is needed for the reason that the subject may be said to be somewhat outside the scope of the ordinary proceedings of the Institute.

In addition, however, to its legal aspects, which will doubtless appeal to a profession which demands from its members a not inconsiderable knowledge of law, I venture to think there are other grounds on which the subject may be considered suitable for discussion.

The Public Trustee Act brings into being a Department, which, from its intimate connection with the affairs of the people and the large financial interests involved, may in course of time rank only second to some of the great Departments of the State, and, from this point of view alone, the subject is of interest.

Moreover, it has a close connection with the work of many Actuaries owing to the large investments made by Life Offices in reversionary securities, this kind of investment being peculiarly affected by questions concerning the trusteeship.

When, further, it is remembered that Insurance Companies form one of the classes of Corporate Bodies which may be authorized by the Treasury and Public Trustee to undertake the duties of Custodian Trustees, I think it will be admitted that the subject is one with which we are practically concerned.

The transference to the State of a number of obligations which had previously been borne by private individuals, has raised, perhaps not unnaturally, in some quarters a certain amount of criticism.

This paper is not, however, intended to deal with the subject from the point of view of opponents of the system, but is mainly concerned with the changes introduced by the administration of trusts by the Public Trustee, with particular reference to the effect of such changes on the reversionary securities of Life Offices.

There are but few references in the *Journal* to the subject of the appointment of a Public Trustee.

In vol. xxvi, p. 349, a short account is given of a Bill

introduced into the House of Commons with this object, which, however, did not become law.

Mention may also be made of a remark by Mr. Barrand, in his paper on "Some Legal Points arising in Life Assurance Practice," vol. xxxiii, p. 233.

Referring to the subject of policies issued for the purpose of providing Estate Duty, he speaks of "the coming by-and-by "when we shall be blessed in this country with a Public "Trustee."

The prophecy thus made over twelve years ago has been fully justified.

EARLY HISTORY OF TRUSTS.

The development of the system of trusts in this country presents features of interest, and it may not be thought out of place to give at the outset a short summary of the steps which have led eventually to the appointment of a State official with power to undertake these obligations.

Trusts were originally introduced into the Roman Law in order to avoid certain stringent regulations which beset the institution of a legal heir. They were known as *fidei commissa*, and could only be effected by will.

In England they also arose through an attempt to evade the law, but in this country could be created *inter vivos* as well as by will.

An Act, known as the Statute of Mortmain, was passed with the object of preventing the alienation of land to religious houses. In order to obviate the difficulty caused by this Statute, the custom arose when it was desired to make a gift to a monastery, to alienate the land to a friend to hold to the use of the monastery, and the King's Chancellor would give effect to the wish thus expressed. It was found that the use thus created was more convenient than the legal estate, and the practice was extended to other purposes, and rapidly grew in favour. The Statute of Uses, 1536, attempted to curtail the practice by enacting that any person to whom a use was granted should have the legal estate. As, however, the Statute did not apply to personal property, or to a use upon a use, it failed in its object, and the Court of Equity still continued to enforce the equitable estate in a trust in favour of a beneficiary.

It will be seen that the main idea which was considered to underlie a trust was that of personal confidence in the trustee, and this view was so strongly insisted upon that it was originally held that a Corporation could not be a trustee, for as it was

quaintly put, "how could that confidence which was essentially implied be reposed in a soulless entity."

When this aspect of the matter is considered, it is, perhaps, not surprising that in this country of long-standing traditions the system of private trustees should so long have held sway, and that the frequently onerous and difficult duties of a trustee should have been carried on by private individuals, in many cases without remuneration.

Amendments
of the Law
relating to
Trustees. Prior to 1883, the law relating to the administration of trusts was in a very unsatisfactory state, and often inflicted great hardship and expense upon beneficiaries.

It was the practice, when any question arose in the administration of a trust, for a person interested to apply for, and as a matter of course obtain, an order for general administration by the Court. This involved endless delay and expense, for the Court would not give an answer to the question asked, which possibly might have been settled by a competent lawyer at once, but was compelled to supervise the whole administration of the trust, and to institute accounts and enquiries with regard to matters which had no relation to the point at issue. A much-needed reform was introduced by the institution of the form of procedure known as an Originating Summons. This is an extremely simple method of procedure, whereby any person interested under a Deed or Will may apply to a Judge in Chambers for the determination of any question arising under the instrument, or a declaration of the rights of persons interested.

Thus questions such as those relating to maintenance or advancement can be dealt with by an Originating Summons.

It is now the exception rather than the rule for an Order for General Administration to be made, and it is difficult to realize the enormous saving of time and money which must have resulted from the improved procedure.

In recent years, moreover, there has been considerable amelioration in the law relating to the liabilities of trustees.

Perhaps the greatest hardship, under which a trustee used to labour, was that he was not allowed to plead the Statute of Limitations as a defence to an action for breach of trust.

The Trustee Act of 1888 now permits this defence to be set up in the case of an honest breach of trust.

Also, debts incurred through honest breaches of trust are now provable in Bankruptcy under the Act of 1883, and an order of discharge operates to release the bankrupt from such debts.

A further step in the direction of bettering the position of a trustee was taken under the Judicial Trustees Act 1896 which introduced the rule that the Court may relieve a trustee from personal liability for a breach of trust provided he has acted reasonably and honestly.

It is certainly curious that, almost contemporaneously with the introduction of these reforms, the demand should have arisen for the appointment of an Official Trustee.

The suggestion of the establishment of such a Department first took practical shape in 1886, when a Bill was introduced in the House of Commons by Sir Howard Vincent with the object of "Meeting the difficulty which both public bodies, and private individuals frequently experience in finding suitable trustees, and "to enable the State under certain reservations to guarantee the "fulfilment of all trusts placed in the Public Trust Office." The Bill, however, did not obtain a second reading.

The question was not dropped, and several Bills with similar objects were introduced in subsequent years, all of which failed to become law.

Select Committee on Trusts, 1895. In 1895 a Select Committee was appointed to enquire into the liabilities to which persons are exposed under the present law as to the administration of trusts, and whether any further legislative provision might be made for securing the adequate administration of trusts, without the necessity of subjecting private trustees and executors to the risks which they now run.

In their Report the Committee stated "that the evidence " puts it beyond question that large sums of money are annually " misappropriated by private trustees," but further pointed out that the majority of cases occur in those trusts where either under the terms of the original appointment or by the death of a co-trustee, the property is vested in a single person.

They also found that the difficulty of inducing competent persons to undertake the office of trustee was very real and increasing. They expressed the opinion that a case had been made out in favour of the establishment of a system under which private trusts can be administered, if so desired, under the control of some official or judicial authority.

Judicial Trustee Act, 1896. The outcome of the Report of the Committee was the Judicial Trustee Act 1896. This Act which appears to be based upon the Scottish system of appointing a Judicial factor to administer a trust, aims at giving the trust

property the same security as would be afforded by a general administration order of the Court.

Upon application by the settlor, or by an existing trustee or beneficiary, the Court may, at its discretion, appoint a person to administer the trust, who may be either an official of the Court or a person nominated in the application. Such a trustee is known as a Judicial Trustee, and if not an official of the Court must give security. He may receive remuneration for his services, and have his accounts audited once a year, but in all cases is subject to the control and supervision of the Court as an officer thereof.

There is no provision in the Act whereby a beneficiary would be indemnified in the event of breach of trust, and it was apparently considered that the safeguards provided would be sufficient.

The institution of the Judicial Trustee cannot be said to be a success. The fact that it is necessary to apply to the Court before the appointment can be made, would render the scheme unacceptable to many settlors. Moreover, should the application be opposed, the Court, in the exercise of its discretion, will not appoint a Judicial Trustee in case of an existing trust where no charge of improper conduct is brought against a trustee.

ESTABLISHMENT OF PUBLIC TRUSTEE.

The Public Trustee Act was introduced into the House of Lords by the Lord Chancellor in 1906, and came into operation on 1 January 1908. The Public Trustee is a Corporation Sole, with perpetual succession and an official seal. The main idea underlying the Act is the appointment of a Government official who is empowered to undertake the duties of a private trustee.

He is to have "All the same powers, duties and liabilities" "and be entitled to the same rights and immunities, and be" "subject to the orders of the Court as a private trustee acting in" "the same capacity." Every trust administered by him is to be kept entirely distinct, and the funds are to be invested as authorized by the trust instrument or by the law for the investment of trust funds.

There is no guarantee as to the security of capital or interest from the point of view of market fluctuations, but any increase of the value of the funds of a particular estate will increase the value of such estate, while any fall in the value of the investments will diminish the value of the estate. There is,

however, a State Guarantee with respect to any breach of trust that may occur.

Powers of the Public Trustee. It is not necessary that the Public Trustee should be appointed sole executor or trustee, but he may act jointly with any person or body of persons in any capacity he is entitled to fulfil. He is given wide power to act as administrator of the estate of a deceased person, and special provisions are made to enable him to conduct the administration of estates of small value in an economical manner. He may act as Custodian Trustee, and is given power to secure an audit and investigation of the accounts of any trust on behalf of any trustee or beneficiary.

Appointment as Original Trustee. The majority of express trusts arise either under a will or a settlement, and the Public Trustee is appointed in the same manner, and by the same persons, as a private trustee. With regard to wills it is not necessary for the testator to obtain the consent of the Public Trustee before appointing him as executor or trustee, and his definite consent will not be given until the death of the testator. As, however, testators are often desirous of knowing whether any difficulty is likely to arise, he will, if particulars of the proposed appointment are forwarded, and are satisfactory, give an intimation to the effect that there is nothing in the nature of the estate or terms of the will which would lead him to decline the office.

Although it is quite optional for testators to inform the Public Trustee that they are desirous he should act for them, it is some evidence of the possibilities of the department that he has been informed that in estates of an estimated capital value of over £25,000,000 he has been appointed executor or trustee.

Appointment under Transferred Trusts. When a trustee dies, or is desirous of being discharged the Public Trustee may be appointed to fill up his place. The appointment will be made under an express power in the trust instrument, or by the Statutory power under Section 10 of the Trustee Act, 1893, by the surviving or continuing trustees or the personal representatives of the last surviving or continuing trustee.

Unless appointed by the Court the Public Trustee cannot apparently be appointed as an additional trustee, except in the case where a vacancy in the number of trustees is being filled up.

When he has been appointed in any trust, a co-trustee may retire by executing a deed stating that he is desirous of being

discharged, and the ordinary limitations to the power of retirement that there must be two trustees remaining, and that the consent of the co-trustees and any person empowered to appoint new trustees must be obtained, are removed.

When it is proposed to appoint the Public Trustee to act in an existing trust, it is necessary that notice should be given to all persons beneficially interested who may be resident within the United Kingdom, and these beneficiaries have the right within 21 days to apply to the Court to prohibit the appointment. A case in this connection has recently been decided, and a remark of Mr. Justice Parker to the effect that trustees before availing themselves of the Public Trustee should try to find some other members of the family to accept the trust, has been the subject of some comment, and appears to have been given a wider significance than the full statement of the learned Judge would warrant.

The case is that of *re Hope Johnstones Settlement Trusts*, 1909, L.T.R. 369, in which a settlor, who was also a beneficiary under the settlement, quarrelled with the trustees of the deed, and they threatened to retire from the trust and appoint the Public Trustee in their place.

The Settlor issued a summons to restrain them from so doing. In giving judgment for the Settlor, Mr. Justice Parker said "The settlement was evidently framed and designed to protect the Settlor against himself. He much doubted whether the Settlor would have consented to make the settlement if the trustee suggested had been a public official instead of members of his own family. Trustees of a settlement of this kind accepted very onerous duties, and might well wish to shift them on to another's shoulders, but before availing themselves of the Public Trustee they should try to find some other member of the family who would accept the trust."

It might reasonably be considered that the idea of protecting the Public Trustee from trusts of this nature was not entirely absent from the mind of the learned Judge, and to read into these remarks a general condemnation of the practice of private trustees retiring in favour of the Public Trustee does not seem to be justified.

During the first year of the working of the Department the majority of trusts which were undertaken were existing trusts transferred to the Public Trustee. This is as might be expected, for with regard to original trusts arising under wills, testators

who had executed their wills previously to the passing of the Act would have found private executors or trustees willing to act, and the option of appointing the Public Trustee would not have been open. It seems probable that, in the future, the number of cases in which he will be appointed original trustee will exceed the transferred trusts.

Power to Decline Trusts. An important principle in a scheme designed primarily to meet the difficulties experienced in obtaining private trustees is that of the right of the Department to decline trusts. It would have been obviously impossible to frame the Act upon lines which should make it obligatory for the Public Trustee to accept every trust submitted to him. The principle adopted is that of giving discretion with regard to trusts that may be accepted, with the important proviso that no trust may be declined solely upon the ground of small value.

This qualification seems quite reasonable, for it is probably with regard to small trusts that the greatest difficulty of finding competent trustees is experienced.

Amongst the points taken into consideration in deciding whether a trust shall be accepted are: The mode of investment and condition of the trust property; the situation, tenure, and character of any land comprised in the property; the places of abode and circumstances of the beneficiaries, and the liabilities attaching to the property. The Annual Report of the Department gives no particulars as to the number of cases it has been found necessary to decline, but I understand these are very few in number.

With regard to trusts which involve the carrying on of a business, the creditors of the business have a right to claim payment of their debts against the trustee himself. Such trust will therefore only be accepted by the Public Trustee in exceptional circumstances, and except with the consent of the Treasury a business will only be carried on for a short period, not exceeding 18 months, and with an ultimate view to its sale, disposition or winding up.

Transfer of Property to Public Trustee. No special methods are adopted to transfer the trust property to the Public Trustee. If he be appointed under a will or letters of administration, it will devolve upon him as executor or administrator. With regard to new or transferred trusts, a draft of the deed to effect the appointment would be prepared by a Solicitor and submitted for

consideration, the property comprised in the trust estate being set out in a schedule to the draft. If the trust be accepted, the property would be transferred by the appropriate methods adopted in the case of a transfer to private trustee.

For the purpose of identifying stocks and shares belonging to a particular estate, and thus enabling dividends to be remitted direct to beneficiaries, it is necessary that the transfer deed should be ear-marked with a particular name or letter. In the case of inscribed stock a form of lodgment must be obtained, to the effect that the Public Trustee agrees to accept the transfer of the stock. This will be duly ear-marked, and must be presented at the time of the transfer, or when power of attorney is applied for.

Administration
of Estates of
Small Value. Some reference may be made to the provisions for the administration in an economical manner, of estates of under £1,000 in value. Upon the Public Trustee undertaking to administer such an estate, all the trust property other than stock vests in him, and he has the right to transfer, or call for the transfer of any stock. For the purposes of the administration he is given power to take an informal opinion of the High Court, and has all the administrative powers and authorities of a Master of the Supreme Court acting in the administration of an estate.

The Section of the Act dealing with these provisions is, however, not free from difficulty, and some doubts have arisen upon its construction. Under the practice at present adopted it is necessary for a grant of probate or letters of administration to be obtained before application can be made to the Public Trustee to undertake the administration. It is also doubtful whether the Section applies to trusts as well as estates of deceased persons.

The wording of the Section is as follows: "Any person who "in the opinion of the Public Trustee would be entitled to apply "to the Court for an order for the administration by the Court "of an estate . . . may apply to the Public Trustee to administer "the estate." The fact of the insertion of the words "in the "opinion of the Public Trustee", coupled with the powers and authorities given under the Section, would suggest that the intention was to give the Public Trustee power to decide whether an applicant was a proper person, without the necessity of such applicant taking out probate or letters of administration. The effect of the construction adopted is to detract somewhat from the value of the provisions, and in view of the manifest advantages

of the Section to the class of persons it is intended to benefit it is a matter of regret that there is any ambiguity on the point.

Limitations of Powers of Public Trustee. There are certain restrictions on the powers of the Public Trustee which may be briefly mentioned. He is debarred from acting as trustee in trusts made exclusively for religious or charitable purposes, in any trust under a deed of arrangement for benefit of creditors, and in any trust made by way of security for money, such as a trust for debenture holders, and from administering an insolvent estate.

It is further provided that the Act shall not extend to Scotland or Ireland. The opinion has been expressed that the effect of this limitation is to place these countries, as far as the Act is concerned, in the position of foreign countries. In England, the fact that a person is domiciled abroad is considered as an objection to his fitness for the office of trustee, as he is not amenable to the jurisdiction of the Court. Mr. Fulton, in his work on the "Law of the Public Trustee", page 77, states, however, that "there would appear to be nothing in "the laws of Scotland or Ireland to prevent the Public Trustee being appointed as executor of a will or as "trustee of a settlement." He further considers that except in the case of land the English Courts would have jurisdiction. The question may, however, arise as to the law which would govern the rights of beneficiaries under such trusts. The general rule in respect to real estate is that the supremacy of the law of the country in which the land is situated is recognized. In the case of personal property under marriage contracts the tendency is to apply the law of the domicile of the husband at the time of the marriage, and under wills that of the domicile of the testator at the date of the will (Rattigan, "Private International Law", p. 137). The expressed intention of the parties is, however, the fundamental guide to rules of construction, and in the case of *Chamberlain v. Napier* 1880, 15 Ch. D 614 it was inferred from the trusts of the marriage contract that one set should be construed as being Scotch and the other as English. It is conceived that the Public Trustee would not accept a Scotch or Irish trust of personal property unless the intention of the parties as to the law which should apply was clearly expressed.

An important element in the Public Trustee Act is the State Guarantee to indemnify beneficiaries against losses that may be caused through breach of trust. The Guarantee is limited to the liability which the Public Trustee,

if he were a private trustee, would be personally liable to discharge, and in no way applies to losses which the trust funds may sustain in consequence of depression in the value of investments, provided such investments were authorized. It may be observed that the liability of the Public Trustee is less than that of a private trustee, for not only is he entitled to all the rights and immunities of a private trustee, but his liability is further restricted. Thus he may take advantage of the Statute of Limitations under the Trustee Act 1888, and of the relief afforded by the Judicial Trustee Act, and in addition is exempt in those cases where the liability is one to which he has not contributed, or which neither he nor his officers could by the exercise of reasonable diligence have averted.

It seems possible that these latter exemptions have been inserted to meet the case of loss to the trust funds through fraud or forgery, for although it has been held that a trustee, even though remunerated, is not liable for loss if the trust property be stolen, provided he took reasonable care of it, yet by a curious anomaly, he has been held liable if he has been induced by fraud or forgery to hand the trust property to the wrong person.

In considering the State Guarantee one is reminded of the somewhat similar guarantee existing under the Land Transfer Act. In a recent case, Attorney General *v.* Odell, 1906, 2 A.C. 47, where a claim was made against the Land Registry, although the Registrar awarded compensation, yet the Treasury as guarantors of the Fund disputed the claim, which was eventually disallowed by the Court of Appeal.

The Treasury were thus shewn to be technically correct in their opposition, but, as pointed out by Mr. Hart in his paper on "Land Registration" (*J.I.A.*, vol. xl, p. 267), "in this instance "the real danger of officialism came out, for the claim was one "of small amount and might well have been treated in a "practical way without expensive litigation."

At the time of writing no claim has been made on the Treasury in respect of the Public Trustee, and one may express an opinion that, should a claim arise, it would be treated in a practical manner and technical difficulties would not be raised. It will be remembered that the Department is not run for profit, the fees charged having to be regulated according to the expenses incurred. Moreover, it has no compulsory powers, and thus stands in a different position than the Land Registry. Probably

due weight would be given to these considerations in the event of a claim occurring.

No special fund to insure the Treasury against loss has yet been set aside out of the fees received. Owing, however, to the practice of only charging half the capital fee upon acceptance of a trust, the Department is gradually accumulating a reserve by way of deferred fees payable upon the determination of trusts.

Through the courtesy of Mr. C. J. Stewart, the Organization of the Department. Public Trustee, I have had an opportunity of seeing some details of the organization of the Department.

The work connected with the administration of trusts is carried out in an eminently business-like manner, and is organized on lines calculated to procure that personal attention which is so necessary in the case of a trust, and at the same time to take advantage of the experience in financial matters possessed by the Department.

Each case is allotted to an officer whose duty it is to keep in close touch with its legal and business aspects, while a general supervision is exercised over all trusts by the Public Trustee and the chiefs of his staff.

All correspondence relating to any particular trust is kept in a separate file, and separate account books are kept in respect to each trust, shewing capital and income accounts. There are separate mortgage and investment departments, from which may be ascertained the total holding of the Public Trustee in any particular class of security. Each class is again subdivided into the particular estates interested, so that, if circumstances arise which may seem to render it desirable to realize any particular class of investment, it is possible to notify at once the trustees and beneficiaries interested, and to ascertain their views.

A feature which should appeal to life tenants, is that adopted of forwarding to them an annual account shewing the amount of investment, the dividends receivable, the amount of income tax deductible, and the various dates in the year when such dividends become due.

Arrangements have been made with the Bank of England, and many of the leading public, municipal and commercial bodies to open separate accounts in respect of each trust estate held by the Public Trustee, so that it is possible to pay dividends direct to the beneficiaries. Where this can be done the income fee is limited to 1 per-cent, this being paid direct to the Public Trustee by the beneficiary.

Objections to
the Department.

Reference may be made to some of the objections which have been urged against the establishment of a Public Trust Office. The principal points upon which criticism is directed are :

- (1) The extension of officialism involved.
- (2) The tendency in a Government Department to become rigid and bound by rules and traditions, with the consequence that it is not likely to give satisfaction in the conduct of trusts which require discretion and close personal attention.

With regard to the first of these objections, which was raised in a Report of the Special Committee of the Law Society, it is difficult to understand why it should be applied to a Government Department worked on a voluntary system and without compulsory powers. Possibly it was feared that the compulsory system might be extended to the Public Trust Office in the event of its not proving a success. Where officialism takes the form of establishing salaried officials to do work which had previously been performed by, and contributed to the support of, private individuals, there is doubtless ground for opposition. In the case of the Public Trustee, the duties which have been undertaken are those which generally had been performed by private persons without remuneration, and often for the sake of friendship. Duties connected with a trust which require the services of solicitors, brokers, bankers, and others, are still performed by those individuals, for it is not the practice of the Public Trustee to act himself in these varied capacities, but he is authorized to employ agents in the administration of a trust.

The fact of the immediate success of the department shows, I think, that the fear of officialism has very little foundation.

As regards the second objection, there is little doubt that a tendency exists for a Government Department to become bound by red tape, and to rely largely upon precedent, and experience alone can show whether this will be avoided in the Public Trust Office.

Perhaps the chief safeguard against the danger is the fact that it is recognized, for it is stated in the Annual Report, that it has been the special aim to organize the Department upon such lines as effectively to avoid shortcomings in this direction.

Mention may also be made of a fear that has been expressed that a Government Department in possession of large funds

might attract the attention of some future Chancellor of the Exchequer desirous of fresh sources of revenue.

Possibly there might be some ground for such fear in a system such as that adopted in New Zealand, where the interest rates payable may be altered from time to time by the Government, and a proportion of the profits earned may be transferred to the Consolidated Fund.

The method adopted in this country of keeping all funds belonging to particular estates quite separate, should, however, render any such action as that suggested extremely unlikely.

CUSTODIAN TRUSTEES.

An interesting experiment made in the Public Trustee Act is the creation of the office of Custodian Trustee.

The suggestion emanated from the Incorporated Law Society, and was made with a view to preserving the personal element in the administration of trusts, and at the same time to afford security to the trust funds.

Private individuals are not empowered under the Act to undertake the duties of a Custodian Trustee, the office being restricted to the Public Trustee or certain corporate bodies.

These bodies are defined as being any such incorporated banking or insurance or guarantee trust Company or friendly society or body corporate established for charitable or philanthropic purposes as may be approved by the Public Trustee and Treasury.

Custodian Trustees are entrusted with the custody of the trust securities, the ownership being vested in them as in a sole trustee. They are not concerned with the management of the trust property, such as changing investments or supervising property held as security, these duties being vested in private persons known as Managing Trustees. They must, however, concur in and perform all acts necessary to enable the managing trustees to exercise their powers of management, provided such acts would not involve them in personal liability or amount to a breach of trust. The fees that may be charged by companies acting in this capacity are limited to the fees charged by the Public Trustee.

In framing the regulations under which companies are licensed to undertake the duties of Custodian Trustees, the Treasury has not ignored the adage *quis custodiet ipsos custodes* and somewhat comprehensive rules have been drawn up.

They must agree to keep separate accounts of every trust, showing the mode of investment of the capital, the place of custody of the securities, and the manner of dealing with the income. Annual accounts must be furnished to the Public Trustee, and all monies held by them as Custodian Trustees must be kept in a separate account at a bank, to whom notification must be given that such money is trust money. The latter provision does not apply to a bank acting itself as Custodian Trustee. The annual returns to be forwarded to the Public Trustee consist of a Revenue Account and Balance Sheet. The Revenue Account has not been framed with a view of forming a connecting link between the Balance Sheets of successive years or of showing the dividends that are received upon Trust funds or any fresh capital that may be received.

The receipts must include the fees received for acting as Custodian Trustee, and any interest or dividends upon a special fund which may be set up to meet liabilities. On the expenditure side must be shown commission and expenses of management. The balance sheet must show as a liability the capital value of trusts in respect of which the company is acting as Custodian Trustee. Assets are to be scheduled under various classes ; land being specified as freehold, leasehold and copyhold, while house property must be given as either freehold or leasehold. Investments are to be given at cost price, but apparently there is no provision for showing any gain or loss which may be made when a change of investments occurs, and, presumably, the capital value of estates shown as a liability will be altered to meet such fluctuations.

It is optional for companies to set up a special fund to meet liabilities which may be incurred through acting as Custodian Trustee. It may, however, be presumed that in most cases, the general resources of a company, excluding in the case of an assurance office, the specially allocated funds, would furnish ample security. Companies are not permitted to advertise the fact that they have been approved as Custodian Trustees unless such announcement is accompanied by a statement that no liability attaches to the Consolidated Fund in respect of any act or omission on the part of the company.

Before a Company can be approved it must satisfy the Public Trustee that under its Deed of Settlement or Memorandum of Association it has power to act as an ordinary trustee, and, moreover, its Registered Office must be situated within the

jurisdiction of the Chancery Division of the High Court. Thus a company whose registered office was in Scotland would not be licensed to act as Custodian Trustee.

The dual control involved in the creation of the office of Custodian Trustee introduces a new element into the law of trusts.

Hitherto it has been a rule of equity that trustees must not delegate their duties, and this rule has been stringently enforced. For instance, if trust money is to be invested, or a valuation of property to be made, the trustees must themselves choose the broker or valuer, and not delegate the duty to another, or even one of themselves. It is not apparent from the Act whether the managing trustees have still to exercise their judgment with regard to duties of this nature or whether they may be left in the hands of the Custodian Trustee as if he were sole trustee.

Again it has hitherto been the general but not invariable rule that the legal estate in the trust property should be vested in the trustees, who have had the duty of defending the legal title to the property, as well as managing the trusts. When a Custodian Trustee is appointed the managing trustees will be divested of the legal estate and left with merely controlling powers, the equitable estate being still in the beneficiaries.

There is no mention in the Act as to how the power of management is to be exercised in the event of the Custodian Trustee refusing to obey the directions of the managing trustees, or dealing with the property without their authority. A Custodian Trusteeship may, however, be terminated by the Court upon the application of the Custodian Trustee or that of any managing trustee or beneficiary, provided that the Court is satisfied that it is the general wish of the beneficiaries or is expedient on other grounds.

The position of a Custodian Trustee with regard to breaches of trust is not entirely satisfactory. Although he is not liable if he act on the written statement of the managing trustees as to any matter of fact upon which the title to the trust property depends, yet he is apparently liable if he concur in an act which amounts to a breach of trust or involves a personal liability, and it will, therefore, be necessary that careful attention should be paid to the terms of the trust before giving concurrence to any act.

A scheme of this nature seems likely to give rise to many difficult questions in practice, and it cannot be said that it has fulfilled expectations or gives promise of being a success.

In addition to the expense to the estate involved through the appointment of a Custodian Trustee, there still remains the possibility that it may become necessary to appoint new managing trustees.

Moreover, it may be assumed that the difficulty of finding private persons willing to be managing trustees will be greater than in the case of obtaining ordinary trustees, when it is remembered that such persons, while incurring the liabilities incidental to a trusteeship, will be deprived of the ownership of the trust property.

From the point of view of the management of the trust property the prospect is not encouraging, for there will doubtless be a tendency when a Custodian Trustee is appointed for the Managing Trustees to endeavour to leave the supervision of the property in his hands, with results that might not be entirely satisfactory to beneficiaries.

THE NEW ZEALAND SYSTEM.

The most important instance of the constitution of a Public Official to administer trusts occurs in New Zealand, and the system there adopted possesses features of interest.

The Public Trust Office was instituted in 1872, and the Public Trustee acts with an Advisory Board consisting of the Minister of Finance and other members of the Ministry. The consent of the Board is required before any appointment can be accepted. The range of duties that may be undertaken is very wide, and includes power to act as trustee, executor, administrator, guardian, committee, agent or attorney. In fact, it has been said that the Public Trustee may assume any power which one person may delegate to another.

He is entitled, should he think fit, to apply for the administration of the estate of any person dying intestate who is domiciled in New Zealand or possesses property there, with special power to act in intestate estates of under £250 value without taking out letters of administration.

Trustees, administrators, and executors may, unless expressly prohibited, appoint the Public Trustee to act instead of themselves, and he may exercise certain powers with respect to the estates of deceased persons, pending the grant of probate or letters of administration to the person entitled.

The management of estates of insane persons is by Statute placed in the Public Trust Office.

He may act in any of his various capacities for the Government, Court, or for any public or private corporate body.

He is trustee for several Government Superannuation Funds, manages native land reserves, and it is not unusual for local bodies to accumulate sinking funds with him in order to pay off loans on maturity.

The Common Fund. Perhaps one of the most interesting features of the scheme is the system adopted in the investment of trust funds.

Unless expressly prohibited by the terms of the trust instrument, all investments are placed in one Common Fund, and such investments are not to be made on account of, or belong to, any particular estate. The interest payable to the persons entitled is fixed from time to time by the Governor in Council, and must not exceed a higher rate than 5 per-cent on sums under £3,000, or on sums exceeding this amount 5 per-cent on the first £3,000, and 4 per-cent on the excess. Both capital and interest are guaranteed by the State, and any deficiency in the lawful claims is met out of the Consolidated Fund.

Investments made otherwise than in the Common Fund are not guaranteed, and any loss or deficiency in capital or interest is borne by the particular estate.

The securities in which the Public Trustee may invest the Common Fund are limited to the following classes :

Government Securities of the United Kingdom or any Colony or Dependency.

Debentures issued by any local authority secured either upon general or special rates, or real estate held in fee simple. Mortgages of real estate in New Zealand to the extent of three-fifths of the estimated value of the property.

Fixed deposits in certain authorized Banks.

The total funds held by the Public Trustee on 31 March 1908 amounted to about £2,445,000, over £2,000,000 of this sum being invested in mortgages, the average rate of interest earned on the total fund being over 4½ per-cent.

The interest rates payable to beneficiaries with investments in the Common Fund were raised on October 1905 to 4½ per-cent on sums under £3,000, and 4 per-cent on any excess over the first £3,000.

A capital fee is charged on the realization of an estate, varying in rate according to the value of the estate or nature of the property. A fee of 5 per-cent may be charged on gross income.

Considerable profits have been made, one fourth of which is accumulated to form an assurance or reserve fund. The remaining three-fourths is at the disposal of the Minister of Finance for payment into the Consolidated Fund, but no part of these profits has yet been transferred.

Although reversions to trust funds in which the capital value is so well secured and free from market fluctuations would seem to be attractive investments, I understand there are but few dealings in this class of security in New Zealand. This may possibly be due to the high rate of interest prevailing. The Public Trust Office has authority to make advances to beneficiaries, the rate of interest charged being 5 per-cent, and no costs being charged in connection with the transaction. The sums advanced are, however, restricted in amount, and if larger amounts are required it is usual to mortgage or sell the security privately. In such cases the rate of interest ruling is higher even than 5 per-cent.

Full information with regard to a trust is given by the Public Trustee to a possible purchaser or mortgagee on production of the written request of the beneficiary, and a record is kept of every charge or assignment upon notice being given to the office.

I am indebted for many of the above details to Mr. J. W. Poynton, the Public Trustee of New Zealand, whose kindness in giving the information I wish to acknowledge.

Official Trustee
in India. Brief mention may be made of a system of administration of trusts under official supervision which has been in existence in India since 1864. The Official Trustee there has to give security and to furnish annual returns to the Court. On his death or retirement all property and interests held by him vest in his successor immediately upon appointment.

There was apparently no State Guarantee until 1902, when it was enacted that the Official Trustee should receive fixed remuneration in place of fees, and all civil liability in respect of his office should be undertaken by the State.

COMPARISON OF ENGLISH AND NEW ZEALAND SYSTEMS.

In contrasting the New Zealand and English systems, attention will probably be centred upon a comparison of the comprehensive state guarantee as to capital and interest, with the limited guarantee against breaches of trust.

The New Zealand system is characterised by a boldness and breadth of view which is admirable in many respects. It may be said to be somewhat analogous to a guaranteed deposit bank, in which deposits may be made for indefinite periods, and in which the rate of interest is not immediately dependent upon market conditions.

Whether such a system would be applicable to this country is, I venture to think, doubtful.

Any scheme whereby the full capital value of trust funds is guaranteed to the beneficiaries must necessarily be accompanied by limitations in the powers of investment.

The trend of legislation in England during recent years has been in an opposite direction, and has been to widen the area of investments available for trust funds (unless expressly forbidden by the trust instrument).

Thus, under the Trustee Act 1893, investments may be made in certain securities of purely commercial undertakings, such as Railway companies. It would be obviously impossible for the State to guarantee to beneficiaries the full capital value of investments made in this class of security. Even with regard to British Government Securities, one can imagine the position of a department which had to pay to beneficiaries, the cost price of investments made, say, in Consols some years ago.

In fact any guarantee of this nature would tend to restrict investments to those classes in which the capital value is fixed, such as mortgages and fixed loans, while investments subject to market fluctuations would be avoided.

This tendency is shown very strongly in the list of investments held by the New Zealand Public Trust office, by far the greater proportion of which are placed in loans to local bodies and mortgages of real estate.

A further point in connection with the guarantee as to capital is, that when the rate of interest is high and prices of securities ruling low, there might be a tendency among beneficiaries, provided they were all *sui juris*, and absolutely entitled, to determine the trust and re-invest the funds. In other words, there might be a selection exercised against the Public Trustee in the direction of a withdrawal of funds at inconvenient periods. I understand this selection has not been noticed in New Zealand, where only a few trusts have been prematurely determined, and then for reasons other than that stated above.

In that country, however, the rate of interest allowed on the

funds is comparatively high, and there would probably be fewer opportunities for taking advantage of conditions favourable to an exchange. The fact that a somewhat heavy fee is payable upon withdrawal of capital would also act as a deterrent to such a course.

Although the scheme is attractive by reason of its simplicity of working, and the fact that no uncertainty exists as to the actual value of the trust estate, I think it would be unsuitable to this country for the following reasons: The indefinite obligation incurred by the State, the limitation of investments, and the possible selection that might be exercised against the department.

Moreover, the rates of interest that could be allowed might not prove attractive to settlors or testators.

REVERSIONARY INTERESTS.

Reversionary Securities. A trustee is seldom a party to a contract for the sale or mortgage of a Reversionary Interest, and from this fact arises many of the disadvantages and difficulties which beset this form of investment.

Holding the legal estate in the trust property for the benefit of the *Cestui que trust*, he must give him any information respecting investments if asked to do so, but is under no obligation to give information as to incumbrances the *Cestui que trust* may have created, and is able to deal with the property within the terms of the trust instrument or as authorized by law, without any reference to a mortgagee or assignee of the equitable interest.

In view of the tendency which has arisen to place trust funds in the hands of the Public Trustee, it may be useful to review some of the points which arise in connection with investments in Reversionary Securities in the hands of private trustees, and to consider how they will be affected by the substitution of an Official Trustee for a private individual.

One of the main drawbacks to this class of security is the fear of improper dealings with the trust funds.

This is admirably summed up by Mr. Coutts in his paper on "Reversionary Securities as Investments" (*J.I.A.*, vol. xl, p. 318), in which he says, "Perhaps the most important general objection to "reversions as investments is the fact that, unless the fund is in "Court, it is under the control of trustees generally unknown to "the office before purchase, who may fraudulently misappropriate

" the investments, or through sheer incapacity or a desire to " increase the income of the life tenant, reinvest in unsatisfactory " securities and appreciably diminish the capital value." In fairness however to trustees, I think it will be admitted that where the fund is in the hands of two responsible persons, these objections are more apparent than real.

Enquiries of Trustees. In the absence of express stipulation a trustee is not bound to answer enquiries made by a probable assignee or mortgagee with regard to notices of incumbrances on the fund which may have been received.

The law on the subject rests on judicial decision and has been summed up as follows :

- (1) Trustees are not bound to answer enquiries made by a stranger about to deal with the *Cestui que trust*.
- (2) If they choose to answer they need give no more than an honest opinion according to their knowledge and belief without any obligation to make enquiries.
- (3) If they answer honestly they incur no liability unless they make a statement amounting to a warranty, or one so clear and unambiguous that they are prevented by estoppel from afterwards denying it.

It is necessary, moreover, that satisfactory answers respecting enquiries as to previous notice of incumbrances should be received from all the trustees before proceeding with the transaction. A case bearing on this point is that of *Ward v. Duncombe* 1893, A.C. 369. Proposed mortgagees of a reversionary share of trust fund made due enquiries as to previous incumbrances of the two trustees. One trustee having knowledge of a prior settlement gave an evasive answer; the other trustee replied honestly that he knew of no incumbrances. The transaction was carried through, and the trustee who knew of the prior settlement afterwards died. It was contended that the notice of the mortgage which had been given to the remaining trustee should give the mortgagees priority over the settlement. The Court of Appeal decided otherwise, Lord Herschell remarking " that if the trustees or one of them decline to answer, an " intending incumbrancer must take upon himself the risk of " whatever prior incumbrances there might chance to be for he " would be dealing with property which he had no sufficient " ground for concluding was at the disposal of the *Cestui que trust*."

Specific Performance.

While dealing with the question of trustees answering enquiries mention may be made of the question as to whether, in the event of the trustees refusing to give information, specific performance of a contract to purchase a reversion would be enforced. This question would more probably arise under an auction contract, as in the case of a sale or mortgage by private treaty satisfactory assurances on the point would first be required.

I do not know of any reported decision on the point, but with regard to the general question it will be remembered that specific performance will not usually be enforced where damages would be an adequate remedy. It is difficult to conceive of a case in connection with reversionary interests where damages would not be an adequate remedy.

The remark of Lord Herschell in the case of *Ward v. Duncombe*, mentioned above, has, however, a bearing on the point. A title to property which the purchaser or mortgagee had no sufficient ground for concluding was at the disposal of the *Cestui que trust* would certainly be doubtful.

Although a Court of Equity may not affirm a title to be bad yet it may be considered to be so doubtful that it would not compel a purchaser to take it.

The trend of modern decisions seems to be that, apart from stipulations as to title, the Court will not compel an unwilling purchaser to accept a title which is doubtful as to fact or law.

Notice to Trustees.

Upon completion of the purchase or mortgage of a Reversionary Interest it is necessary that notice should be served upon each one of the trustees.

Notice to one only of the trustees is not sufficient, for when one of the trustees had received notice, and the others had not, the latter are not liable for anything they may do in ignorance, *Low v. Bouverie*, 1891, 3 Ch. 82. Some doubt has been expressed as to whether it is legally necessary for fresh notice to be given to a new trustee upon the death or retirement of an existing trustee.

In the case of *re Wasdale Britten v. Partridge*, 1899, 1 Ch. 163; also *J.I.A.*, vol., xli, p. 148, it was held that an assignee of a reversionary interest in a trust fund who has given notice to all the trustees in existence at the time of the assignment, is under no obligation to give further notice, and is entitled to priority over a subsequent assignee who has taken his assignment after the death or retirement of all those trustees. On the other hand,

while a new trustee has been held liable to make good moneys paid by him *bond fide* to a beneficiary when the papers relating to the trust contain notice of an encumbrance created by the beneficiary depriving him of the right to receive the money, yet the liability of the trustee depends entirely upon his shirking the duty of search, and, if no amount of search would have disclosed the notice, he would not be liable (*Hallows v. Lloyd*, 39 Ch. 396).

It might happen that notice of a charge might be lost or mislaid. If the original trustees, in existence at the time the charge was created, died or retired without informing the new trustees, there is the danger that the fund might be paid to the beneficiary or a subsequent encumbrancer. As a precautionary measure it seems advisable that further notice should be given on the appointment of a new trustee.

Further points which often restrict dealings with reversionary interests occur when there is only a sole trustee, or when the beneficiary selling his share is also a trustee. The objection to a sole trustee is, that whether he was originally appointed to act alone or has become so by survivorship, he has the absolute and unlimited control at law over the property, and the danger of misappropriation of the trust funds is consequently increased.

The fact of a beneficiary being a trustee is open to objection on the ground that in case of breach of trust he will in general be liable to indemnify his co-trustee to the extent of his beneficial interest. This latter point is not of so much consequence when the trustee is a life tenant, and is selling or mortgaging his life interest, as where the trustee is a reversioner.

Changes in Investments. Generally speaking, information of any change in the investments of the fund will be obtained by receiving a notice warning off the *distringas*. Occasionally, however, a notice in lieu of *distringas* cannot be placed upon some of the securities, and if the fund be in the hands of private trustees information as to a change of investments can only be obtained by enquiry, or by notification from the solicitors. It is, of course, necessary that a purchaser should have knowledge of any proposed change in order to be in a position to take steps to prevent the re-investment being made in unauthorized securities, and also for valuation purposes.

GENERAL CONSIDERATIONS.

Considering the preceding questions from the point of view of the trust funds being held by the Public Trustee, it is obvious,

in the first place, that fear of misappropriation of the funds is at once removed. Moreover, it is not likely that a Public Department would be influenced by a life tenant to invest in securities which yield a high rate of interest, to the detriment of the capital value. With regard to answering enquiries, the practice of the Department is governed by the Public Trustee Rules. These state that, upon application in writing, information respecting the trust property may be given to any applicant who is interested in the trust property, or who holds the authority of any person interested. The rules in respect to this point are very comprehensive, and allow copies to be taken of an entry in the Register relating to the trust or estate, or so far as the interest of the applicant is affected by any account, notice, or other document in the custody of the Public Trustee. It must not be overlooked that the Department or Consolidated Fund is exempt from any liability in respect to any inaccuracy in the information supplied. I believe, however, that it is very usual for private trustees to limit their liability in this respect.

There will be a further simplification from the point of view of a Life Office with respect to giving notices of assignments or charges, for the Public Trust Office being a continuing office, notice once given will be sufficient.

It is difficult to see any reason for objection to the purchase or mortgage of a reversionary interest on the ground that the Public Trustee is sole trustee. In fact, when he acts as co-trustee, it will eventually happen, unless the trust be previously determined or a fresh trustee appointed, that he will be in the position of sole trustee through the death or retirement of his co-trustee.

With regard to changes in investments, I understand that the Public Trustee will, if requested at the time notice is given, inform assignees or incumbrancers of any change in the investments of a trust fund.

This should prove of advantage to Life Offices in regard to those investments upon which distinguishing notices cannot be placed. In this connection it is often observed that where a Government Department is established by Statute, there is a tendency for many of their decisions, except in matters of great importance, to be accepted without question, and for a large body of practice to spring up which might or might not be warranted on strictly legal grounds.

If giving information to incumbrancers with respect to

changes in investments becomes the settled practice of the Department, it might be worth while to consider the question as to whether it was necessary to place distringas notices on stocks and shares held by the Public Trustee.

An important point to a purchaser of reversionary interests is the question of the fees charged.

Under the Public Trustee Act, the fees are to be arranged from time to time to produce an annual amount sufficient to meet expenses, and to insure the Consolidated Fund against loss. Probably only experience can show whether the present scale of fees is adequate or not, but should it prove insufficient, the question would arise whether the fees on trusts which had actually been accepted would be increased.

With regard to the capital fee payable upon the withdrawal of funds, the Public Trustee Rules provide that this shall be at the rate per-cent upon which the fee for the acceptance of the whole trust property was calculated. The Public Trustee may, however, agree to commute the Withdrawal Fee for its present value at 3 per-cent interest, and in this way any doubt on the part of the purchaser of a Reversion could be eliminated.

With regard to Income Fees, which would affect the case of the purchaser of a Life Interest, I understand that arrangements have been made, whereby in many cases these can also be commuted.

A further point that may be mentioned is that, while in calculating the price that can be given for the purchase of a Reversion or Life Interest to funds in the hands of the Public Trustee allowance can be made for future fees payable, there is a possibility, if the funds be in the hands of private trustees, of the trust being transferred to the Public Trustee, and the fees, falling on the estate, a purchaser would consequently suffer.

In concluding this aspect of the subject it does not seem probable that there will be any tendency for reversionary securities of the better class to realize higher prices by reason of the funds being in the hands of the Public Trustee. Probably the fees payable will be looked upon as a premium for the improved security. Doubtless, however, many securities, which, in the hands of private trustees would prove unacceptable for questions concerning the trusteeship, would become available as investments for Life Offices if the trust estates were administered by the Public Trustee.

CORPORATE TRUSTEES.

Closely connected with the subject of this paper is that of Corporate Bodies transacting the duties of Executors and Trustees, and a brief reference thereto may not be thought out of place.

It may be noted that the Public Trustee is a particular type of Corporate Trustee, created by Statute, and possessing special powers.

In the United States the administration of trusts by companies has attained large proportions; in many of the Colonies, trusts are often administered in this manner.

In England the system does not appear to have attained any growth until a comparatively recent date and it is only during the last few years that any Life Assurance Companies have commenced to transact the business.

Despite the comparative youthfulness of the Corporate Executor and Trustee, it was pointed out by Lord Avebury in the House of Lords, December 1906, on the occasion of a debate on the Public Trustee Bill: "That as some of our most important banks and insurance companies had undertaken to act as trustees, there was not the same pressing need for the Bill." In view, however, of the large amount of property that is administered under trusts, one may express the opinion that the Public Trustee and the Corporations may be considered to be, not so much in competition, as supplemental to each other, and it is to the advantage of testators and settlors to be able to choose the method most advantageous to the estate.

The first question, which would probably arise in the case of a Company intending to carry on trustee business, is that relating to their power to act in this capacity. Such power may be contained in the Charter incorporating the Company, Private Act of Parliament, Deed of Settlement or Memorandum of Association. The power should be expressly stated, and it would not seem advisable to rely upon any general words which may be contained in a clause framed to enable the Company to carry on other kinds of business. The tendency of the Court with regard to objects which are ancillary to the main object of the Company was shown in the recent case of *re Palace Restaurants, The Times*, 2 September 1909, where a petition by debenture-holders for compulsory winding-up was granted on the ground that the main object for which the Company was established had failed.

A further case having an indirect bearing on the point is that of *Blythe v. Birtley*, *The Times*, 1 December 1909, where an injunction was granted to restrain the trustees of the Royal Co-operative Collecting Society from acting upon a resolution passed and confirmed for the conversion of the Society into a Company. The principle reason given for granting the injunction was that the proposed objects of the Company (which included *inter alia* trustee business) were so far beyond those specified in the rules of the Society.

If a Company registered under the Companies Acts possess no express power, advantage may be taken of Section 9 of the Companies (Consolidation) Act, 1908, which permits the alteration of the Memorandum in certain directions. Thus by special resolution subsequently confirmed by the Court, power may be obtained to carry on some business or businesses, which under existing circumstances may conveniently or advantageously be combined with the business of the Company. In the case of a Life Assurance Company it is assumed that this definition would include trustee and executor business.

Companies which have power to act in these capacities work under certain legal restrictions, based upon the practice of the Court, which, although not vital in their nature, might with advantage be removed.

These disabilities are set out by Mr. E. K. Allen in his work on "Corporate Executors and Trustees." Among them may be mentioned that a Company cannot act as co-executor with an individual, although it may act as co-trustee. Also a corporation aggregate is not permitted to take the oath necessary before a grant of Probate can be obtained.

This latter restriction renders it necessary when a Corporation acts as executor, to appoint an individual, usually an officer of the Company, and known as a Syndic, to receive administration with the will annexed. Should the terms of the will constitute a trust it becomes necessary, when the office of executor merges into that of trustee, for the trust property to be transferred from the Syndic to the Corporation. If, however, investments existing at the time the estate is taken over have to be sold, this entails no disadvantage, as the new investments may be made in the name of the Corporation.

It appears that Parliamentary action would be necessary to obtain an alteration in the law relating to Companies acting as Executors and Trustees. An attempt made in 1905 by a

Company to obtain greater facilities was refused by Parliament on the ground that it was not advisable to grant an amendment of the general law by a Private Bill.

While admitting the validity of this argument it is not without precedent that such a change has taken place. It was the general law under Lord Campbell's Act that in estimating the pecuniary damages sustained by reason of death, the amount of any assurances against accident should be taken into account. Before the general law on the point was altered several Companies had obtained private acts abrogating this rule with respect to their policies.

In view of the present tendency of the Legislature as shown in the Assurance Companies Act, 1909, and also the rules relating to Custodian Trustees it seems probable that if an Act were passed dealing with Corporate Trustees it would contain provisions with regard to the publication of Revenue Accounts and Balance Sheets, although possibly the necessity for a deposit to be made with the Paymaster-General would be omitted.

Turning particularly to Life Assurance Companies, it would appear that as one of the principal duties of a trustee consists in the judicious investment of the trust funds, such Companies are particularly fitted for the administration of trusts.

The two first canons enunciated in connection with the investment of Life Assurance Funds are: (1) The primary necessity of securing the safety of the capital. (2) That the highest rate of interest consistent with such safety should be obtained. These principles apply equally well to the investment of trust funds for (subject to the terms of the trust instrument, or the law for the investment of trust funds) by following them the interests of both reversioner and life tenant will be secured.

Moreover, unless a Life Company anticipates a decrease in its funds in the near future, a large proportion of its investments may be made in what may be called "long term securities." This class of security is, in most cases, advisable for the investment of trust funds, for the capital being generally locked up during the lives of existing persons, the charges incurred through changes of investment might, unless the re-investments be made with great skill, result in loss to the reversioners and life tenants.

A further advantage is that Life Companies have an unique experience in the management of large properties, and are, therefore, peculiarly fitted to undertake the control of landed

estates, house property, and investments of this nature often included in trusts.

When a Life Office acts as trustee it seems not improbable that it may occasionally be asked to purchase the share of a *cestui que trust*. Although a trustee is not incapable of purchasing the interest of a beneficiary, yet the Court regards such transactions with great jealousy, and the burden of proving the validity of the transaction will lie upon the person obtaining the benefit of it, and, if impeached, the contract cannot stand unless the trustee can show conscientiously and affirmatively that the parties were at arms' length, that the transaction was for the benefit of the beneficiary, and that full information was given to the beneficiary of the value of the property, the nature of his interest, and the circumstances of the transaction. (*William v. Scott*, 1900, A.C. 499).

Prior to the passing of the Sales of Reversions Act 1867, the obligation was considered to lie upon the purchaser of a reversion to shew that he had given full value for it. This Act abolished the setting aside of sales of reversions merely on the ground of undervalue, but still leaves this question open to be raised if there are other equitable grounds for relief. It would therefore appear that for a Corporate Trustee to purchase such interests by private treaty would be undesirable, for if a question as to the validity of the sale be afterwards raised, the question as to the adequacy of the purchase money might also be brought forward, and it was established that the market price of a reversionary interest, and not the estimate of actuaries, was the criterion by which the Court decided the question of undervalue.

The question of granting a loan to a beneficiary upon the security of his interest stands, however, on a different footing, and it was decided in the case of *Newman v. Newman*, 28 Ch. D. 674, that a trustee may take a fair mortgage from his beneficiary, and in that case may rely upon his possession of the legal estate to give him priority over prior mortgages of whose claims he had no notice.

CONCLUSION.

The justification for the establishment of a State Department to administer trusts rests, I venture to think, on the ground that it is the right and duty of the State to protect the interests of its members.

It may be a matter of opinion as to how far such protection should extend in the direction of interference with the rights of

private contract, but where the necessity exists the principle of the right of the State to give protection is admitted.

The policyholders of an Assurance Company, the members of a Friendly Society, the shareholders of a trading Company, are safeguarded by enactments which rely principally upon the element of publicity to attain their end, but which generally leave the enforcement of their provisions to the initiative of the members.

No such protection exists in the case of beneficiaries of a trust, who are frequently in ignorance of the exact terms of the trust instrument, and often, as in the case of minors, under some legal incapacity.

The fact that a trust is of the nature of an equitable obligation rather than a contract, and that the persons to be benefited may not be parties to it, strengthens the view that there should be some means whereby their interests may be efficiently safeguarded.

The necessity for supervision to be exercised over trust funds was recognized by the Judicial Trustees Act 1896, which provides for an annual audit of the accounts of a judicial trustee by an officer of the Court. A similar method is adopted in Scotland where the judicial factor sends accounts once a year to the office of the Accountant of the Court.

To apply some such scheme to trusts administered by private persons would probably be considered too stringent, and in addition to causing expense would render it still more difficult to obtain individuals willing to accept the duties of trustees.

The plan adopted under the Public Trustee Act has been framed on lines calculated to involve as little interference as possible with private rights. In respect of trusts administered by private individuals the right has been given to obtain an audit and investigation of the trust accounts on behalf of any trustee or beneficiary, without the necessity of an application to the Court. With regard to new trusts a settlor or testator may still appoint whom he will to be trustee. Moreover the Public Trustee occupies no privileged position, but is subject to the same duties and liabilities and to the control and orders of the Court as a private trustee. He must, wherever practicable, take into consideration the wishes of the creator of the trust, or of other trustees or beneficiaries, either expressed or implied by the practice of the creator of the trust or its previous management.

To put the matter briefly, the State undertakes, if a trust is

placed with them, to follow directions, to protect the interests of beneficiaries, and to provide a guarantee against loss by breach of trust.

While, perhaps, capable of improvement in certain directions, such as a more liberal guarantee, or some simplification with regard to the transference of the trust property to the Department, it was probably inevitable that in a voluntary scheme, largely dependent for its success upon the goodwill of various professions, the ordinary law with regard to trusts should have been adopted with but slight modification.

Judging from the results of the Department, which after being two years in existence is administering current trusts of about £5,500,000 in value, there can be but little doubt that the scheme was needed and is acceptable to both settlors and testators.

In concluding this paper the author is only too conscious that many points relating to the subject have not been touched upon. If, however, the hope that it will promote an interesting discussion is realised, its purpose will have been amply fulfilled.

ABSTRACT OF THE DISCUSSION.

MR. T. F. ANDERSON pointed out that the Judicial Trustee Act of 1896, had been made little use of in practice, but its provisions seemed to be useful, and did not involve heavy fees. So far as the Public Trustee was concerned, he was disposed to think that the charges were too low for small estates, but that, with trusts of £5,000 and over, the fees were sufficient to give a margin of profit. He also pointed out the possibility of difficulties arising if the Public Trustee exercised his right of obtaining a Judge's direction in small cases, without consulting the opposite sides. He presumed the practice was to give notice to the other parties, but thought that, to avoid the possibility of trouble arising, this notice should be made statutory. So far as the State guarantee was concerned, he would much like to see it made more clear and definite. As it stood at present, the expression or qualification "exercise of reasonable diligence," was likely to lead to disputes.

As regards Custodian Trusteeships, he was strongly of opinion that they should be avoided. The duties were badly defined, and, if an office were so appointed, it meant that the company did all the work of a full trustee for inadequate fees. He also alluded to the appointment by the Public Trustee of auditors to go through trust accounts. He presumed that, in the exercise of the powers conferred on the Public Trustee, he charged the expenses on the

estate, or on the beneficiaries applying for the audit, or on the trustee, according as the audit was a fair trust expense, or the beneficiaries were unreasonable in their demand, or the trustees had committed breaches of trust.

MR. J. R. HART said the author had given examples of how reversions would be improved by the substitution of a Public Trustee for private trustees, and had mentioned the risk of misappropriation of the funds, and of a private trustee investing at a rate of interest which would endanger the capital value of the fund. There were one or two other risks which he would like to mention. A trust fund very often included bearer bonds, and companies dealing with reversions had to make provision for their custody. A very common case was one where the beneficiary was one of the trustees. He thought it was a very good thing for companies that the Public Trustee should be substituted for private trustees, as those elements of risk which at present existed in connection with a very large number of reversions would be obviated. The rate of interest companies were now getting, which he considered was very much too low, having regard to the real risks, would then be really an adequate return. Turning more to the general question of the appointment of the Public Trustee, he noticed there had been a good deal of criticism in the legal papers with reference to the absence of the personal element. It was said that the interest of kinship of private trustees was very often a valuable thing; it enabled those private trustees to carry out the wishes of the testator. He believed the Public Trustee tried to bring that personal interest to bear as far as was possible.

Another criticism directed against the Public Trustee was that of "officialism." He thought there might be a disadvantage in that because there might be a want of elasticity. Very often disputes arose, and private people came together and agreed to a compromise. A Government Department could not feel justified in adopting such a course. On the question of assurance companies taking up the particular business, there was one opinion expressed by Mr. Sharman, on pages 334 and 335, from which he would venture to differ. The author stated: "A further advantage is that life companies have an unique experience in the management of large properties, and are, therefore, peculiarly fitted to undertake the control of landed estates, house property and investments of this nature, often included in trusts." With one or two exceptions, he was inclined to think that life companies had not the machinery to manage estates; they could collect ground rents and similar rents, but his experience was that if a company got one or two foreclosed mortgages they were very glad to get rid of them as soon as possible.

MR. R. R. TILT said the members had before them a practical paper on a subject which was of interest to Reversionary Departments of Life Offices and also to Reversionary Companies, whose investments in trust funds represented perhaps some five or six million pounds. The subject, in fact, touched the whole *raison d'être* of reversionary offices. He wished, therefore, he was more competent

to criticise the appointment of the Trustee and express a more definite opinion. There were one or two points, however, which had occurred to him and to which he might refer. The principal benefit connected with the Public Trustee, as stated in the paper, seemed to be the State guarantee against breach of trust. He thought that in the trusts which were dealt with by members of the Institute, the companies so protected themselves that the risk of breach of trust was small ; they put distingases on the stocks and took other precautions that minimised the risk of fraud. There was one class of trust investment which presented risk, namely, the investment of trust funds on mortgages of property. He believed it was the custom of the Public Trustee, in taking over a trust to employ the solicitors of that trust, and he did not know to what extent the Public Trustee accepted mortgage securities. It might very well be that in a trust managed in the solicitor's office the solicitor would take roseate views of the property belonging to some of his clients and invest the money of other clients on its security, having first obtained a valuation. Valuations which he had seen had not impressed him as being of great value : they were often little more than multiplication sums. The rent was taken and multiplied by 16 or 20, and on the strength of that valuation the solicitor would lend two-thirds of the value—of course in a *bonâ file* manner and with every desire to further the interests of his clients. But mistakes, and serious mistakes, had been made, as they all knew, as to the values of mortgage securities.

If the Public Trustee relied to any great extent on valuations thus made, he was inclined to think it would be better to have private trustees, who would probably be directly or indirectly beneficially interested in the trust, a fact which should make them watch the mortgages closely with a personal interest ; if the Public Trustee took investments of that class, change of time or other circumstances might produce depreciation in these mortgages, which might get overlooked, whilst pigeon-holed valuations would protect the trustee. He should think that many of the losses of trust funds, irrespective of depreciation of Stock Exchange investments, had been due to bad mortgage securities. The provision for the investment of trust funds in mortgages was one of the weak points of trusts. He was glad to hear that the Trustee was willing to give notice of changes of investment from time to time to all those interested. That was a matter which was of great convenience to the purchasers of reversions. There might be some little risk that the Public Trustee having, under the advice of his brokers, formed a favourable opinion of a new issue, could render the flotation too easy with the large funds at his command : that is, the stock could be issued at a higher price than would be the case if the Public Trustee ignored it. However, he no doubt had careful advice and there was not perhaps much in the point.

MR. E. K. ALLEN (Principal Clerk in the Public Trustee Department) said it was a matter of regret to the Public Trustee that many pressing matters prevented his attending the meeting. The Public

Trustee, however, welcomed criticism from men of business, and particularly so when it came from gentlemen connected with life offices, because he knew it would be sagacious and well founded. On the question of the State guarantee, he was aware that in the Act there were some ambiguous words to the effect that the Public Trustee and his officers were not to be found liable to contribute to any loss if they had not contributed to it and could not by any "reasonable diligence" have averted it. Those words, he believed, were put in by Lord St. Aldwyn, when he was Sir Michael Hicks-Beach, into a former Public Trustee Bill, and were taken up by the Government, when, later, the Bill became a Government measure. The matter was not quite clear, even after seeing Lord St. Aldwyn's explanation of what he meant the effect of the words to be. The opinion seemed to be that the words did not and would not, if ever they came before the Courts for adjudication, excuse the Public Trustee, over and above the excuse which would be meted out to a private trustee. A private trustee was excused if he had acted reasonably and honestly, and ought, under all the circumstances, to be excused. It was thought, taking into account the proper reluctance of the Court of Chancery to excuse breaches of trust, that they would be very reluctant to give any enlarged meaning to those words in favour of the Public Trustee, and he thought that was very likely to be the trend of the decisions.

Some point had been made by Mr. Anderson as to the obscurity of Section 3 to afford a simple and cheap procedure for the administration of small estates under £1,000, where people were poor people, and as to whether notice of applications to the Judge was given to the beneficiaries. The Courts of Chancery had been slow to give that section its natural development, and it might be that some future Act would be necessary to give those Courts the clearer indication which they seemed to need, that the section did mean what it appeared to say. With regard to the bearing of the Public Trustee towards insurance companies, the Public Trustee Department had already observed that stop orders were not now so much used as formerly. Where the Public Trustee Department had taken over a trust, and the reversioners, or the people who had advanced money, asked the Department to give them notice of any change of investment or change of trustees, the Department was always glad to do so. That, he thought, was an advantage, and satisfactory to insurance companies and reversionary societies. Some point had been made that breaches of trust would not be permitted by an official, but he did not think that statement dealt adequately with the position. Surely the Public Trustee should have all the freedom conceded to a private trustee, and he should on such occasions take the view that would be taken by a prudent and common-sense man of business, and under the same safeguards and precautions. Of course, it was known that in some cases, where a breach of trust was of some complexity and difficulty, it could be carried out under the protection of the Court.

With reference to the question of the Public Trustee dealing

with mortgages, he there followed the trend of modern investment. Certainly the Board of Trade returns showed that mortgages were not favoured as they used to be, nor did the Public Trustee favour them unless they were first-class mortgages. In taking over a mortgage, the Public Trustee relied on nobody, neither the family solicitors nor the valuer. The valuer was probably known to him, or he had reliable information as to the valuer's standing and credit, but he also formed his own judgment as a man of experience as to the character of the mortgage, and if there was any doubt he would probably try to increase the margin of security by getting something paid off year by year. In taking over mortgages, the Public Trustee very early went through them personally, and marked the register as to how soon they were to come up for further investigation. This course of procedure in many trusts had brought about a much healthier state of affairs than when first taken over by the Department. He thought there need be no anxiety as to the Public Trustee's sense of duty as to mortgage investments, which no doubt required careful periodical revision, and for which revision the Public Trustee Department seemed, as opposed to the meagre organisation possible with a private trustee, particularly fitted.

The question of audit had been raised, and Mr. Anderson was quite right in his remarks. If the Public Trustee made no order, the cost of the audit fell on the trust estate, and as practice had gone he had in some cases, where the audit was more or less unnecessary, thrown the cost upon the applicants and, at other times, on the defaulting trustees. With regard to assurance companies acting as corporate trustees and executors, he supposed many actuaries would be divided in opinion as to whether they ought to enter into the business, thereby possibly offending vested interests, such as solicitors, who still had a very strong objection to corporate trustees, although, he thought, needlessly, for they should adhere to the legal work arising out of the trust and not to the office of trustee itself. At the same time, some figures had been put into his hands that day from which he understood the value of trust property in this country was estimated to be five hundred millions. The Public Trustee Department only had seven millions of that at present, so he was sure there was room both for the Public Trustee and all insurance companies. In America, where numerous trust companies had been at work for many years, he had gathered from correspondents that the private trust business was of very slow growth, and that they dealt with general financial business, which was more profitable ; and that would probably be the case in this country.

MR. T. B. GILLISON remarked that he might first mention that a very interesting paper on the subject of the Australian Trustee Companies was read by Mr. Robert C. Nesbitt (a London solicitor) to the Provincial Law Society, Liverpool, on 13 and 14 October 1903, in which that gentleman went fully into the question and gave particulars up to that date. The first Trustee Company in Australia was established about the year 1879. There was a very long discussion in the Houses of Parliament in Victoria, where the first

trustee company was established. Evidence was taken both for and against the establishment of these private trustee companies, and eventually the Trustees Executors and Agency Company, as it was called, with its head offices in Melbourne, obtained an Act, and proceeded to do business. In the course of a few years it was followed by others, and there were now in Australia fourteen trustee companies doing business, which had at the present time in their hands estates to the estimated value of thirty-five millions. It might interest the members to know that private trustees in Australia were usually entitled, either under the will, or order of the Court, to make a charge for their services, and he was inclined to think one of the reasons for the establishment of those trustee companies was to do the work for a little less than the private trustee was allowed by the Court. It was not at all unusual for 5 per-cent to be allowed by the Court for the management of an estate. Those trustee companies, therefore, came into the field and fixed certain charges, which were roughly, for moderate estates, about $2\frac{1}{2}$ per-cent on the corpus, and $2\frac{1}{2}$ per-cent on the income, with no charge for any change of investment.

The establishment of such trustee companies in Australia had, of course, been of very great interest to the life companies, for two reasons: first of all, the life assurance companies, in lending upon reversions and life interests in the hands of trustee companies, did so with the greatest confidence, because they knew the estates were properly managed. It was not unusual to charge a higher rate of interest when the reversion was in the hands of private trustees, and frequently a leading company would not do any business at all unless the matter was transferred by the private trustees to one of the trustee companies. The other reason was that the trustee companies were very keen competitors of the life offices in respect of their investments. Having such a large amount of money as thirty-five millions in their hands, they naturally required to find investments for it. The life offices, also with large amounts to invest, came immediately into competition with the trustee companies; and practically those two sets of institutions in Australia controlled the rate of interest on mortgage. There were now comparatively few estates managed privately. In connection with mortgages, which, of course, were the principal form of investment in Australia, he could quite understand the adverse remarks made that evening. Many people in this country did not know how to manage mortgage business. They lent their money from six months to six months, and, if it was in the hands of a private trustee, he allowed the thing to go on indefinitely, and paid very little attention to the property so long as the interest was paid. The trustee companies in Australia, in the first place, did not lend two-thirds (60 per-cent is the maximum fixed by statute): in the second place, they lent the money for a fixed term, namely, three or five years, and the mortgage came up before the board for revision at the end of that time. The system worked most beautifully; and he had never heard of any trouble at all in the office. It was perfectly easy to

get a statement of account. When there was a sum paid on account of a life interest a proper statement of account, showing how the interest was derived, and all the deductions, was immediately sent from the office with the cheque when it was collected, and one knew exactly how the estate was going on. Then in regard to the charges, there were no exceptional charges. The company employed the solicitor to the estate. It employed its own solicitor, at its own cost, to do a great deal of work in getting opinions and things of that sort, and it did not charge the estate with the particular cost of those opinions. There was another point, also, which he would like to mention, namely, when an estate was in the hands of a trustee company in Australia there was no necessity for distringases, or stop-orders, or anything of that sort : the life company simply lent on the estate, and the borrower was put to no expense.

MR. S. G. WARNER remarked that the question of trusts was an exceedingly interesting one. That great jurist, the late Professor Maitland, whose premature death was such a heavy loss to the science of which he was an exponent, had told the story of their origin and development, pointing out how they had their origin long after the development of the Common Law, in an attempt to evade that Law. But the only result was that a new branch of law came into being, with elaborate rules modelled very largely on those of the Common law, and continually approximating towards it ; until eventually the Judicature Act of 1873 profoundly modified the whole position ; and it came as an oddly suggestive climax that a movement originally so begun should culminate in the creation of a public, legal, national official to manage and administer trusts. As had been pointed out, the foundation element in the idea of a trust was personal ; but there were two possible aspects in which this might be regarded : trusts might be a means of binding a family together, or they might be a means of introducing dissension. He questioned whether, if one knew the whole history of trusts in this country, there had not been quite as much of the one as of the other, and it was conceivable, even on that ground, that the importation of an impartial official element might be a thing to be welcomed.

It was true that the idea of a public official did seem at first to suggest the impossibility of that individual solicitude and care, based on intimate knowledge of the circumstances, which had marked the historic system ; but on the other hand there were great risks to be avoided. The two primary reasons, according to the Commission quoted in the paper, why the establishment of such an office as that of Public Trustee was advisable, seemed to be that men were less willing than they used to be to accept trusts, and that when they did so they were more apt to fall under the erroneous impression that the fund existed for their own personal benefit. Probably, in the stress and strain of modern life, so constantly increasing, men did not feel justified in assuming responsibilities and duties so perplexing and extensive, and which might make demands on time and energy at unexpected and inconvenient seasons. If,

then, it might be taken that there was good ground for the creation of the public office, he thought everybody would agree that on the whole the verdict, at all events from the meeting that evening, had been one of approval, because the criticisms had, after all, been comparatively minor ones.

With regard to the question of the Trustee's responsibility, and the unsatisfactory character of such a phrase as "reasonable diligence," it was possible that a more accurate definition would be exceedingly difficult to find. With regard to small estates, it was certainly to be desired that greater facilities should be given than appeared to be given by the Act, or at all events by its interpretation hitherto in practice. It was undoubtedly with regard to the estates of comparatively small value that the chief troubles, difficulties and heartburnings in matters of trusteeship arose. The less money people had, the more important it became to them, and it was the small estates throughout the country which would probably profit most by taking advantage of the appointment of a Public Trustee.

Although it was not immediately connected with the subject of Public Trustee, he would like to say a word or two about some remarks which had been made on the subject of mortgages. If Mr. Gillison meant the private trustee who invested the money of the *cestui que trust* on mortgage and then paid no further attention to it, that was a matter about which he (Mr. Warner) had not the knowledge which would justify the expression of a positive opinion, and Mr. Gillison's remarks might be quite justified; but he thought it was matter of common knowledge that a considerable amount of care was exercised by British life assurance institutions in those matters—that mortgages were by no means entered upon and then hung up or forgotten, but that a very close supervision was exercised periodically, and everything that could possibly be done to watch them, and to intervene if they appeared to be getting in any way unsatisfactory, was done.

A cordial vote of thanks to Mr. Sharman for his paper was then proposed and carried with acclamation.

MR. W. C. SHARMAN, in reply, said that regret had been expressed that he had not dealt with the history of corporate trustees in this country more fully; but that subject was ably discussed in Mr. Allen's work on Corporate Trustees. Mr. Hart mentioned a remark, which had been attributed to the late Lord Justice Selwyn, with regard to it being the main duty of a trustee to commit judicious breaches of trust. He was not aware that that conception of the duty of a trustee was acted upon by the Public Trustee, but it might be advisable that occasionally that view should be taken, and it certainly removed one of the objections which had been urged against corporate and official bodies transacting trustee business. There had been a certain amount of discussion with regard to the liability for breach of trust under the State guarantee. There is one point which might be raised, namely, with regard to the provisions under the Judicial Trustee Act for the relief of trustees in respect to honest breaches of trust. It seemed

possible that, in the case of a trustee who had undertaken the trusteeship for remuneration, those provisions might be more strictly construed by the Court, than in the case of an unremunerated trustee, and consequently the security from breaches of trust would really be greater in the case of a corporate body than in the case of a private trustee. Probably these considerations would not apply to the Public Trustee, as the Department was appointed under statutory powers. He had been very interested in Mr. Gillison's remarks on the subject of the Australian Trustee companies. It certainly was instructive to hear that a higher rate of interest was charged for advances on reversionary securities in the hands of private trustees than those in the hands of trust companies. He had taken the view in his paper that, with regard to reversionary interests, there would probably be no tendency for prices to increase, by reason of the funds being in the hands of the Public Trustee, but with the Australian instance in view that statement might be somewhat modified.

I.—*On the Valuation of the Payment on the Death of a Pensioner of the excess of his Contributions, with or without interest, over his pension payments.*

II.—*On a Method of scheduling particulars for the Valuation, in certain cases, of prospective Pensions based on Terminal Salaries.*

By THOMAS TINNER, F.I.A., of the Comptroller's Department of the London County Council.

[Read before the Institute, 21 March 1910.]

I.

IN many staff pension funds there is a provision that if a pensioner dies before his pension payments amount to the accumulation of his contributions (with or without compound interest) to the date of retirement, the difference shall be paid to his representatives.

Although Mr. H. W. Manly and Mr. George King have dealt with the problem of valuing this benefit, and Mr. Manly has shown how to calculate the rate of contribution to provide for it, the formulas presently to be deduced may be thought of sufficient interest to warrant their publication.

In considering the benefit mentioned above, it will be seen that if—

- (i) the contributions amount to n times the yearly pension,
- (ii) the annuity used in valuing the pension be continuous,
- (iii) the benefit be valued on the assumption that the funds earn no interest—

the value of the benefit at the date of retirement, for each unit of annual pension, will be as follows, assuming the deaths to be equally distributed over the year:

Of l_x persons retiring at age x , d_x will die in the year of age x to $x+1$, and as they will, on the average, die in the middle of the year, there will be payable in respect of them $\left(n - \frac{1}{2}\right)d_x$, being the amount of their contributions less $\frac{1}{2}d_x$ which they will have received as pension.

The number dying in the year of age $x+1$ to $x+2$ will be d_{x+1} , and there will be paid to their representatives $\left(n - 1\frac{1}{2}\right)d_{x+1}$, being the amount of their contributions, less pension payments for $1\frac{1}{2}$ years.

The total payments in respect of those dying in the succeeding years will be $\left(n - 2\frac{1}{2}\right)d_{x+2}$, $\left(n - 3\frac{1}{2}\right)d_{x+3}$, and so on.

Summing these and dividing by l_x , we get as the average amount paid in respect of each pensioner:

$$\left\{ \left(n - \frac{1}{2}\right)d_x + \left(n - 1\frac{1}{2}\right)d_{x+1} + \left(n - 2\frac{1}{2}\right)d_{x+2} + \dots + \frac{1}{2}d_{x+n-1} \right\} \div l_x$$

Now, if

$$d_x = d_{x+1} = d_{x+2} = \dots = d_{x+n-1}$$

the above reduces to

$$\left(\frac{n \cdot n + 1}{2} - \frac{n}{2} \right) \frac{d_x}{l_x} \text{ or } \frac{n^2}{2} \cdot \frac{d_x}{l_x}$$

If the ratio of the contributions to the pension be $n+k$, where k is a fraction, the payments will be

$$\begin{aligned} & \left\{ \left(n + k - \frac{1}{2}\right)d_x + \left(n + k - 1\frac{1}{2}\right)d_{x+1} + \dots \right. \\ & \quad \left. + \left(k + \frac{1}{2}\right)d_{x+n-1} + \frac{k^2}{2} \cdot d_{x+n} \right\} \div l_x. \end{aligned}$$

The correctness of the last term is evident when it is remembered that there will be a balance payable to the representatives of those who die between ages $x+n$ and $x+n+k$. The number dying in this fraction of the year will be kd_{x+n} , and they will, on the average, live for half this time, so that the balance payable to each will be $\frac{k}{2}$ or $\frac{k^2}{2} \cdot d_{x+n}$ in all. If, again, we

assume that $d_x = d_{x+1} = \dots = d_{x+n}$, the above expression becomes

$$\left(\frac{n^2}{2} + nk + \frac{k^2}{2}\right) \frac{d_x}{l_r} = \frac{1}{2}(n+k)^2 \frac{d_x}{l_x}$$

From this it will be seen that the amount to be paid may be expected to vary approximately as the square of the period during which there is a risk of having to make a payment, whether this period be integral or fractional.

The benefit above referred to has been treated as though it were a continuously decreasing temporary assurance. In practice, however, the diminution of the assurance would not be continuous, but would take place at intervals corresponding to the frequency with which the pension was paid. Thus, if the pension were paid monthly and the risk period were n years, the sum assured would be n for the first month, $n - \frac{1}{12}$ for the second month, and so on. As, however, the pension is practically treated as a continuous annuity in the valuation, it follows that any underestimate of liability caused by treating the temporary assurance as continuously decreasing is counterbalanced by an overestimate of the pension payments. To put it in another way, if the pension is payable m times a year, and the pensioner dies $\frac{t}{m}$ of a year (where t is a proper fraction) after receiving r payments of pension, the amount payable to his representatives will be $\left(n - \frac{r}{m}\right)$. Now, the continuous annuity provides for pension payments to the moment of death, and as only $\frac{r}{m}$ will have been paid instead of $\frac{r+t}{m}$ as provided for by the annuity, there will be $\frac{t}{m}$ of the amount provided for by the annuity unused, which, added to the $n - \frac{r+t}{m}$ provided by the annuity, will exactly provide the amount to be paid. It is safe, therefore, to value the benefit as a continuously decreasing temporary assurance.

It has been shown that the sum payable may be expected to vary approximately as the square of the period of risk, and it will be interesting to see whether this is the case with regard to the present value of the benefit. If the period of risk be n years,

and the deaths each year equally distributed over that year, the present value of the assurance will be

$$\frac{d_x \int_0^1 (n-t)v^t dt + rd_{x+1} \int_0^1 (n-1-t)v^t dt + \dots + v^{n-1} d_{x+n-1} \int_0^1 (1-t)v^t dt}{l_x}.$$

Now,

$$\int_0^1 tv^t dt = \frac{\bar{a}_1 - v}{\delta}, \text{ and } \int_0^1 r^t dt = \bar{a}_{\bar{1}}$$

The above expression, therefore,

$$= \left\{ d_x \left(n\bar{a}_{\bar{1}} - \frac{\bar{a}_{\bar{1}} - v}{\delta} \right) + rd_{x+1} \left((n-1)\bar{a}_{\bar{1}} - \frac{\bar{a}_{\bar{1}} - v}{\delta} \right) + \dots \right. \\ \left. \dots + v^{n-1} d_{x+n-1} \left(\bar{a}_{\bar{1}} - \frac{\bar{a}_{\bar{1}} - v}{\delta} \right) \right\} \div l_x$$

The average amount payable to those dying between ages $x+r$ and $x+r+1$ is, as shown above, $\left(n-r-\frac{1}{2}\right)$, and the true equated time of payment after the commencement of the $(r+1)$ th year may be found from the equation

$$\left(n-r-\frac{1}{2}\right)v^e = (n-r)\bar{a}_{\bar{1}} - \frac{\bar{a}_{\bar{1}} - v}{\delta},$$

where e is the equated time. Upon consideration it will be seen that $e < \frac{1}{2}$ and that its value increases with $n-r$. At 4 per-cent, $e=32$ when $n-r=1$, and 48 when $n-r=5$. It will be sufficiently accurate to assume that $e=\frac{1}{2}$, and then the value of the decreasing assurance for n years may be taken as

$$\frac{v^{\frac{1}{2}} d_x \left(n-\frac{1}{2}\right) + v^{1\frac{1}{2}} d_{x+1} \left(n-1\frac{1}{2}\right) + \dots + v^{n-\frac{1}{2}} d_{x+n-1}}{l_x}.$$

Multiplying numerator and denominator by v^x , this becomes

$$\frac{v^{x+\frac{1}{2}} \left(n-\frac{1}{2}\right) d_x + v^{x+1\frac{1}{2}} \left(n-1\frac{1}{2}\right) d_{x+1} + \dots + v^{x+n-\frac{1}{2}} \frac{1}{2} d_{x+n-1}}{v^x l_x},$$

which we may call $(D\bar{A})_x^{(n)}$.

In considering the extent to which we are justified in making the fundamental assumption that the value of the decreasing assurance $(D\bar{A})_x^{(n)}$ varies as the square of n , the

following figures taken from a table of $(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(n)}$ calculated from Mr. Manly's special table of mortality after retirement with interest at 4 per-cent will be interesting. The full table, together with one at 3 per-cent is appended to this paper. I may remark that in constructing the tables the formula given above was used, $d_{x+\frac{1}{2}}$ being taken as equal to $\frac{1}{2}(d_x + d_{x+1})$. The results were adjusted by inspection with a view to graduating them roughly.

$$(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(n)}.$$

x	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
30	.048	.186	.400	.680	1.017	1.401	1.826	2.288	2.779
40	.041	.159	.342	.583	.874	1.207	1.580	1.984	2.418
50	.034	.130	.283	.483	.726	1.007	1.320	1.664	2.032
60	.026	.102	.222	.382	.575	.801	1.051	1.335	1.646

The table shows that although the value of $(\bar{D}\bar{A})_x^{(n)}$ does not vary exactly as the square of n , it does so approximately within a fair range. For instance, $(\bar{D}\bar{A})_{40\frac{1}{2}}^{(4)}$ would = $4(\bar{D}\bar{A})_{40\frac{1}{2}}^{(2)}$ if the law held, but $4(\bar{D}\bar{A})_{40\frac{1}{2}}^{(2)} = .636$, whereas the true value of $(\bar{D}\bar{A})_{40\frac{1}{2}}^{(4)}$ is .583.

Some further comparisons are :

$$4(\bar{D}\bar{A})_{40\frac{1}{2}}^{(3)} = 1.368 = \text{approximate value of } (\bar{D}\bar{A})_{40\frac{1}{2}}^{(6)}, \text{ the true value being } 1.207$$

$$\frac{4}{5}(\bar{D}\bar{A})_{50\frac{1}{2}}^{(5)} = 1.859 = \text{,,} \quad (\bar{D}\bar{A})_{50\frac{1}{2}}^{(5)} = \text{,,} \quad 1.664$$

$$\frac{6}{6}(\bar{D}\bar{A})_{50\frac{1}{2}}^{(6)} = .448 = \text{,,} \quad (\bar{D}\bar{A})_{50\frac{1}{2}}^{(4)} = \text{,,} \quad .483$$

$$\frac{9}{5}(\bar{D}\bar{A})_{60\frac{1}{2}}^{(5)} = .207 = \text{,,} \quad (\bar{D}\bar{A})_{60\frac{1}{2}}^{(3)} = \text{,,} \quad .222$$

Bearing in mind that these comparisons are somewhat extreme, the differences are not very large, and it is easy to tell their direction. If working from one value of n to a higher one, the result is too great, and if working to a lower one, the result is too small.

It will be seen that, in calculating a value of $(\bar{D}\bar{A})^{(n')}$ which is assumed = $\frac{n'^2}{n^2} \cdot (\bar{D}\bar{A})^{(n)}$, it is advisable to minimise the resulting error by working from a value of n as near as possible to n' ; and,

further that n should, if possible, be slightly smaller than n' , so that the resulting error may be on the safe side.

For the sake of simplifying the formulas $(D\bar{A})^{(1)}$ is the value taken as the basis, but, to ensure greater accuracy, it will be wiser to use the approximate value $\frac{(D\bar{A})^{(n)}}{n^2}$ for $(D\bar{A})^{(1)}$, where n is as near as possible to the term for which the value is to be calculated. By so doing the result obtained is, of course, the same as that obtained by using $(D\bar{A})^{(n')}$, as $(DA)^{(n')} = (D\bar{A})^{(n)} \times \frac{n'^2}{n^2}$, which is identical with $\frac{(D\bar{A})^{(n)}}{n^2} \times n'^2$.

We may now deduce approximate values of the benefit in four different cases.

(A) Pension based on average salary and no interest allowed on contributions.

Let the rate of contribution be c per unit of salary, and the pension be $\psi \times (\text{total salary})$. Assume that each year's contribution insures payment (as pension) of ψ times that year's salary, and also the return of the corresponding balance (if any) on death after entering upon pension.

Of r_x persons joining the fund at age x , r_x will retire between ages x and $x+1$, and will each receive a pension of $\frac{1}{2}\psi s_x$, assuming that retirement takes place in the middle of the year and that each member has contributed $\frac{1}{2}cs_x$. The value at retirement of the return of the balance (if any) to each of the r_x persons will be for each unit of pension :

$$(D\bar{A})_{x+1}^{(1)} \times \left(\begin{array}{l} \text{Number of years during} \\ \text{which a balance is payable} \end{array} \right)^2, \text{i.e., } (D\bar{A})_{x+1}^{(1)} \times \left(\frac{\frac{1}{2}c \cdot s_x}{\frac{1}{2}\psi s_x} \right)^2$$

and the total value of these payments at date of joining the fund will be :

$$\begin{aligned} & r_x^2 r_x \left(\frac{1}{2} \psi s_x \right) \left(D\bar{A} \right)_{x+1}^{(1)} \times \left(\frac{\frac{1}{2}c s_x}{\frac{1}{2}\psi s_x} \right)^2 \\ &= \frac{1}{2} v^{\frac{1}{2}} r_x (D\bar{A})_{x+1}^{(1)} s_x \cdot \frac{c^2}{\psi} \\ &= \left(\frac{1}{2} v^{x+\frac{1}{2}} r_x (D\bar{A})_{x+1}^{(1)} \cdot \frac{s_x c^2}{\psi} \right) \div v^x \end{aligned}$$

The value of the benefit, in respect of the first year's contributions, to those who retire between ages $x+1$ and $x+2$, will be

$$\begin{aligned} & v^{1\frac{1}{2}} r_{x+1} \psi s_x (\bar{D}\bar{A})_{x+1\frac{1}{2}}^{(1)} \left(\frac{c s_x}{\psi s_x} \right)^2 \\ &= v^{1\frac{1}{2}} r_{x+1} (\bar{D}\bar{A})_{x+1\frac{1}{2}}^{(1)} \cdot \frac{s_x c^2}{\psi} \\ &= \left\{ v^{x+1\frac{1}{2}} r_{x+1} (\bar{D}\bar{A})_{x+1\frac{1}{2}}^{(1)} \cdot \frac{s_x c^2}{\psi} \right\} \div v^x \end{aligned}$$

and generally for those who retire between ages $x+t$ and $x+t+1$, the value will be

$$\begin{aligned} & v^{t+\frac{1}{2}} r_{x+t} \psi s_x (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \left(\frac{c s_x}{\psi s_x} \right)^2 \\ &= v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} s_x \cdot \frac{c^2}{\psi} \\ &= \left\{ v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{s_x c^2}{\psi} \right\} \div v^x \end{aligned}$$

The total value of the return of the balancees of the first year's contributions will, therefore, be

$$\left\{ \Sigma v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} - \frac{1}{2} v^{x+\frac{1}{2}} r_x (\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \right\} \frac{s_x c^2}{\psi v^x},$$

where the variable, t , is given every possible value from 0 upwards.

A table of $v^{x+\frac{1}{2}} r_x (\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)}$ can now be formed and called $r^{(\bar{D}\bar{A})} C_x$; this can be summed as the ordinary C_x column is summed, and called $r^{(\bar{D}\bar{A})} M_x$. From each value of $r^{(\bar{D}\bar{A})} M_x$ the corresponding value of $\frac{1}{2} r^{(\bar{D}\bar{A})} C_x$ may be subtracted, giving a table which may be called $r^{(\bar{D}\bar{A})} \bar{M}_x$. Each value of $r^{(\bar{D}\bar{A})} \bar{M}_x$ must be multiplied into s_x , giving a product which may be called $r^{(\bar{D}\bar{A}) s} \bar{M}_x$. It will be seen that $r^{(\bar{D}\bar{A})} \bar{M}_x \times \frac{c^2}{\psi v^x}$ represents the value of the return of the balancee of the first year's contributions made by all of the l_x persons joining at age x who die after being pensioned and before the payments of the pension insured by the first year's contribution amount to that year's contribution.

Similarly, it may be shown that the value of the return of the balancee of the second year's contributions made by the l_{x+1} persons surviving out of the l_x who join is $r^{(\bar{D}\bar{A}) s} \bar{M}_{x+1} \times \frac{c^2}{\psi v^x}$; and generally, that the value in respect of the $(t+1)$ th year's contributions

made by the l_{x+t} persons surviving out of the l_x joining is
 $r^r D\bar{A} \cdot \bar{M}_{x+t} \times \frac{c^2}{\psi v^x}$.

The column of $r^r D\bar{A} \cdot \bar{M}_x$ may be summed, giving a column of $r^r D\bar{A} \cdot \bar{R}_x$, and the total value of the benefit will be represented by
 $r^r D\bar{A} \cdot \bar{R}_x \times \frac{c^2}{\psi v^x}$. As each of the l_x persons receives a salary of s_x , the value of the benefit for each unit of present salary will be

$$\begin{aligned} & \frac{r^r D\bar{A} \cdot \bar{R}_x \times \frac{c^2}{\psi v^x}}{l_x s_x} \\ &= \frac{c^2}{\psi} \times \frac{r^r D\bar{A} \cdot \bar{R}_x}{s D_x}, \end{aligned}$$

where $s D_x = v^r l_x s_x$.

If no pension is granted before completing n years' service, the summation will commence at age $x+n$, and the value of the benefit in respect of the first n years' contributions will be

$$\begin{aligned} & v^{n+\frac{1}{2}} r_{x+n} \psi(s_x + s_{x+1} + \dots + s_{x+n-1}) (D\bar{A})_{x+n+\frac{1}{2}}^1 \left(\frac{c(s_x + \dots + s_{x+n-1})}{\psi(s_x + \dots + s_{x+n-1})} \right)^2 \\ & + v^{n+1\frac{1}{2}} r_{x+n+1} \psi(s_x + s_{x+1} + \dots + s_{x+n-1}) (D\bar{A})_{x+n+1\frac{1}{2}}^1 \left(\frac{c(s_x + \dots + s_{x+n-1})}{\psi(s_x + \dots + s_{x+n-1})} \right)^2 \\ & + \dots \\ & + v^{n+t+\frac{1}{2}} r_{x+n+t} \psi(s_x + s_{x+1} + \dots + s_{x+n-1}) (D\bar{A})_{x+n+t+\frac{1}{2}}^1 \left(\frac{c(s_x + \dots + s_{x+n-1})}{\psi(s_x + \dots + s_{x+n-1})} \right)^2 \\ & + \dots \\ & = (\sum v^{n+t+\frac{1}{2}} r_{x+n+t} (D\bar{A})_{x+n+t+\frac{1}{2}}^1) (s_x + s_{x+1} + \dots + s_{x+n-1}) \cdot \frac{c^2}{\psi} \\ & = (\sum v^{x+n+t+\frac{1}{2}} r_{x+n+t} (D\bar{A})_{x+n+t+\frac{1}{2}}^1) (\Sigma s_x - \Sigma s_{x+n}) \cdot \frac{c^2}{\psi v^x} \\ & = \frac{c^2}{\psi v^x} (\Sigma s_x - \Sigma s_{x+n}) r^r D\bar{A} M_{x+n}. \end{aligned}$$

The value of the benefit in respect of contributions made after n years by the l_{x+n} survivors of the l_x persons joining is
 $r^r D\bar{A} \cdot \bar{R}_{x+n} \cdot \frac{c^2}{\psi v^x}$. Adding together these two portions of the benefit, and dividing by $l_x s_x$, the value for each unit of present salary is

$$\frac{c^2}{\psi} \times \frac{(\Sigma s_x - \Sigma s_{x+n}) r^r D\bar{A} M_{x+n} + r^r D\bar{A} \cdot \bar{R}_{x+n}}{s D_x}$$

(b) *Pension based on average salary and compound interest to date of retirement allowed on contributions.*

If compound interest is calculated on the contributions to date of retirement, the foregoing formulas must be suitably modified. Again assume that each yearly contribution insures part of the pension, so that if the proportion of total salary paid as pension is ψ , the payment of the contribution of cs_x between ages x and $x+1$ insures the payment of a pension of ψs_x , and also of the excess of cs_x , accumulated at compound interest, over the payments of the pension of ψs_x on death (after retiring upon pension) before the latter amount to the former.

In any year of age the equated date of payment of the contributions will vary according to the frequency with which they are paid. If they are paid weekly, the equated time, allowing for interest at 4 per-cent, will be .51 of a year after the beginning of the year; if monthly, .54; and if quarterly, .62.

In calculating interest to the date of retirement, the interest should, strictly speaking, be reckoned from the actual date of payment, or, what is the same thing, from the equated date. It will involve but a small error and, moreover, be on the safe side if they are assumed to be paid in the middle of the year. It is assumed that retirements take place in the middle of the year.

The contribution paid in the year of age x to $x+1$, that is cs_x , will, if the rate of interest allowed be j (the rate of interest at which the fund is assumed to accumulate being i), amount to $cs_x(1+j)^t$ on retirement at age $x+t+\frac{1}{2}$, and the period during which there will be a risk of having to pay a balance is $\frac{cs_x(1+j)^t}{\psi s_x}$; the value of this risk at date of retirement will therefore be $(D\bar{A})_{x+t+\frac{1}{2}}^{(1)} \left\{ \frac{cs_x(1+j)^t}{\psi s_x} \right\}^2$ for every unit of pension.

Of l_x persons joining at age x , r_x will retire between ages x and $x+1$, having each paid half a year's contribution, and receiving in respect thereof a pension of $\frac{1}{2} \psi s_x$. The value of the return to these r_x persons is

$$r_x^{\frac{1}{2}} r_x \left(\frac{1}{2} \psi s_x \right) (D\bar{A})_{x+\frac{1}{2}}^{(1)} \left\{ \frac{\frac{1}{2} cs_x}{\frac{1}{2} \psi s_x} \right\}^2$$

which may be written

$$\frac{1}{2} v^{\frac{1}{2}} r_x (\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \cdot \frac{s_x c^2}{\psi}$$

Of the l_x persons joining at age x , r_{x+t} will retire between ages $x+t$ and $x+t+1$, and will each receive in respect of his first year's contribution a pension of ψs_x . The value of the return to these r_{x+t} persons is (t being not less than unity),

$$v^{t+\frac{1}{2}} r_{x+t} \psi s_x (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \left(\frac{c s_x (1+j)^t}{\psi s_x} \right)^2$$

or, $v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} s_x \cdot \frac{c^2 (1+j)^{2t}}{\psi}$

Multiply and divide this expression by $v^x (1+j)^{2x}$ and the result is

$$\left\{ v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} s_x \cdot \frac{c^2 (1+j)^{2(x+t)}}{\psi} \right\} \div v^x (1+j)^{2x},$$

for those retiring in the $(t+1)$ th year. Giving t all possible values and remembering that those who retire between ages x and $x+1$, will on the average have made only half a year's contributions, the total value of the return in respect of the contributions made between ages x and $x+1$ by all the l_x persons will be

$$\begin{aligned} & \left[\frac{1}{2} v^{x+\frac{1}{2}} r_x (\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} (1+j)^{2x} + v^{x+1+\frac{1}{2}} r_{x+1} (\bar{D}\bar{A})_{x+1+\frac{1}{2}}^{(1)} (1+j)^{2(x+1)} + \dots \right. \\ & \quad \left. + v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} (1+j)^{2(x+t)} + \dots \right] \frac{s_x c^2}{v^x (1+j)^{2x} \psi} \\ & = \left[\Sigma \left\{ v^{x+\frac{1}{2}} r_x (\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} (1+j)^{2x} \right\} \right. \\ & \quad \left. - \frac{1}{2} v^{x+\frac{1}{2}} r_x (\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} (1+j)^{2x} \right] \frac{s_x c^2}{v^x (1+j)^{2x} \psi} \end{aligned}$$

A table of $v^{x+\frac{1}{2}} r_x (\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} (1+j)^{2x}$ can now be prepared, and the function may be called $\text{v}^{\frac{1}{2}} \text{D}\bar{A} C_x$. If this table is summed as the ordinary C_x table is summed, the resulting function may be called $\text{v}^{\frac{1}{2}} \text{D}\bar{A} M_x$. Subtract the corresponding value of $\frac{1}{2} \text{v}^{\frac{1}{2}} \text{D}\bar{A} C_x$ from each value of $\text{v}^{\frac{1}{2}} \text{D}\bar{A} M_x$, and the remainder, $\frac{1}{2} \text{v}^{\frac{1}{2}} \text{D}\bar{A} \bar{M}_x$, will be the value of the expression in square brackets. Multiply each value of this last function into the proper value of $\frac{s_x}{(1+j)^{2x}}$,

calling the product ${}^{rj} \text{D}\bar{A} {}^s \bar{M}_x$. Then ${}^{rj} \text{D}\bar{A} {}^s \bar{M}_x \times \frac{c^2}{v^x \psi}$ is the value of the payments to be made in respect of the contributions made between ages x and $x+1$. Similarly, it may be shown that the value of the payments in respect of the contributions made between ages $x+1$ and $x+2$ by the l_{x+1} persons who reach age $x+1$, is ${}^{rj} \text{D}\bar{A} {}^s \bar{M}_{x+1} \times \frac{c^2}{v^{x+1} \psi}$, and the value in respect of the contributions made by the survivors between ages $x+t$ and $x+t+1$ is ${}^{rj} \text{D}\bar{A} {}^s \bar{M}_{x+t} \times \frac{c^2}{v^{x+t} \psi}$.

Summing the respective values, the total value of the benefit in respect of all the contributions made by the l_x persons joining is $\sum {}^{rj} \text{D}\bar{A} {}^s \bar{M}_{x+t} \times \frac{c^2}{v^{x+t} \psi}$. If the table of ${}^{rj} \text{D}\bar{A} {}^s \bar{M}_x$ be summed in the same manner as the ordinary M_x column, and the resulting function called ${}^{rj} \text{D}\bar{A} {}^s \bar{R}_x$, the total value of the benefit may be represented by ${}^{rj} \text{D}\bar{A} {}^s \bar{R}_x \times \frac{c^2}{v^x \psi}$.

To find the value for each unit of present salary, the last result must be divided by $l_x s_x$, giving as a result

$$\frac{{}^{rj} \text{D}\bar{A} {}^s \bar{R}_x \times \frac{c^2}{\psi}}{v^x l_x s_x}$$

$$= \frac{c^2}{\psi} \times \frac{{}^{rj} \text{D}\bar{A} {}^s \bar{R}_x}{{}^s D_x}$$

If no pension is granted before completing n years' service, the summation must commence at age $x+n$.

In respect of the first year's contributions the value will be

$$\begin{aligned} & \{ v^{x+n+\frac{1}{2}} r_{x+n} (\text{D}\bar{A})_{x+n+\frac{1}{2}}^1 (1+j)^{2(x+n)} + v^{x+n+1\frac{1}{2}} r_{x+n+1} (\text{D}\bar{A})_{x+n+1\frac{1}{2}}^1 (1+j)^{2(x+n+1)} \\ & \quad + \dots \} \frac{s_x c^2}{v^x (1+j)^{2x} \psi} \\ & = \{ {}^{rj} \text{D}\bar{A} C_{x+n} + {}^{rj} \text{D}\bar{A} C_{x+n+1} + \dots \} \frac{s_x c^2}{v^x (1+j)^{2x} \psi} \\ & = {}^{rj} \text{D}\bar{A} M_{x+n} \cdot \left(\frac{s_x}{(1+j)^{2x}} \times \frac{c^2}{v^x \psi} \right). \end{aligned}$$

Similarly, in respect of the second year's contributions the value will be

$${}^{rj} \bar{D}\bar{A} M_{x+n} \cdot \frac{s_{x+1}}{(1+j)^{2(x+1)}} \times \frac{c^2}{v^x \psi},$$

and so on. The total value in respect of the first n year's contributions will therefore be

$$\begin{aligned} & {}^{rj} \bar{D}\bar{A} M_{x+n} (v'^{2x} s_x + v'^{2x+1} s_{x+1} + \dots + v'^{2x+n-1} s_{x+n-1}) \frac{c^2}{v^x \psi} \\ & = {}^{rj} \bar{D}\bar{A} M_{x+n} \{ \sum v'^{2x} s_x - \sum v'^{2x+n} s_{x+n} \} \frac{c^2}{v^x \psi}, \end{aligned}$$

where v' is written for $\frac{1}{(1+j)}$.

To this must be added the value of the return of the balances of contributions made after age $x+n$, namely, $\frac{c^2}{v^x \psi} \times {}^{rj} \bar{D}\bar{A} s \bar{R}_{x+n}$.

The sum of these two must now be divided by $l_x s_x$ to give the value of the return for each unit of present salary, and so we obtain the expression,

$$\frac{c^2}{\psi} \times \frac{\{ \sum v'^{2x} s_x - \sum v'^{2x+n} s_{x+n} \}}{s D_x} + {}^{rj} \bar{D}\bar{A} M_{x+n} + {}^{rj} \bar{D}\bar{A} s \bar{R}_{x+n}$$

Where compound interest is calculated on the contributions, the assumption that each year's contribution purchases a separate benefit introduces a small error. If retirement takes place at age $x+t+\frac{1}{2}$, the value of the benefit to all the persons retiring then is, on this assumption,

$$\begin{aligned} & v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}} \left(\psi^{s_x} \left[\frac{cs_x(1+j)^t}{\psi^{s_x}} \right]^2 + \psi^{s_{x+1}} \left[\frac{cs_{x+1}(1+j)^{t-1}}{\psi^{s_{x+1}}} \right]^2 \right. \\ & \quad \left. + \dots + \frac{1}{2} \psi^{s_{x+t}} \left[\frac{\frac{1}{2} cs_{x+t}}{\frac{1}{2} \psi^{s_{x+t}}} \right]^2 \right) \\ & = v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}} \left\{ s_x \cdot \frac{c^2 (1+j)^{2t}}{\psi} + s_{x+1} \cdot \frac{c^2 (1+j)^{2t-1}}{\psi} + \dots + \frac{1}{2} s_{x+t} \cdot \frac{c^2}{\psi} \right\}. \end{aligned}$$

The true value would be obtained by applying to the total amount of pension granted to each individual, namely,

$\psi(s_x + s_{x+1} + \dots + \frac{1}{2}s_{x+t})$, the value of the decreasing assurance for the time during which there will be a liability to make a payment. This time will be

$$\frac{c \left[s_x(1+j)^t + s_{x+1}(1+j)^{t-1} + \dots + s_{x+t-1}(1+j) + \frac{1}{2}s_{x+t} \right]}{\psi(s_x + s_{x+1} + \dots + s_{x+t-1} + \frac{1}{2}s_{x+t})}$$

This expression must be squared and multiplied into $(\bar{DA})_{x+t+\frac{1}{2}}^{(1)}$. The true value of the total benefit payable to persons retiring at age $x+t+\frac{1}{2}$ is, therefore,

$$v^{t+\frac{1}{2}} r_{x+t} \psi(s_x + s_{x+1} + \dots + s_{x+t-1} + \frac{1}{2}s_{x+t}) (\bar{DA})_{x+t+\frac{1}{2}}^{(1)}$$

$$\times \left\{ \frac{c \left[s_x(1+j)^t + s_{x+1}(1+j)^{t-1} + \dots + s_{x+t-1}(1+j) + \frac{1}{2}s_{x+t} \right]^2}{\psi(s_x + s_{x+1} + \dots + s_{x+t-1} + \frac{1}{2}s_{x+t})} \right\}$$

$$+ \frac{c^2 \left[s_x(1+j)^t + s_{x+1}(1+j)^{t-1} + \dots + s_{x+t-1}(1+j) + \frac{1}{2}s_{x+t} \right]^2}{\psi(s_x + s_{x+1} + \dots + s_{x+t-1} + \frac{1}{2}s_{x+t})} (B)$$

The comparative degree of error can be ascertained by taking the ratio of (A) to (B).

(A) may be written in the form :

$$v^{t+\frac{1}{2}} r_{x+t} (\bar{DA})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{c^2}{\psi} (1+j)^{2(x+t)} \left\{ v'^{2x} s_x + v'^{2x+1} s_{x+1} + \dots + v'^{2x+t-1} s_{x+t-1} + \frac{1}{2} v'^{2x+t} s_{x+t} \right\}$$

$$= v^{t+\frac{1}{2}} r_{x+t} (\bar{DA})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{c^2}{\psi} (1+j)^{2(x+t)} \left\{ \Sigma v'^{2x} s_x - \Sigma v'^{2x+t} s_{x+t} + \frac{1}{2} v'^{2x+t} s_{x+t} \right\}$$

(B) may be written :

$$r_{x+t} (\bar{DA})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{c^2}{\psi} (1+j)^{2(x+t)} \frac{\left\{ v'^x s_x + v'^{x+1} s_{x+1} + \dots + v'^{x+t-1} s_{x+t-1} + \frac{1}{2} v'^{x+t} s_{x+t} \right\}^2}{s_x + s_{x+1} + \dots + s_{x+t-1} + \frac{1}{2} s_{x+t}}$$

$$= v^{t+\frac{1}{2}} r_{x+t} (\bar{DA})_{x+t+\frac{1}{2}}^{(1)} \frac{c^2}{\psi} (1+j)^{2(x+t)} \cdot \frac{\left\{ \Sigma v'^x s_x - \Sigma v'^{x+t} s_{x+t} + \frac{1}{2} v'^{x+t} s_{x+t} \right\}^2}{\left(\Sigma s_x - \Sigma s_{x+t} + \frac{1}{2} s_{x+t} \right)}$$

Finally, the ratio reduces to

$$\frac{\left(\Sigma v'^{2x} s_x - \Sigma v'^{2(x+t)} s_{x+t} + \frac{1}{2} v'^{2(x+t)} s_{x+t} \right) \left(\Sigma s_x - \Sigma s_{x+t} + \frac{1}{2} s_{x+t} \right)}{\left(\Sigma v'^x s_x - \Sigma v'^{x+t} s_{x+t} + \frac{1}{2} v'^{x+t} s_{x+t} \right)^2}$$

If tables of $v'^{2x} s_x$, $v'^x s_x$ and s_x are prepared, with their summations, the value of this ratio can be easily calculated for different combinations of x and t .

The following table, based on Mr. Manly's hypothetical salary scale, shows the value of this ratio for certain combinations of x and t , x being the age at entry, and t , the duration :

x	t			
	10	20	30	40
25	1.015	1.054	1.120	1.211
35	1.014	1.054	1.117	...
45	1.015	1.052
55	1.013

(c) *Pension based on final salary, no interest being allowed on contributions.*

The next case to be considered is that in which the pension is a proportion of the final salary corresponding to the number of years' service, i.e., where, after n years' service, the pension is equal to $n\psi$ of the final salary, ψ being, of course, the proportion of final salary allowed for each year of service.

Again, assume that each year's contribution purchases its own proportion of the pension (in this case, ψ of final salary) and, further, that only completed years count for pension.

As regards the first year's contributions, of r_x persons joining at age x , r_x will retire between ages x and $x+1$, but will not receive a pension, as they will not have served for the whole of it. The r_{x+1} who retire in the following year will each receive a pension of ψs_{x+1} , and the time during which there will be a risk of having to return a balance will be

$\frac{cs_x}{\psi s_{x+1}}$. The value of the payments to the representatives of the r_{x+1} persons will be

$$v^{1\frac{1}{2}} r_{x+1} \psi s_{x+1} (\bar{D}\bar{\Lambda})_{x+1}^{(1)} \left(\frac{cs_x}{\psi s_{x+1}} \right)^2$$

which may be put in the form,

$$v^{1\frac{1}{2}}r_{x+1}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \cdot \frac{c^2 s_x^2}{\psi s_{x+1}}.$$

The value of the payments in respect of the r_{x+2} persons retiring in the following year will be

$$v^{2\frac{1}{2}}r_{x+2}\psi s_{x+2}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \left(\frac{cs_x}{\psi s_{x+2}} \right)^2 \text{ or, } v^{2\frac{1}{2}}r_{x+2}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \cdot \frac{c^2 s_x^2}{\psi s_{x+2}}$$

and, generally, in respect of the r_{x+t} persons retiring in the $(t+1)$ th year,

$$v^{t+\frac{1}{2}}r_{x+t}\psi s_{x+t}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \left(\frac{cs_x}{\psi s_{x+t}} \right)^2 \text{ or, } v^{t+\frac{1}{2}}r_{x+t}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \cdot \frac{c^2 s_x^2}{\psi s_{x+t}}.$$

Multiplying and dividing each of these expressions by v^x , and taking the sum, the following expression is obtained :

$$\begin{aligned} & \left\{ v^{x+1\frac{1}{2}}r_{x+1}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \times \frac{1}{s_{x+1}} + v^{x+2\frac{1}{2}}r_{x+2}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \times \frac{1}{s_{x+2}} + \dots \right. \\ & \quad \left. + v^{x+t+\frac{1}{2}}r_{x+t}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \times \frac{1}{s_{x+t}} \dots \right\} \frac{c^2 s_x^2}{\psi v^x} \\ & = \left\{ \sum v^{x+1\frac{1}{2}}r_{x+1}(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \times \frac{1}{s_{x+1}} \right\} \frac{c^2 s_x^2}{\psi v^x}, \end{aligned}$$

which becomes

$$(\Sigma^{r(\bar{D}\bar{A})/ls} C_{x+1}) \frac{c^2 s_x^2}{\psi v^x} \text{ on writing } {}^{r(\bar{D}\bar{A})/ls} C_x \text{ for } v^{x+\frac{1}{2}}r_x(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \times \frac{1}{s_x}.$$

Let $\Sigma^{r(\bar{D}\bar{A})/ls} C_x = {}^{r(\bar{D}\bar{A})/ls} M_x$, and the value of the return in respect of the first year's contribution is represented by the expression

$${}^{r(\bar{D}\bar{A})/ls} M_{x+1} \cdot \frac{c^2 s_x^2}{\psi v^x}.$$

Similarly, subject to the correction to be explained presently, the value in respect of the second year's contributions will be ${}^{r(\bar{D}\bar{A})/ls} M_{x+2} \cdot \frac{c^2 s_{x+1}^2}{\psi v^x}$, and, generally, the value in respect of the $(t+1)$ th year's contributions will be ${}^{r(\bar{D}\bar{A})/ls} M_{x+t+1} \cdot \frac{c^2 s_{x+t}^2}{\psi v^x}$.

But this does not allow for the return in respect of the half-year's contributions assumed to be paid in the year of retirement. To allow for this, approximately, there may be added

$\frac{1}{2} {}_{r(\bar{D}\bar{A})ls} C_x \cdot \frac{c^2 s_x^2}{\psi v^x}$ for the first year; $\frac{1}{2} {}_{r(\bar{D}\bar{A})ls} C_{x+1} \cdot \frac{c^2 s_{x+1}^2}{\psi v^x}$ for the second year, and so on.

The full expression will then be :

First year's contribution :

$${}_{r(\bar{D}\bar{A})ls} M_{x+1} \cdot \frac{c^2 s_x^2}{\psi v^x} + \frac{1}{2} {}_{r(\bar{D}\bar{A})ls} C_x \cdot \frac{c^2 s_x^2}{\psi v^x} = \left({}_{r(\bar{D}\bar{A})ls} M_x - \frac{1}{2} {}_{r(\bar{D}\bar{A})ls} C_x \right) \frac{c^2 s_x^2}{\psi v^x}$$

Second year's contribution :

$${}_{r(\bar{D}\bar{A})ls} M_{x+2} \cdot \frac{c^2 s_{x+1}^2}{\psi v^x} + \frac{1}{2} {}_{r(\bar{D}\bar{A})ls} C_{x+1} \cdot \frac{c^2 s_{x+1}^2}{\psi v^x} = \left({}_{r(\bar{D}\bar{A})ls} M_{x+1} - \frac{1}{2} {}_{r(\bar{D}\bar{A})ls} C_{x+1} \right) \frac{c^2 s_{x+1}^2}{\psi v^x}$$

($t+1$)th year's contribution :

$${}_{r(\bar{D}\bar{A})ls} M_{x+t+1} \cdot \frac{c^2 s_{x+t}^2}{\psi v^x} + \frac{1}{2} {}_{r(\bar{D}\bar{A})ls} C_{x+t} \cdot \frac{c^2 s_{x+t}^2}{\psi v^x} = \left({}_{r(\bar{D}\bar{A})ls} M_{x+t} - \frac{1}{2} {}_{r(\bar{D}\bar{A})ls} C_{x+t} \right) \frac{c^2 s_{x+t}^2}{\psi v^x}$$

The process of working will be seen to be as follows :

Construct a table of ${}_{r(\bar{D}\bar{A})ls} C_x$, and sum it, obtaining a table of ${}_{r(\bar{D}\bar{A})ls} M_x$. From each value of ${}_{r(\bar{D}\bar{A})ls} M_x$ subtract half the corresponding value of ${}_{r(\bar{D}\bar{A})ls} C_x$, and multiply the difference into s_x^2 , calling the product, say, ${}_{r(\bar{D}\bar{A})ls} \bar{R}_x^s$. Sum this last table and call the result ${}_{r(\bar{D}\bar{A})ls} \bar{R}_x^s$. Then the value of the return in respect of all the contributions will be $\frac{c^2}{\psi v^x} \cdot {}_{r(\bar{D}\bar{A})ls} \bar{R}_x^s$.

To find the value for each unit of present salary this must be divided by $l_x s_x$, giving as the final expression for the benefit,

$$\frac{c^2}{\psi} \cdot \frac{{}_{r(\bar{D}\bar{A})ls} \bar{R}_x^s}{s D_x}$$

If no pension is granted before serving n years, the summation will commence at age $x+n$, and the value of the return in respect of all the contributions made in the first n years, will be :

$$\begin{aligned} {}_{r(\bar{D}\bar{A})ls} M_{x+n} &\left(\frac{c^2 s_x^2}{\psi v^x} + \frac{c^2 s_{x+1}^2}{\psi v^x} + \dots + \frac{c^2 s_{x+n-1}^2}{\psi v^x} \right) \\ &= \frac{c^2}{\psi v^x} (\Sigma s_x^2 - \Sigma s_{x+n}^2) {}_{r(\bar{D}\bar{A})ls} M_{x+n} \end{aligned}$$

The value in respect of the contributions made after n years will be $\frac{c^2}{\psi v^x} \times {}_{r(\bar{D}\bar{A})ls} \bar{R}_{x+n}^s$.

Taking the sum of these two expressions, and dividing by $l_x s_x$, the value for each unit of present salary will be :

$$\frac{c^2}{\psi} \cdot \frac{(\sum s_x^2 - \sum s_{x+n}^2) \cdot {}^{r(D\bar{A})ls} M_{x+n} + {}^{r(D\bar{A})ls} \bar{R}_{x+n}^s}{{}^s D_x}$$

The result of assuming that each year's contribution purchases a separate pension and return of balance (if any) on death after entering upon pension is to introduce an error. If retirement takes place at age $x+t+\frac{1}{2}$, the value, on this assumption, of the benefit to all persons then retiring, is :

$$\begin{aligned} & v^{t+\frac{1}{2}} r_{x+t} (D\bar{A})_{x+t+\frac{1}{2}}^{(1)} \left\{ \psi s_{x+t} \left(\frac{cs_x}{\psi s_{x+t}} \right)^2 + \psi s_{x+t} \left(\frac{cs_{x+1}}{\psi s_{x+t}} \right)^2 \dots \right. \\ & \quad \left. + \psi s_{x+t} \left(\frac{cs_{x+t-1}}{\psi s_{x+t}} \right)^2 + \frac{1}{2} \psi s_{x+t} \left(\frac{cs_{x+t}}{\psi s_{x+t}} \right)^2 \right\} \\ & = v^{t+\frac{1}{2}} r_{x+t} (D\bar{A})_{x+t+\frac{1}{2}}^{(1)} \times \frac{c^2}{\psi s_{x+t}} \left\{ s_x^2 + s_{x+1}^2 + \dots + s_{x+t-1}^2 + \frac{1}{2} s_{x+t}^2 \right\} \\ & = v^{t+\frac{1}{2}} r_{x+t} (D\bar{A})_{x+t+\frac{1}{2}}^{(1)} \times \frac{c^2}{\psi s_{x+t}} \left(\sum s_x^2 - \sum s_{x+t}^2 + \frac{1}{2} s_{x+t}^2 \right) \dots \quad (A) \end{aligned}$$

The true value is :

$$\begin{aligned} & v^{t+\frac{1}{2}} r_{x+t} t \psi s_{x+t} \cdot (D\bar{A})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{\left\{ cs_x + cs_{x+1} \dots + cs_{x+t-1} + \frac{1}{2} cs_{x+t} \right\}^2}{t \psi s_{x+t}} \\ & = v^{t+\frac{1}{2}} r_{x+t} (D\bar{A})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{c^2}{\psi s_{x+t}} \cdot \frac{\left(\sum s_x - \sum s_{x+t} + \frac{1}{2} s_{x+t} \right)^2}{t} \dots \quad (B) \end{aligned}$$

The extent of the error can be ascertained by taking the ratio of (A) to (B), which reduces to

$$\frac{t \left(\sum s_x^2 - \sum s_{x+t}^2 + \frac{1}{2} s_{x+t}^2 \right)}{\left(\sum s_x - \sum s_{x+t} + \frac{1}{2} s_{x+t} \right)^2}$$

The subjoined table gives specimen values of this ratio.

x	t			
	10	20	30	40
25	.970	1.022	1.057	1.097
35	.961	1.000	1.043	...
45	.957	1.015
55	1.003

It will be observed that the values of the ratio are very near to unity on the average, so close indeed, that the error involved is practically negligible.

(d) *Pensions based on final salary and compound interest to date of retirement allowed on contributions.*

The last case is that in which the pension after t years is ψs_x of the final salary, and compound interest at rate j is calculated on the contributions to the date of retirement.

Reasoning as before, we may find the value of the return in respect of the first year's contributions made by l_x persons joining at age x . Of these, r_x will retire in the first year, but will not receive a pension, as they will not have served for a year. The r_{x+1} who retire in the second year will each receive a pension of ψs_{x+1} , and the period during which there will be a risk of having to pay a balance will be $\frac{c(1+j)s_x}{\psi s_{x+1}}$. The value of the payments to the representatives of these r_{x+1} persons will be

$$\begin{aligned} & v^{1\frac{1}{2}} r_{x+1} \psi s_{x+1} (\bar{D}\bar{A})_{x+1\frac{1}{2}}^{(1)} \left\{ \frac{c(1+j)s_x}{\psi s_{x+1}} \right\}^2 \\ &= v^{1\frac{1}{2}} r_{x+1} (\bar{D}\bar{A})_{x+1\frac{1}{2}}^{(1)} \cdot \frac{c^2(1+j)^2 s_x^2}{\psi s_{x+1}} \\ &= \left\{ v^{x+1\frac{1}{2}} r_{x+1} (\bar{D}\bar{A})_{x+1\frac{1}{2}}^{(1)} \cdot \frac{c^2(1+j)^{2(x+1)} s_x^2}{\psi s_{x+1}} \right\} \div v^x (1+j)^{2x} \\ &= \left\{ v^{x+1\frac{1}{2}} r_{x+1} (\bar{D}\bar{A})_{x+1\frac{1}{2}}^{(1)} (1+j)^{2(x+1)} \times \frac{1}{s_{x+1}} \right\} \left(\frac{c^2 s_x^2}{v^x (1+j)^{2x} \psi} \right) \end{aligned}$$

Similarly, it may be shown that the value of the payments to the representatives of those pensioned in the $(t+1)$ th year will be

$$\begin{aligned} & v^{t+\frac{1}{2}} r_{x+t} \psi s_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \left\{ \frac{c(1+j)^t s_x}{\psi s_{x+t}} \right\}^2 \\ &= v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{c^2(1+j)^{2t} s_x^2}{\psi s_{x+t}} \\ &= \left\{ v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} (1+j)^{2(x+t)} \times \frac{1}{s_{x+t}} \right\} \left(\frac{c^2 s_x^2 t'^{2x}}{v^x \psi} \right), \end{aligned}$$

where $v' = \frac{1}{(1+j)}$.

For $v^{x+\frac{1}{2}}r_x(D\bar{A})_{x+\frac{1}{2}}^{(1)}(1+j)^{2x} \times \frac{1}{s_x}$, write $r_j(D\bar{A}, l)C_x$.

The value of the return in respect of the first year's contribution will be (subject to the correction shown hereafter)

$$\begin{aligned} & (r_j(D\bar{A}, l)C_{x+1} + r_j(D\bar{A}, l)C_{x+2} + \dots + r_j(D\bar{A}, l)C_{x+t} + \dots) \frac{c^2 s_x^2 v'^{2x}}{v^x \psi} \\ & = r_j(D\bar{A}, l)M_{x+1} \times \frac{c^2 s_x^2 v'^{2x}}{v^x \psi}. \end{aligned}$$

The value of the return in respect of the second year's contributions may be shown to be

$$r_j(D\bar{A}, l)M_{x+2} \times \frac{c^2 s_{x+1}^2 v'^{2(x+1)}}{v^x \psi}$$

and in respect of the $(t+1)$ th year's contributions,

$$r_j(D\bar{A}, l)M_{x+t+1} \times \frac{c^2 s_{x+t}^2 v'^{2(x+t)}}{v^x \psi}$$

No allowance has been made for contributions made in the year of retirement, and the allowance may be made approximately by adding, in respect of the first year's contributions,

$$\frac{1}{2} r_j(D\bar{A}, l)C_x \times \frac{c^2 s_x^2 v'^{2x}}{v^x \psi};$$

in respect of the second year's contributions,

$$\frac{1}{2} r_j(D\bar{A}, l)C_{x+1} \times \frac{c^2 s_{x+1}^2 v'^{2(x+1)}}{v^x \psi};$$

and, generally, in respect of the $(t+1)$ th year's contributions,

$$\frac{1}{2} r_j(D\bar{A}, l)C_{x+t} \times \frac{c^2 s_{x+t}^2 v'^{2(x+t)}}{v^x \psi};$$

The complete expression will therefore be,
for the first year's contributions,

$$\begin{aligned} & \left(r_j(D\bar{A}, l)M_{x+1} + \frac{1}{2} r_j(D\bar{A}, l)C_x \right) \frac{c^2 s_x^2 v'^{2x}}{v^x \psi} \\ & = \left(r_j(D\bar{A}, l)M_x - \frac{1}{2} r_j(D\bar{A}, l)C_x \right) \frac{c^2 s_x^2 v'^{2x}}{v^x \psi}; \end{aligned}$$

for the second year's contributions,

$$\begin{aligned} & \left(r_j(D\bar{A}, l)M_{x+2} + \frac{1}{2} r_j(D\bar{A}, l)C_{x+1} \right) \frac{c^2 s_{x+1}^2 v'^{2(x+1)}}{v^x \psi} \\ & = \left(r_j(D\bar{A}, l)M_{x+1} - \frac{1}{2} r_j(D\bar{A}, l)C_{x+1} \right) \frac{c^2 s_{x+1}^2 v'^{2(x+1)}}{v^x \psi}, \end{aligned}$$

and so on.

The process will be seen to be as follows :

Construct a table of $rj \cdot D\bar{A} \cdot ls C_x$ and sum it, obtaining a table of $rj \cdot D\bar{A} \cdot ls M_x$. From each value of $rj \cdot D\bar{A} \cdot ls M_x$ subtract the corresponding value of $\frac{1}{2} rj \cdot D\bar{A} \cdot ls C_x$, multiplying the remainder into $s_x^2 v'^{2x}$. Call the last result $rj \cdot D\bar{A} \cdot ls \bar{M}_x^s$. The value of the return in respect of the first year's contributions will then be $rj \cdot D\bar{A} \cdot ls \bar{M}_x^s \times \frac{c^2}{v^x \psi}$; of the second year's, $rj \cdot D\bar{A} \cdot ls \bar{M}_{x+1}^s \times \frac{c^2}{v^{x+1} \psi}$, etc. The total value of the return in respect of all the contributions will be the sum of these expressions, or $(\Sigma rj \cdot D\bar{A} \cdot ls \bar{M}_x^s) \times \frac{c^2}{v^x \psi}$.

The table of $rj \cdot D\bar{A} \cdot ls \bar{M}_x^s$ can be summed as the ordinary M_x table is summed, and called $rj \cdot D\bar{A} \cdot ls \bar{R}_x^s$. The total value of the return in respect of the l_x persons joining will then be $rj \cdot D\bar{A} \cdot ls \bar{R}_x^s \times \frac{c^2}{v^x \psi}$, and dividing this by $l_x s_x$, the value of the return for each unit of present salary is

$$\begin{aligned} & \frac{c^2}{\psi v^x} \cdot \frac{rj \cdot D\bar{A} \cdot ls \bar{R}_x^s}{l_x s_x} \\ &= \frac{c^2}{\psi} \cdot \frac{rj \cdot D\bar{A} \cdot ls \bar{R}_x^s}{s D_x}. \end{aligned}$$

If no pension is granted before completing n years' service, the value in respect of the first n years' contributions will be as follows :

First year's contributions,

$$(rj \cdot D\bar{A} \cdot ls C_{x+n} + rj \cdot D\bar{A} \cdot ls C_{x+n+1} + \dots) \frac{c^2 s_x^2 v'^{2x}}{v^x \psi},$$

Second year's contributions,

$$(rj \cdot D\bar{A} \cdot ls C_{x+n} + rj \cdot D\bar{A} \cdot ls C_{x+n+1} + \dots) \frac{c^2 s_{x+1}^2 v'^{2x+1}}{v^{x+1} \psi},$$

$$\vdots \quad \vdots \quad \vdots$$

n th year's contributions,

$$(rj \cdot D\bar{A} \cdot ls C_{x+n} + rj \cdot D\bar{A} \cdot ls C_{x+n+1} + \dots) \frac{c^2 s_{x+n-1}^2 v'^{2x+n-1}}{v^x \psi}.$$

The sum of these is

$$\begin{aligned} & \frac{c^2}{v^x \psi} \cdot rj \cdot D\bar{A} \cdot ls M_{x+n} (v'^{2x} s_x^2 + v'^{2x+1} s_{x+1}^2 + \dots + v'^{2x+n-1} s_{x+n-1}^2) \\ &= \frac{c^2}{v^x \psi} \cdot rj \cdot D\bar{A} \cdot ls M_{x+n} \{ \sum (v'^x s_x)^2 - \sum (v'^{x+n} s_{x+n})^2 \} \end{aligned}$$

The value in respect of contributions made after n years will be $\frac{c^2}{v^x \psi} \cdot {}^{rj \text{ D}\bar{A}} \text{ls} \bar{R}_{x+n}^s$. Adding the two expressions together and dividing by $l_x s_x$, the value of the return in respect of all the contributions for each unit of present salary is represented by

$$\frac{c^2}{\psi} \cdot \frac{\left\{ \sum (v'^x s_x)^2 - \sum (v'^{x+n} s_{x+n})^2 \right\} {}^{rj \text{ D}\bar{A}} \text{ls} M_{x+n} + {}^{rj \text{ D}\bar{A}} \text{ls} \bar{R}_{x+n}^s}{s \text{D}_x}.$$

The extent of the error introduced by treating each contribution separately may be found in the same way as before. The formula is based on the assumption that the value of the return on retirement in the $(t+1)$ th year is

$$\begin{aligned} v^{t+\frac{1}{2}} r_{x+t} (\text{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} & \left\{ \psi s_{x+t} \left(\frac{cs_x(1+j)^t}{\psi s_{x+t}} \right)^2 + \psi s_{x+t} \left(\frac{cs_{x+1}(1+j)^{t-1}}{\psi s_{x+t}} \right)^2 \dots \right. \\ & \left. + \psi s_{x+t} \left(\frac{cs_{x+t-1}(1+j)}{\psi s_{x+t}} \right)^2 + \frac{1}{2} \psi s_{x+t} \left(\frac{cs_{x+t}}{\psi s_{x+t}} \right)^2 \right\} \\ = v^{t+\frac{1}{2}} r_{x+t} (\text{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} & \cdot \frac{1}{\psi s_{x+t}} \cdot c^2 \left\{ (s_x \overline{1+j^t})^2 + (s_{x+1} \overline{1+j^{t-1}})^2 \dots \dots \right. \\ & \left. + (s_{x+t-1} \overline{1+j})^2 + \frac{1}{2} s_{x+t}^2 \right\} \\ = v^{t+\frac{1}{2}} r_{x+t} (\text{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} & \cdot \frac{1}{\psi s_{x+t}} \cdot c^2 (1+j)^{2x+t} \left\{ (s_x v'^x)^2 + (s_{x+1} v'^{x+1})^2 \dots \dots \right. \\ & \left. + (s_{x+t-1} v'^{x+t-1})^2 + \frac{1}{2} (s_{x+t} v'^{x+t})^2 \right\} (\Lambda) \end{aligned}$$

The true value is :

$$\begin{aligned} v^{t+\frac{1}{2}} r_{x+t} t \psi s_{x+t} (\text{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} & \left\{ \frac{cs_x(1+j)^t + cs_{x+1}(1+j)^{t-1} \dots + cs_{x+t-1}(1+j) + \frac{1}{2} cs_{x+t}}{t \psi s_{x+t}} \right\}^2 \\ v^{t+\frac{1}{2}} r_{x+t} (\text{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} & \cdot \frac{1}{\psi s_{x+t}} \cdot \frac{c^2 \left\{ s_x(1+j)^t + s_{x+1}(1+j)^{t-1} \dots + s_{x+t-1}(1+j) + \frac{1}{2} s_{x+t} \right\}^2}{t} \\ v^{t+\frac{1}{2}} r_{x+t} (\text{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} & \cdot \frac{1}{\psi s_{x+t}} \cdot c^2 (1+j)^{2x+t} \\ & \times \frac{\left\{ s_x v'^x + s_{x+1} v'^{x+1} \dots + s_{x+t-1} v'^{x+t-1} + \frac{1}{2} s_{x+t} v'^{x+t} \right\}^2}{t} (\text{B}) \end{aligned}$$

The ratio (A) : (B) :

$$\begin{aligned}
 &= \frac{t \left\{ (s_x v'^x)^2 + (s_{x+1} v'^{x+1})^2 + \dots + (s_{x+t-1} v'^{x+t-1})^2 + \frac{1}{2} (s_{x+t} v'^{x+t})^2 \right\}}{\left(s_x v'^x + s_{x+1} v'^{x+1} + \dots + s_{x+t-1} v'^{x+t-1} + \frac{1}{2} s_{x+t} v'^{x+t} \right)^2} \\
 &= \frac{t \left\{ \sum (s_x v'^x)^2 - \sum (s_{x+t} v'^{x+t})^2 + \frac{1}{2} s_{x+t} v'^{x+t} \right\}}{\left\{ \sum (s_x v'^x) - \sum (s_{x+t} v'^{x+t}) + \frac{1}{2} s_{x+t} v'^{x+t} \right\}^2}
 \end{aligned}$$

Values of this ratio are shown in the following table:

x	t			
	10	20	30	40
25	.953	.976	.988	1.015
35	.953	.980	1.016	...
45	.954	1.010
55	1.003

It may be noted that the formulas relating to pensions based on final salary may be used for pensions based on the average of the last r years by substituting the required average values for the values of s_x in $\frac{1}{s_x}$ in the foregoing.

When the pension is based on the final salary, the period during which there is a risk of having to pay a balance will obviously decrease, as time elapses, from $\frac{cs_x}{\psi s_{x+1}}$ to $\frac{cs_x}{\psi s_{x+t}}$, where $x+t$ is the limiting age. If the values of these two expressions be n' and n'' respectively, the value of the benefit will be too small if reckoned from $(D\bar{A})^{(n')}$; that is to say, if $(D\bar{A})^{(1)}$ is taken as $\frac{(D\bar{A})^{(n')}}{n'^2}$, because we should then be working from a higher value of n to a lower one (see p. 349).

Conversely, if $(D\bar{A})^{(1)}$ is taken as $\frac{(D\bar{A})^{(n'')}}$, the value of the benefit will be in excess of the true value. Consequently, we should work from a value of n between n' and n'' , as far as can be judged, inclining to the lower rather than the higher limit, in order to be on the safe side.

Each of the expressions for the value of the return on death after pension is of the form $c^2 \times$ (a coefficient which is independent of c), and if the expression is introduced into the equation of benefit this equation becomes a quadratic in c , from which the value of c can be calculated. Thus, in the simple case where the benefits consist of a pension of $\psi \times$ (total salary to date of retirement), and the payment on death, after being pensioned, of the amount by which the total contributions exceed the total pension payments, the equation is

$$c \cdot (\text{Present value of future salary}) = \psi \cdot (\text{Present value of pension of total salary to date of retirement}) + c^2 \cdot \frac{r(D\bar{A})_s \bar{R}}{\psi^s D}.$$

It might be convenient to calculate first the rate of contribution without allowing for the special benefit, and afterwards to calculate the contribution required to allow for it. If, in the example given above, the former rate were, say, c' , the true rate, c , would be found from the equation

$$(c - c')(\text{Present value of future salary}) = c^2 \cdot \frac{r(D\bar{A})_s \bar{R}}{\psi^s D}.$$

The formulas may, perhaps, seem to be too elaborate, but it is submitted that they have the following advantages :

(1) The values of the functions can be calculated, by a straightforward process, by a clerk without actuarial knowledge, and when this is done the calculation of c is a simple matter.

(2) The nature of the errors resulting from the use of the formulas is known. They are :

(a) The error resulting from the assumption that the value of $(D\bar{A})^{(n)}$ varies as the square of n . This error would be very small if a value of n near to the true value were chosen. A near value could be ascertained from the value of c without the special benefit.

(b) The error resulting from assuming that the return in respect of each year's contributions can be calculated separately. The extent of this is indicated in the table given in each of the last three cases, and an approximate correction can be made if desired.

Although the formulas were devised for the purpose of determining the rates of contribution, they can be used in a valuation.

The value of the return in respect of future contributions can be found by using the formulas already given, but it will be

necessary to modify them in order to value the benefit in respect of past contributions of active members and in respect of members in receipt of pensions at the date of the valuation.

If the pension is equal to $\psi \times (\text{total salary})$, the pension in respect of past salary will be $\psi \times (\text{total past salary})$, and the contributions will be $c \times (\text{total past salary})$. The number of years after retirement during which there will be a risk of having to make a payment will be $\frac{c}{\psi}$. The value of the benefit will therefore be

$$\psi (\text{total past salary}) \cdot \frac{\{\sum (v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)}\}}{l_x} \cdot \left(\frac{c}{\psi}\right)^2$$

where x is the age at the date of valuation. This may be written :

$$\begin{aligned} & \frac{c^2}{\psi} \cdot (\text{total past salary}) \cdot \frac{\{\sum (v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)})\}}{v^x l_x} \\ &= \frac{c^2}{\psi} (\text{total past salary}) \frac{v^x \bar{D}\bar{A}_x M_x}{\bar{D}_x}. \end{aligned}$$

If no return can be made before a later age, say $(x+n)$, the value will be :

$$\frac{c^2}{\psi} \cdot (\text{total past salary}) \cdot \frac{v^x \bar{D}\bar{A}_x M_{x+n}}{\bar{D}_x}.$$

If the pension is equal to $\psi \times (\text{total salary})$, and compound interest at rate j is allowed on the contributions to date of retirement, the ratio of contributions and interest, at date of valuation, to pension in respect of past service, will be :

$$\frac{\text{Past contributions} + \text{interest}}{\psi (\text{total past salary})} = K, \text{ say.}$$

Now, if the member retires in the year following the date of the valuation, K will have increased to $K(1+j)^{\frac{1}{2}}$; if he retires in the second year, it will have increased to $K(1+j)^{\frac{1}{2} \cdot 2}$; and if retirement takes place in the $(t+1)$ th year, it will have increased to $K(1+j)^{t+\frac{1}{2}}$. At the moment of his retirement in the $(t+1)$ th year, the value of the return for every unit of pension will be

$$(\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \{K(1+j)^{t+\frac{1}{2}}\}^2$$

$$= (\bar{D}\bar{A})_{x+t+\frac{1}{2}} (1+j)^{2(t+1)} K^2.$$

The value of the return in respect of past contributions will therefore be

$$\begin{aligned}
 & \frac{\psi (\text{total past salary}) \left\{ \sum (v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^1) \right\} \left\{ (1+j)^{2(t+\frac{1}{2})} K^2 \right\}}{l_x} \\
 = & \psi (\text{total past salary}) K^2 (1+j) \cdot \frac{\left\{ \sum v^x r^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^1 (1+j)^{2(x+t)} \right\}}{v^x l_x} - (1+j)^{2x} \\
 = & \psi (\text{total past salary}) K^2 (1+j) \cdot \frac{\left(\sum_{t=0}^{rj(\bar{D}\bar{A})} C_{x+t} \right) \div (1+j)^{2x}}{v^x l_x} \\
 = & \frac{[\text{Past contributions + interest}]^2}{\psi (\text{total past salary})} \cdot \frac{rj(\bar{D}\bar{A}) M_x}{D_x} \times v'^{2x-1}
 \end{aligned}$$

The left-hand factor is, of course, obtained by substituting for K the value given above. If no pension is granted within m years from the date of entry and the duration is less than m , say n , $rj(\bar{D}\bar{A}) M_{x-n+m}$ should be substituted for $rj(\bar{D}\bar{A}) M_x$ in the above formula. K will vary according to duration of membership, and, in cases of the same duration, according to the rate at which the salary has increased in the past in each individual case. The differences arising from the latter cause would be very small, and in applying the last formula it would be sufficiently accurate to use the aggregate of the total past contributions and interest, and the aggregate of the total past salaries, of all aged x at the date of valuation who entered at the same age. In practice it might be thought sufficient to combine the total past contributions and interest, and the total past salaries of all persons aged x at the date of valuation irrespective of duration.

If the pension is at the rate of ψ of the final salary for each year of service, the value of the return in respect of the past contributions can be found as follows: Let x be the age at date of valuation, and n the duration of membership. Take the case of an individual member. It is assumed that his present salary, which may be called s'_x , will be constant for the first year after valuation; $s'_x \times \frac{s_{x+1}}{s_x}$ for the second year, and, generally,

$s'_x \times \frac{s_{x+t}}{s_x}$ for the $(t+1)$ th year. If he retires in the $(t+1)$ th year, his pension in respect of service to age x will be, $n\psi s'_x \times \frac{s_{x+t}}{s_x}$; and the time after retirement during which there

will be a risk of having to pay a balance will be

$$\frac{\text{Past Contributions}}{n\psi s'_x \times \frac{s_{x+t}}{s_x}} = \frac{\text{Past Contributions}}{n\psi s'_x} \times \frac{s_x}{s_{x+t}}$$

The value of this at date of retirement will be

$$\begin{aligned} n\psi s'_x \times \frac{s_{x+t}}{s_x} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} & \left\{ \frac{\text{Past Contributions}}{n\psi s'_x} \times \frac{s_x}{s_{x+t}} \right\}^2 \\ & = \frac{(\text{Past Contributions})^2}{n\psi s'_x} \cdot (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{s_x}{s_{x+t}}. \end{aligned}$$

The present value of the return will therefore be—

$$\begin{aligned} \frac{(\text{Past Contributions})^2}{n\psi s'_x} \cdot \frac{\sum_{t=0}^{\infty} \left\{ v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{1}{s_{x+t}} \right\} s_x}{l_x} \\ = (\text{Past Contributions})^2 \cdot \frac{\sum_{t=0}^{\infty} \left\{ v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \cdot \frac{1}{s_{x+t}} \right\} s_x}{n\psi s'_x \cdot v^x l_x} \end{aligned}$$

Writing $r \cdot (\bar{D}\bar{A})^{ls} C_x$ for $v^{x+\frac{1}{2}} r_x (\bar{D}\bar{A})_{x+\frac{1}{2}}^{(1)} \cdot \frac{1}{s_x}$

and $r \cdot (\bar{D}\bar{A})^{ls} M_x$ for $\sum r \cdot (\bar{D}\bar{A})^{ls} C_x$,

we obtain the formula

$$\frac{(\text{Past Contributions})^2}{n\psi (\text{present salary})} \times \frac{r \cdot (\bar{D}\bar{A})^{ls} M_x s_x}{D_x}.$$

If no pension is paid before m years have been served, $r \cdot (\bar{D}\bar{A})^{ls} M_{x-n+m}$ must be written for $r \cdot (\bar{D}\bar{A})^{ls} M_x$ in the above formula.

In actual working, all cases of the same age and duration could be grouped, and $\frac{(\text{Aggregate of past Contributions})^2}{(\text{Total present Salaries})}$ substituted for $\frac{(\text{Past Contributions})^2}{(\text{Present Salary})}$.

If the pension is at the rate of ψ of the final salary for each year of service, and compound interest at rate j to date of retirement is allowed on the contributions, the value of the return in respect of past contributions may be found as follows, x being the age at the valuation date and n the duration. If the member retires in the $(t+1)$ th year, his present salary of s'_x will have increased to $s'_x \times \frac{s_{x+t}}{s_x}$, and his pension in respect of service to age x will be $n\psi s'_x \times \frac{s_{x+t}}{s_x}$. The period after retirement during which there will be a risk of having to pay a balance will be:

$$\frac{(1+j)^{t+\frac{1}{2}} (\text{Past contributions} + \text{interest})}{n\psi s'_x \times \frac{s_{x+t}}{s_x}}$$

$$= \frac{(\text{Past contributions} + \text{interest})(1+j)^{t+\frac{1}{2}}}{n\psi s'_x} \cdot \frac{s_x}{s_{x+t}},$$

and the value of the return, at date of retirement, will be :

$$\begin{aligned} s'_x \times \frac{s_{x+t}}{s_x} \cdot \left[v^{t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \left\{ \frac{(\text{Past contributions} + \text{interest}) (1+j)^{t+\frac{1}{2}} \cdot \frac{s_x}{s_{x+t}}}{n\psi s'_x} \right\}^2 \right] \\ \frac{(\text{Past contributions} + \text{interest})^2}{n \cdot \psi s'_x} \cdot (1+j)s_x \cdot \frac{\left\{ v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} (1+j)^{2t} \cdot \frac{1}{s_{x+t}} \right\}}{v^x l_x} \\ \frac{(\text{Past contributions} + \text{interest})^2}{n \cdot \psi s'_x} \cdot (1+j)s_x \\ \times \frac{\left[v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} (1+j)^{2(x+t)} \cdot \frac{1}{s_{x+t}} \right]}{D_x} - (1+j)^{2x} \end{aligned}$$

But,

$$v^{x+t+\frac{1}{2}} r_{x+t} (\bar{D}\bar{A})_{x+t+\frac{1}{2}}^{(1)} \cdot (1+j)^{2(x+t)} \cdot \frac{1}{s_{x+t}} = r_j (\bar{D}\bar{A}) l_s C_{x+t}.$$

(see page 363).

The value of the return in case of retirement in the $(t+1)$ th year is, therefore,

$$\frac{(\text{Past contributions} + \text{interest})^2}{n\psi s'_x} \cdot (1+j)s_x \cdot \frac{r_j (\bar{D}\bar{A}) l_s C_{x+t} v'^{2x}}{D_x}$$

where $v' = \frac{1}{1+j}$. Summing the expression for all possible values of t , commencing at 0, we get as the value of the return :

$$\frac{(\text{Past contributions} + \text{interest})^2}{n\psi s'_x} \cdot s_x \cdot \frac{rj^{D\bar{A}/ls} M_x \cdot v'^{2x-1}}{D_x}.$$

If no pension is granted before completing m years' service and n is less than m , $rj^{D\bar{A}/ls} M_{x-n+m}$ must be substituted for $rj^{D\bar{A}/ls} M_x$ in the above formula.

All cases of the same age and duration can be grouped, so that the (Past Contributions+Interest) may represent the total of the accumulations and s'_x the total present salaries.

The valuation of the return in cases where pensions have been entered upon is a simple matter. Taking all the cases where the nearest age of the pensioner at the date of valuation is $x + \frac{1}{2}$, they can be grouped according to the number of months during which there will be a balance payable on death. Let P_1 be the total of the annual pensions payable in cases where the risk will expire in one month, P_2 the total where it will expire in two months, and so on.

The value in cases where the risk will expire in t months is $P_t(D\bar{A})_{x+\frac{1}{2}}^1 \left(\frac{t}{12}\right)^2 = \frac{(D\bar{A})_{x+\frac{1}{2}}^1}{144} \cdot (t^2 P_t)$. The total value in respect

of pensioners now aged $x + \frac{1}{2}$ will therefore be

$$\frac{(D\bar{A})_{x+\frac{1}{2}}^1}{144} \cdot (P_1 + 4P_2 + 9P_3 + \dots)$$

II.—On a method of scheduling particulars for the valuation, in certain cases, of prospective pensions based on terminal salaries.

In his paper on "Staff Pension Funds" (*J.I.A.*, vol. xxxix, p. 129), Mr. George King, in describing his method of dealing with pensions based upon terminal salaries, advocated arranging the particulars according to ages at entry and duration, all those entering at a given age being entered on one sheet on which they

are classified according to duration. In effect, the valuation schedule takes the following form :

Age at Entry, x .

Age attained (1)	Total Salaries at date of Valuation (2)	Valuation Factor (3)	$\frac{(4)}{=(2) \times (3)}$
x	$_x S_x$	$\frac{_x M^{ra}}{^s D_x}$	$_x S_x \times \frac{_x M^{ra}}{^s D_x}$
$x+1$	$_x S_{x+1}$	$\frac{_x M^{ra}_{x+1}}{^s D_{x+1}}$	$_x S_{x+1} \times \frac{_x M^{ra}_{x+1}}{^s D_{x+1}}$
$x+2$	$_x S_{x+2}$	$\frac{_x M^{ra}_{x+2}}{^s D_{x+2}}$	$_x S_{x+2} \times \frac{_x M^{ra}_{x+2}}{^s D_{x+2}}$
$x+3$	$_x S_{x+3}$	$\frac{_x M^{ra}_{x+3}}{^s D_{x+3}}$	$_x S_{x+3} \times \frac{_x M^{ra}_{x+3}}{^s D_{x+3}}$
⋮	⋮	⋮	⋮

In this table the symbol $_x S_{x+t}$ is used to denote the total salaries of those members now aged $x+t$ who entered at age x .

It will be observed that in preparing the valuation factors every numerator is divided by its appropriate value of $^s D_x$, and that each value of $^s D_x$ is employed once for every age at entry up to and including x . It occurred to me, therefore, that if the particulars could be classified in the first place according to age attained, and secondly according to duration, it would only be necessary to divide once by each value of $^s D_x$.

The work could be arranged as under:

Age attained, x .

Duration (1)	Total Salaries at date of Valuation (2)	Numerator of Valuation Factor (3)	$\frac{(4)}{=(2) \times (3)}$
n	$_{x-n} S_x$	$_{x-n} z M^{ra}_x$	$_{x-n} S_x \times _{x-n} z M^{ra}_x$
$n-1$	$_{x-n+1} S_x$	$_{x-n+1} z M^{ra}_x$	$_{x-n+1} S_x \times _{x-n+1} z M^{ra}_x$
$n-2$	$_{x-n+2} S_x$	$_{x-n+2} z M^{ra}_x$	$_{x-n+2} S_x \times _{x-n+2} z M^{ra}_x$
⋮	⋮	⋮	⋮

The total of column (4) could be divided by sD_x , which would thus be used once only.

If the proportion of the terminal salary paid as pension varies irregularly with the duration, there seems very little advantage, if any, to be gained by arranging the particulars according to ages attained ; but where the proportion varies directly as the duration, much labour can be saved by so doing if there are many ages at entry, and it is easy to allow for the pension not exceeding a given maximum.

In a valuation which I had before me recently the pension was at the rate of one-sixtieth of terminal salary for each completed year of service, not exceeding forty-sixtieths, no pension being given on retirement before completing ten years' service.

The formulas used were :

$$(a) \quad \frac{10^{ra}M_{x-n+10}^{ls} + {}^{ra}R_{x-n+11}^{ls} - {}^{ra}R_{x-n+41}^{ls}}{{}^sD_x} \times \frac{1}{60},$$

for durations of ten years or less ;

$$(b) \quad \frac{n^{ra}M_x^{ls} + {}^{ra}R_{x+1}^{ls} - {}^{ra}R_{x-n+41}^{ls}}{{}^sD_x} \times \frac{1}{60},$$

for durations from 11 to 39 inclusive ;

$$(c) \quad \frac{40^{ra}M_x^{ls}}{{}^sD_x} \times \frac{1}{60},$$

for durations 40 and upwards.

In these formulas x is the age at the valuation date and n the duration.

To apply these formulas a schedule in the following form (in which, for simplicity, the total of the salaries paid to members aged x with duration n years is denoted by ${}_nS_x$ instead of ${}_{x-n}S_x$, Mr. King's symbol) may be used :

Age attained, x.

Total salaries col. (3) {
" " " " " (S)

Log (E)
Less Log 8D_r

Grand total

In describing the use of this form it will be convenient to deal with the durations in sections. The liability (omitting the denominator sD_x) in respect of durations exceeding 10 years is evidently :

(1) For durations of 40 years or over :

$$\begin{aligned} {}_{45}S_x \times 40^{ra} M_x^{ls} \\ + {}_{44}S_x \times 40^{ra} M_x^{ls} \\ \vdots \\ + {}_{40}S_x \times 40^{ra} M_x^{ls} \\ = (\sum_{n=40}^{n=45} n S_x) 40^{ra} M_x^l \end{aligned}$$

(2) For durations between 40 and 10 years :

$$\begin{aligned} {}_{39}S_x 39^{ra} M_x^{ls} + {}_{39}S_x^{ra} R_{x+1}^{ls} - {}_{39}S_x^{ra} R_{x-39+41}^{ls} \\ + {}_{38}S_x 38^{ra} M_x^{ls} + {}_{38}S_x^{ra} R_{x+1}^{ls} - {}_{38}S_x^{ra} R_{x-38+41}^{ls} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ + {}_{11}S_x 11^{ra} M_x^{ls} + {}_{11}S_x^{ra} R_{x+1}^{ls} - {}_{11}S_x^{ra} R_{x-11+41}^{ls} \\ = (\sum_{n=11}^{n=39} n S_x \times n) {}^{ra} M_x^{ls} + \sum_{n=11}^{n=39} n S_x^{ra} R_{x+1}^{ls} - \sum_{n=11}^{n=39} n S_x^{ra} R_{x-n+41}^{ls} \end{aligned}$$

The coefficient of ${}^{ra} M_x^{ls}$ may easily be obtained by summing column (3) as the C_x column is summed, but downwards instead of upwards. To assist the reader in following the description of the working the symbols printed in columns (3) and (4) may be useful.

The values of $_n S_x$ from $n=45$ to $n=40$ can first be cast, and the sum placed in column (4); ${}_{39}S_x$ can now be added, then ${}_{38}S_x$, and so on until the end of the column is reached.

Now, ${}_{40}S_x$ will be contained in $\sum_{n=t}^{n=45} n S_x$ for all values of t from 40 to 11, so that it will be contained in 30 different values. If, therefore, the column of $\sum_{n=t}^{n=45} n S_x$ is cast, $30 {}_{40}S_x$ will be contained in the total. Similarly, every value of $_n S_x$ from $n=41$ to $n=45$ will also be contained 30 times in the total.

As regards the remaining terms, ${}_{39}S_x$ will be involved 29 times, ${}_{38}S_x$, 28 times, and so on. The total of column (4) will thus be :

$$30({}_{45}S_x + {}_{44}S_x \dots + {}_{40}S_x) + 29 {}_{39}S_x + 28 {}_{38}S_x \dots + 2 {}_{12}S_x + {}_{11}S_x$$

Before casting write $10\Sigma_{n=11}^{n=45} S_x$, or ten times the last value in column (4), beneath that value, that is

$$10(45S_x + 44S_x \dots + 43S_x) + 10_{39}S_x + 10_{38}S_x \dots 10_{12}S_x + 10_{11}S_x.$$

If column (4) is now cast, the total will be the coefficient of ${}^{ra}M_x^{ls}$, namely,

$$40(45S_x + 44S_x \dots + 40S_x) + 39_{39}S_x + 38_{38}S_x \dots + 12_{12}S_x + 11_{11}S_x.$$

If the lowest value of n is greater than 11, say t , the summation will stop at $\Sigma_{n=t}^{n=45} nS_x$, which can be multiplied into $t-1$, and the product entered before the cast is made.

The coefficient of ${}^{ra}R_{x+1}^{ls}$ is the total of column (3) from ${}_{39}S_x$ to ${}_{11}S_x$ inclusive.

To obtain $\Sigma_{n=11}^{n=39} (nS_x {}^{ra}R_{x-n+41}^{ls})$ the logarithms of nS_x should be entered in column (5) from ${}_{39}S_x$ downwards. Now ${}_{39}S_x$ has to be multiplied by ${}^{ra}R_{x-39+41}^{ls}$, that is, by ${}^{ra}R_{x+2}^{ls}$; ${}_{38}S_x$ has to be multiplied by ${}^{ra}R_{x-38+41}^{ls}$, that is, by ${}^{ra}R_{x+3}^{ls}$, and so on. It will be seen, therefore, that by placing a table of $\log {}^{ra}R_x^{ls}$ against the logarithms of nS_x , so that $\log {}^{ra}R_{x+2}^{ls}$ is against ${}_{39}S_x$, $\log {}^{ra}R_{x+3}^{ls}$ will be opposite ${}_{38}S_x$ and all the other required values of $\log {}^{ra}R_x^{ls}$ will be in their proper positions respectively. The two sets of logarithms can then be added laterally, and the totals, which will be the logarithms of $nS_x {}^{ra}R_{x-n+41}^{ls}$, entered in column (6), after which the natural numbers can be entered in column (7) and cast. At the younger ages attained nS_x will be zero for the higher values of n ; and it may be observed that the number of multiplications into ${}^{ra}R_{x-n+41}^{ls}$ on any sheet will be small because such multiplication will be required only in the cases of those entering more than 40 years before the age for compulsory retirement.

For durations of ten years and under, it is necessary to have a table of $\log [10^{ra}M_{x+10}^{ls} + {}^{ra}R_{x+11}^{ls} - {}^{ra}R_{x+41}^{ls}]$ or, what is the same thing, a table of $\log [10^{ra}M_x^{ls} + {}^{ra}R_{x+1}^{ls} - {}^{ra}R_{x+31}^{ls}]$. This table can be placed beside column (9) with $\log [10^{ra}M_x^{ls} + {}^{ra}R_{x+1}^{ls} - {}^{ra}R_{x+31}^{ls}]$ against $\log {}_{10}S_x$, when the other values required will be in their proper positions. The logarithms can be added laterally and entered in column (10), after which the natural numbers can be entered in column (11) and cast.

The remainder of the work, comprising the multiplication of ${}^{ra}R_{x+1}^{ls}$ and ${}^{ra}M_x^{ls}$ by their respective coefficients, the summarising of the figures, and the division by sD_x , is indicated at the foot of the schedule.

Although the formulas relate to pensions based on final salaries, the schedule could be used equally well for valuing pensions based on the average salaries of the last t years.

To avoid large numbers it will be found advisable to divide all the values in the service table by 1,000 or even by 10,000. It is not unusual to divide s_x by 100, but this is not so powerful.

The schedule will no doubt appear very complicated at first, but in practice it works smoothly and men without actuarial knowledge experience no difficulty in using it.

Obviously the greatest advantage is obtained from its use when there is a wide range of ages at entry, and it is apparent that if all members entered at the same age no advantage would result from using it, but, on the contrary, the work would be increased.

In the fund for the valuation of which the form was devised, the members entered at ages from 18 to 50, and it will illustrate the advantage of using it if the number of operations involved by its use is compared with the number required by Mr. King's method, assuming that retirement is compulsory at age 65.

By Mr. King's method a separate valuation table has to be made for each age at entry.

The first step is to compute the values of

$$\log v^{x+\frac{1}{2}} r_x \bar{a}_{x+\frac{1}{2}} z_x \times .01 = \log \bar{z} C_x^{ra}$$

which is common to all the valuation tables, and in which z_x is the terminal salary. After this the procedure in the construction of each valuation table and its application is as follows (*J.I.A.*, vol. xxxix, p. 184) :

- (1) $\log \bar{z} C_x^{ra} + \log \kappa_t$ (where κ_t is the proportion of terminal salary taken as pension after t years' service),
- (2) Take out $\bar{z} C_x^{ra} \times \kappa_t = \text{antilog of (1)},$
- (3) $\bar{z} M_x^{ra} = \sum \bar{z} C_x^{ra},$
- (4) Log $\bar{z} M_x^{ra},$
- (5) Subtract $\log s D_x$ from (4), obtaining $\log \bar{z} F_x^{ra},$
- (6) Take $\log y S_x,$
- (7) Add $\log \bar{z} F_x^{ra},$
- (8) Take antilogs,
- (9) Cast.

The above shows that, for the valuation for entry age x , there are nine operations for every age from x to 65 inclusive, or $9(65 - x + 1)$. But this does not allow for the fact that, as no pension is granted before completing ten years' service, $\log \bar{z} M_x^{ra}$ is constant for all durations up to and including 10. For

durations 0 to 9, therefore, the first four operations are unnecessary and the number of operations for each age at entry is reduced by 40, making a total of $9(65-x+1)-40$.

The number of entry ages from 18 to 50 inclusive is 33, and if, in the last expression, we give x every value from 18 to 50 and sum the results, the total is

$$(9 \times 66 - 40)33 - 9 \times \frac{18+50}{2} \times 33 = 8,184.$$

When using the schedule it is necessary to construct tables of $\log^{ra}M_x^{ls}$, $\log^{ra}R_x^{ls}$ and $\log(10^{ra}M_x^{ls} + {}^{ra}R_{x+1}^{ls} - {}^{ra}R_{x+31}^{ls})$.

To calculate the number of operations involved by the use of the schedule is a lengthy and complicated task, and readers of this paper will not desire to have the calculations set out in detail. I have, however, calculated approximately the number of operations involved in the three cases where the maximum ages at entry are 50, 40 and 30 respectively, the youngest age at entry being 18 and retirement compulsory at 65. For comparison the numbers by Mr. King's method are shown also. In each case the numbers refer to operations subsequent to the preparation of the table of $\log^z C_x^{ra}$ which, or its equivalent, is required by both methods.

	MAXIMUM AGE AT ENTRY					
	50		40		30	
	Mr. King's Method	New Method	Mr. King's Method	New Method	Mr. King's Method	New Method
Summations . . .	726	733	621	638	416	443
Taking logs and anti-logs . . .	3,564	1,564	2,944	1,344	1,924	1,124
Lateral additions of 2 numbers . . .	2,838	611	2,323	501	1,508	391
Casts . . .	1,056	2,344	851	2,024	546	1,504
Total . . .	8,184	5,252	6,739	4,507	4,394	3,462

The table clearly shows that the use of the schedule effects a great reduction in the number of operations to be performed, but to obtain a clear notion of the extent of the saving of work it is necessary to bear in mind the nature of the operations. In the

in the above table the word "summation" is used to denote the addition of a series of numbers in the same manner as the ordinary C_x and D_x columns are summed, where, as each number is added, the total to that point is written down; column (4) of the schedule is an instance. The addition of each number in the column counts as one operation, so that if there were twenty numbers to be summed the addition of the twenty would count as twenty operations. The numbers opposite "lateral additions of two numbers" are the numbers of pairs of numbers added together. The word "cast" is used to describe the simple addition of a column of figures in which only the total of the whole is entered; if there are 20 numbers to be added, the addition of the column is reckoned as twenty.

Now it is evident that casting is very much quicker and easier than any of the other operations; and although it is difficult to make an exact comparison, it will be safe to assume that the work of casting a column of figures is less than one-half, and probably less than one-third, of that involved in performing the same number of operations of any of the other three classes.

On the basis of casting being equivalent to half the labour involved in the other operations the comparison works out as follows:

	MAXIMUM AGE AT ENTRY.					
	50		40		30	
	Mr. King's Method	New Method	Mr. King's Method	New Method	Mr. King's Method	New Method
Total number of operations	8,184	5,252	6,739	4,507	4,394	3,462
Less one-half of casts	528	1,172	425	1,012	273	752
	7,656	4,080	6,314	3,495	4,121	2,710

If the labour of casting be considered equivalent to one-third of the labour involved by an equal number of the other operations the comparison is more striking, namely:

Total number of operations	8,184	5,252	6,739	4,507	4,394	3,462
Less two-thirds of casts	704	1,563	567	1,349	364	1,002
	7,480	3,689	6,172	3,158	4,030	2,460

Value of a Temporary Assurance commencing at n and continuously decreasing for n years.

(Based on Mr. H. W. MANLY's Special Mortality Table for obtaining Values of Annuities on the Lives of Invalid Pensioners.)

$$(\bar{DA})_{x+\frac{1}{2}}^{(n)}$$

Interest 4 per cent.

x	$n =$						
	1	2	3	4	5	6	7
29	.049	.188	.405	.688	1.027	1.416	1.846
30	.048	.186	.400	.680	1.017	1.401	1.826
31	.048	.184	.395	.671	1.004	1.384	1.803
32	.047	.181	.390	.662	.989	1.365	1.779
33	.046	.178	.384	.652	.975	1.345	1.755
34	.045	.175	.378	.643	.961	1.326	1.730
35	.045	.173	.372	.633	.947	1.306	1.705
36	.044	.170	.366	.623	.932	1.286	1.680
37	.043	.167	.360	.613	.917	1.266	1.655
38	.042	.164	.354	.603	.903	1.247	1.630
39	.042	.162	.348	.593	.889	1.227	1.605
40	.041	.159	.342	.583	.874	1.207	1.580
41	.040	.156	.336	.573	.859	1.188	1.555
42	.040	.153	.330	.563	.845	1.169	1.530
43	.039	.150	.324	.553	.831	1.150	1.505
44	.038	.147	.319	.544	.817	1.130	1.480
45	.037	.145	.313	.534	.803	1.111	1.454
46	.037	.142	.307	.524	.788	1.091	1.428
47	.036	.139	.301	.514	.772	1.070	1.401
48	.035	.136	.295	.504	.757	1.049	1.374
49	.034	.133	.289	.493	.741	1.028	1.347
50	.034	.130	.283	.483	.726	1.007	1.320
51	.033	.128	.277	.473	.711	.986	1.293
52	.032	.125	.271	.463	.696	.966	1.267
53	.032	.122	.265	.453	.680	.946	1.241
54	.031	.119	.259	.443	.666	.926	1.215
55	.030	.117	.253	.433	.652	.906	1.189
56	.030	.114	.247	.423	.637	.885	1.162
57	.029	.111	.241	.413	.622	.865	1.136
58	.028	.108	.235	.403	.607	.844	1.109
59	.027	.105	.229	.393	.592	.823	1.080
60	.026	.102	.222	.382	.575	.801	1.051
61	.025	.099	.215	.370	.559	.779	1.032
62	.025	.096	.209	.360	.543	.763	1.018
63	.024	.094	.205	.355	.543	.770	1.033
64	.023	.091	.201	.353	.547	.781	1.054

Value of a Temporary Assurance commencing at n and continuously decreasing for n years.

(Based on MR. H. W. MANLY'S Special Mortality Table for obtaining Values of Annuities on the Lives of Invalid Pensioners.)

x	$(\bar{D}\bar{A})_{x+\frac{1}{2}}^{(n)}$							Interest 3 per-cent.
	n =							
x	1	2	3	4	5	6	7	
29	.049	.190	.409	.697	1.043	1.441	1.883	
30	.049	.188	.405	.689	1.032	1.426	1.863	
31	.048	.186	.400	.680	1.019	1.409	1.841	
32	.047	.183	.394	.671	1.006	1.390	1.817	
33	.046	.180	.388	.661	.991	1.370	1.791	
34	.046	.177	.382	.651	.976	1.350	1.765	
35	.045	.174	.376	.641	.961	1.330	1.740	
36	.044	.171	.370	.631	.946	1.310	1.715	
37	.044	.168	.363	.620	.931	1.290	1.689	
38	.043	.165	.357	.610	.916	1.270	1.663	
39	.042	.163	.351	.600	.902	1.250	1.638	
40	.041	.160	.345	.591	.888	1.231	1.613	
41	.041	.157	.339	.581	.874	1.211	1.588	
42	.040	.155	.333	.571	.860	1.191	1.563	
43	.039	.152	.327	.560	.845	1.171	1.538	
44	.038	.149	.321	.550	.830	1.151	1.512	
45	.038	.146	.315	.541	.815	1.131	1.485	
46	.037	.143	.310	.531	.800	1.110	1.457	
47	.036	.140	.304	.521	.784	1.088	1.429	
48	.035	.137	.298	.511	.768	1.067	1.402	
49	.035	.134	.292	.500	.753	1.046	1.375	
50	.034	.132	.286	.490	.738	1.025	1.348	
51	.033	.129	.280	.480	.723	1.004	1.321	
52	.032	.126	.274	.469	.708	.983	1.294	
53	.032	.123	.268	.459	.693	.963	1.268	
54	.031	.120	.262	.449	.678	.943	1.241	
55	.030	.118	.256	.439	.663	.922	1.213	
56	.030	.115	.249	.428	.647	.900	1.184	
57	.029	.112	.243	.417	.631	.878	1.155	
58	.028	.109	.237	.407	.615	.856	1.127	
59	.028	.106	.231	.397	.600	.835	1.100	
60	.027	.103	.225	.387	.584	.814	1.075	
61	.026	.100	.218	.376	.568	.794	1.055	
62	.025	.097	.212	.366	.556	.783	1.047	
63	.024	.095	.207	.359	.552	.783	1.054	
64	.023	.092	.204	.359	.557	.797	1.077	

ABSTRACT OF THE DISCUSSION.

MR. E. C. THOMAS said that Mr. Tinner had provided the members with an extremely ingenious solution of a complicated problem and had thereby earned their hearty congratulations and thanks. The benefit to be valued was a deferred decreasing assurance. A man retiring on a pension came into possession of an immediate annuity of, say, 1 per annum for life, together with an assurance commencing at n and decreasing by 1 per annum to zero at the end of n years. If n were integral, and the decrements took place at the end of each year, the total number of yearly assurances of 1 would obviously be the sum of the first n natural numbers, *i.e.*, $n(n+1) \div 2$. By considering the decrement to be taking place continuously, we had the sum of a series commencing at $n - \frac{1}{2}$ and terminating at $\frac{1}{2}$, which was clearly equal to $(n^2 \div 2)$. If we assumed that over a short term (and n had never a high value) $C_x = C_{x+1} = C_{x+2}$, &c., we could write as the value of our temporary assurance $(n^2 \div 2) \times A_{x-1}^1$ approximately. Now an assurance of 1 decreasing continuously to zero at the end of 1 year was approximately equal to an ordinary term assurance of 1 for $\frac{1}{2}$ year. Calling such an assurance $(D\bar{A})_x^{(1)}$ we could express our decreasing assurance for n years (commencing at n) as $n^2 \times (D\bar{A})_x^{(1)}$.

It was this function $n^2 \times (D\bar{A})_x^{(1)}$ which formed the starting point for the whole of the subsequent work. This was the actual benefit (apart from the pension itself) which emerged on retirement. In problem 1 (the simplest form) it entered into the formula in exactly the same way as the annuity values for the pension benefit, and if in Mr. Manly's original formula for the present value of a pension (based on average salary) you substituted this function for a_x , you would arrive at Mr. Tinner's expression. In this particular formula, n was the quotient of $(c \div \psi)$, and was constant for all values of x . The expression for the *ordinary* retirement benefit gave the value for every unit of potential pension and would be applied to S (total salary) $\times \psi$. Mr. Tinner had brought ψ into his working, and by cancelling it against $(c^2 \div \psi^2)$ produced the constant $(c^2 \div \psi)$ in his final result. This constant could be expressed as $c \times (c \div \psi)$ where $(c \div \psi) = n$, and as the whole expression was applied to the total salary, and as, moreover, total salary $\times c =$ total contributions, it took the form of total contributions $\times n$ times $\frac{r(D\bar{A})_x^{(1)} R_x}{s D_x}$. Bearing in mind that $(D\bar{A})_x^{(1)}$ was a term assurance for $\frac{1}{2}$ year, we arrived at the conclusion that the expression represented a retiring benefit of $(n \div 2)$ times an assurance for 1 year of the whole of the contributions. Thus we had clearly an approximation to the value of an assurance commencing, on retirement, with the whole of the contributions, and continuously decreasing to zero at the end of n years. The question at once arose as to the object and advantage gained by using the function $n^2 \times (D\bar{A})_x^{(1)}$ in preference to the decreasing assurance for the exact term. One advantage was immediately obvious, if it was considered

that n was generally fractional, and that it was very much simpler to construct a table of decreasing term assurances for one year, or any integral number of years, than a fractional period. For this reason he should have no hesitation in using the formula in practice. The labour would not deter him, in fact he always considered it an economy of skilled labour to put the problem into such a form, that a clerk, with a knowledge of logarithms, could under clear and definite instructions relieve one of the arithmetical work. The objection that $(\bar{D}\bar{A})_x^{(1)}$ was only an approximation was practically removed if we adopted Mr. Tinner's alternative method of calculating $\bar{D}\bar{A}_x^{(1)}$ by dividing the corresponding value of $\bar{D}\bar{A}_x^{(n)}$ by n^2 , where n' was the nearest integer to n . Suppose that the term of assurance, i.e. ($c \div \psi$), was $3\frac{1}{3}$. You took Mr. Tinner's value of the $(\bar{D}\bar{A})$ function for 3 years and divided by 3^2 , i.e., 9, and called this the value of $(\bar{D}\bar{A})_x^{(1)}$, instead of using the actual calculated value. By this means one obtained almost as exact a result as if the correct fractional (DA) had been used. Mr. Manly gave an approximate formula for this benefit (*J.I.A.*.. vol. xlii, p. 20), in which he assumed an average retiring age. If he had calculated his decreasing assurance function for each separate age, the formula would have been practically the same as Mr. Tinner's.

The author had taken us further than this comparatively simple case, and had applied his methods to three other cases, involving assurances based on contributions with compound interest and pensions based on final salary, and here we got into deeper water. There was, however, a wonderful symmetry in all these formulas. Taking the next case developed by Mr. Tinner, where the pension was a function of average salary, and the returns on death after superannuation were based on the total contributions accumulated with compound interest; a formula was given in the *Journal* (vol. xxxviii, p. 276), for the return on death before superannuation of the whole of the contributions, accumulated at the rate j per annum. If Mr. Tinner's present formula was compared with that referred to it would be found, allowing for the differing circumstances, that there was a wonderful family likeness. The author's first column in this case was exactly the same as in his previous problem $\times (1+j)^{2x}$. The death benefit formula referred to used the ordinary $C_x \times (1+j)^{x+\frac{1}{2}}$, or more correctly, as afterwards noted, $C_x \times (1+j)^x$. The factor $(1+j)^{2x}$ entered into Mr. Tinner's formula because $[(1+j)^x]^2$ was part of the co-efficient of $DA^{(1)}$; that is to say $(1+j)^x$ came in twice, once to fix the number of years' assurance, and again to determine the amount of commencing assurance. The next step was in each case to form the corresponding M column by summation. Omitting the final payment correction, the succeeding operation was in the earlier problem to multiply by $\frac{s_x}{(1+j)^x}$ and in the later one by $\frac{s_x}{(1+j)^{2x}}$; the $(1+j)$ factor is in each case required at this stage to correct the over-accumulation which has up to the present in effect started from age 0 instead of

the entrance age, and as Mr. Tinner in his formula has already used $(1+i)^{2x}$ in the numerator, he now has to use the same function in the denominator. From this modified M column the corresponding R column was in each case obtained by summation.

If we glanced now at the author's third formula, where the contributions were returnable without interest, but the pension was based on final salary, we found an analogy between this, and his first formula involving average salary, similar to that between Mr. Manly's retirement benefit formulas for final and average salary respectively. It had to be remembered here that the benefit was still a function of average or total salary, because the total contributions were assured at the commencement, and that the final salary only came in as a divisor, *i.e.*, to fix the term. We therefore found that the fundamental column of the first problem had to be divided instead of multiplied by s_x , and in order to get an effective s_x into the numerator the corresponding M column had to be multiplied by s_x^2 .

The fourth problem, involving final salary and interest on contributions, produced a formula which was a logical combination of Nos. 2 and 3. He found, on reading the paper, that by taking Mr. Tinner's columnar results, and endeavouring to find a satisfactory reason, consistent with previous work, for their composition, he was better able to follow the somewhat intricate mathematical demonstrations leading up to those results, and it was in the hope that his experience would be helpful to others that he had ventured to make the foregoing remarks, but he recognised fully the great ingenuity and painstaking labour which Mr. Tinner had taken in developing his ideas. There was one case that had not been dealt with by Mr. Tinner, and that was where the percentage of average or final salary allowed as a pension was a varying one. Doubtless this had been omitted, in order not to overburden the paper. In this case, strictly speaking, each entry age had to be dealt with separately, and the function $(1 \div \psi_t)$, instead of being placed outside the bracket would enter into the initial column "C" as an additional factor.

Coming now to the practical use of the formulas, he had already stated that, as regards No. 1, he should have no hesitation in using it. With regard to the others he did not feel so sure. They were very neat formulas, but difficult to follow and probably more difficult to retain, and, moreover, there was an inherent weakness in them. As Mr. Tinner pointed out, they each involved a systematic error. It is true that this error was so small as to be negligible, and the trouble that the author had taken to forestall objections by measuring its extent was beyond all praise. At the same time, there would probably be an uneasy feeling in using these methods, without investigating the extent of the error in the particular case, as this would certainly vary in different circumstances. Mr. Tinner had followed the course remarked upon by Mr. Lidstone in the discussion on Mr. King's paper (*J.I.A.*, vol. xxxix, p. 193), and by means of which most of the successful Pension Fund formulas had been obtained, of considering each year's contribution separately and following its course through the various years

of assurance. Now it was in these three later cases of the author's that this method, as a strictly scientific device, broke down. Where interest was involved, or the pension was based on final salary, the assurance term in regard to any particular contribution was not simply a function of that contribution and (or) of its accumulations but of other contributions.

Bearing in mind that this benefit was usually comparatively small in value, probably some might feel that there was in all this sufficient excuse for using a method which, while avowedly approximate, would be sufficiently clear and simple to enable its working to be followed in all its stages, and to feel satisfied with the result. In a valuation of a fund which had come before him on two occasions, where the return with interest was allowed, he had used a method which had at least the merit of requiring comparatively little labour. The working for the evaluation of the benefit for the whole society only took one-and-a-half pages of foolscap paper. He fixed on an average age at retirement, and then accumulated the whole of the existing members' contributions, past and future, up to this retiring age on the assumption that all would live, and divided this figure by the total pensions payable on the same assumption. This quotient gave the term for the decreasing assurance, which he then valued by the average $(D_{x+n} \div D_x)$ obtained from the actual figures of the valuation. He felt sure that this method gave fairly good results, and, as the value of this benefit in the particular case was only a little over 2 per cent. of the value of the ordinary retiring benefit, it was obvious that no decided advantage would be obtained by a straining after greater accuracy. Each fund of course presented problems of its own, and sometimes similar problems appeared in different guises; each actuary must, therefore, select or invent for himself the means of surmounting his own difficulties. By this time, there was certainly no lack of material from which to select, and the additions which Mr. Tinner had now provided would take a very important place among that previously available.

The latter part of the paper was devoted to the exposition of a valuation schedule of Mr. Tinner's own design. He could not help thinking the form would have been more self-explanatory if it had been printed the other way round on the paper, so that he could have shown all his columns on the same level. It was an ingenious device, by means of which he obtained the sum of t total salaries by a simple summation of the salaries themselves, thus saving the labour of multiplying each line by its appropriate value of t . The schedule shown was eminently suitable for the particular circumstances mentioned; and it was, of course, not feasible to produce a form that would be applicable to all cases. He gathered that Mr. King, in publishing his suggested schedules (*J.I.A.*, vol. xxxix, pp. 189-192), intended to meet the general rather than any particular case. The ordinary fund had some complexities, but often, he should think, not sufficient to require Mr. King's extended methods. and in regard to the case cited by Mr. Tinner, his method would certainly result in a great saving of work. By scheduling the members under age attained,

and by operating with the commutation columns, instead of valuation factors or multipliers, he obviated the necessity of dividing each function on each line by D_x . Speaking for himself, however, he might say that every fresh experience he had, whether from the theoretical or practical side, had confirmed him in a predilection for factors, rather than commutation columns, and although it seemed perhaps an unnecessary increase in the work to divide by D_x ten or twenty times instead of only once, he had always felt that it was well worth the extra labour. When you had the actual valuation factors before you, you could generally tell by inspection whether they were approximately correct, and you could also form an intelligent idea of what your final results should be. With commutation values, which had no meaning until divided by their appropriate denominator, you were to a large extent working in the dark, and the facility for detecting large errors at any stage short of the final one disappeared.

MR. GEORGE KING said that the paper was divided into two parts, the first of which was extremely complicated and one which would require very careful study and very careful weighing, to see how far it could really be made use of as a saving of labour, because he thought that even by empirical methods a very good approximation could be obtained to the value of the benefit in question. It had to be remembered that the weight of the benefit on the finances of pension funds was very much affected by various circumstances. To begin with, the contributions of members were very often badly graded according to entry age; at some ages too much was paid, and at some ages too little. If the members paid the whole of the contributions that would be very inequitable, but seeing as a rule the employer paid something like half, it might be taken that he paid, not for individual members, but for the benefit of the staff as a whole, and that his contributions might be taken to supplement those ages at entry where the payments were too small, and could therefore be taken away to a certain extent from those ages at entry where the contributions were perhaps too high. That was one reason for the difficulty of valuing the benefit. Another was that it also varied very much from other causes, such as the rate of early pensioning. The same ratio did not apply in the earlier years as in the later years, and in those funds where there was a very great amount of early pensioning, the value of the benefit would be very different from what it was in the case of funds where there was but little early pensioning, and the pensions mostly were taken at the normal pension age. He had found that the weight of the benefit varied very much in different funds; in some it was very small indeed, perhaps not more than 3 per-cent of the value of the prospective pensions; in others it had gone up to as much as 8 or 9 per-cent, or perhaps even more. It was sometimes difficult to explain why that should be so, and it required to be very carefully looked at.

With regard to the second part of the paper, he thought Mr. Tinner's suggestions of a different way of tabulating the data might

in some cases be useful, but it should be remembered that he applied them only to what might be called the terminal salaries, whether the actual final salary, or whether the average of the last two, three, or five years. He did not know whether Mr. Tinner had investigated the matter to find whether he could make his method applicable to the average salary plan? Funds now were much more numerous with average salary than with terminal salary pensions, and in fact there were several funds he had dealt with in days gone by which used to have the terminal salary system, but which had changed to the average, and personally he always tried to introduce, where possible, the average plan, because it was so much more stable and so much less apt to experience violent fluctuations through changes in rates of superannuation and salary and so on. Really, however, one became a little alarmed at seeing the immense number of operations that had to be performed. However, he ventured to think that Mr. Tinner rather exaggerated the number of operations in his (Mr. King's) method, for the simple reason that he hardly ever used logarithms at that stage, but found it very much quicker to use the arithmometer.

Another point in that connection was that, if the pension factor was used, there were very much smaller multipliers than when using the commutation columns. It would be observed that in Mr. Tinner's table on page 373, he gave the M that was required as the numerator of the valuation factor, but he (Mr. King) used only the factor itself after dividing by the D; and that generally was a number having three significant figures, or four at the outside. As a matter of fact, the labour in valuing the funds was due to the analysis of the cards and the tabulating of the data, and taking out the experience, and once that was effected he felt that the work was nearly over, the calculation of the factors themselves being not much more than a question of hours, and the multiplying a very short process. But all the same he did not dispute that there might be shorter ways of making tabulations and doing the work, and he would welcome anything of that kind, because it was very important to save labour to the utmost possible extent. There was also, he thought, one possible advantage in the author's new method of tabulation, which he did not think Mr. Tinner had mentioned, namely, that it might make it easier to get out the grand final summary which was required for valuing future contributions, returns of contributions, and so on. With his own method of using entry ages, tabulating first by ages at entry, and then by ages attained, he had to pick out all the ages attained in the intermediate schedule and sum them to get the final grand summary. With Mr. Tinner's method, he thought it was quite possible that that work might be obviated, because the attained age was at once found in the schedule.

MR. A. W. WATSON said he wished to express some measure of disagreement with one suggestion of Mr. King. This dealt in a measure with the politics of pension funds, and he should not like it to go forth as an expression of anything like a unanimous actuarial view of the contributions to such funds. It was true that one might

regard the contribution of the employers as a contribution made to the whole fund and intended to make good the deficiency in the contributions of the members taken as a whole, but his experience was that the members of the funds themselves did not regard the contribution as one which was paid for the whole fund, but that each member regarded the $2\frac{1}{2}$ per-cent, or other stipulated proportion, which was ostensibly paid by the employer on his behalf, as a contribution made in supplement of his own salary, and as, generally speaking, the men at the bottom were paying less insufficiently than the men high up, any suggestion to the men of the lower grades that the contribution ostensibly paid on their behalf was really intended to secure the solvency of the whole fund and was not needed for them particularly, he was quite sure would be met with resentment.

With regard to the relative importance of the function which Mr. Tinner had investigated, it was quite true that the function was of less significance than one might imagine it to be from the extent of Mr. Tinner's work, because all such funds were influenced by the increase of salary; he ventured to say that there was far greater scope for error of approximation in the use of a scale of salaries than would be compensated by extreme accuracy in the valuation of this particular benefit. He quite admitted that the use of a salary scale was necessary in very many cases, but he thought it ought to be admitted also that there was probability of departure from the estimates in the actual working, and that consequently a certain amount of latitude in respect of the valuation of subsidiary benefits was admissible. There should be no affectation of accuracy which could not be sustained when the methods were microscopically examined. In saying that he did not wish to detract from either the skill or industry of Mr. Tinner, whose extremely interesting formulas would prove educative to the student and mentally stimulating to the practising actuary, who would be certainly tempted by some of them to reconsider his customary working processes.

MR. H. W. MANLY, referring first to Mr. Watson's observations with regard to the disposal of the contributions of the employer in the funds, admitted it was a difficult subject, but said his own view of the matter was that if the member who remained in the service and then retired received the full value, as a deferred annuity, of his own contributions and of the similar contributions of the employer, he could not ask for more. Profit was made out of the withdrawals, when the contributions of the member were returned without interest, and also on death in the service, when the contributions of the member and the like contributions of the employer were returned without interest, and he thought that those profits might be legitimately applied to make up the possible deficiencies of the contributions of the other members, who, according to the theoretical calculations, did not pay enough. He knew there had been, and he supposed there still was, a feeling amongst the members that the contributions of the employers were individual, and that was unfortunately justified by an early rule which was adopted in the funds, that on death the contributions of the member, and the

contributions of the employer on his behalf, should be returned to the representatives. But in recent years there had been a tendency, where there had been shown to be a large deficiency, for the employer to agree to subscribe more, and sometimes much more, than the members. To avoid any question about the rule, a supplementary fund formed by these extra contributions has been created, but if it was going to be suggested that these extra contributions were contributed individually, he was afraid it would bring about a very difficult position. When the employer guaranteed interest at 4 per-cent, he thought the member on retirement received ample value for his contributions if he were secured the deferred annuity which twice his contributions would purchase at that rate of interest.

With regard to the paper itself, when he first read it he looked on Mr. Tinner as a very bold man, and thought that possibly he had gone beyond him (Mr. Manly) in audacity. But on looking at the paper further he discovered that he had not gone beyond his own suggestion in the paper which Mr. Thomas had referred to, where he assumed that the values in the C_x column of the commutation tables were constant for three or four years. Now Mr. Tinner asked the members to accept the proposition that in a Life Table representing the mortality amongst invalid lives the values of d were constant for any group of consecutive lives up to seven. Granting that, then the whole of Mr. Tinner's formulas naturally followed and might be accepted as correct. For the greater part of an ordinary table that proposition might be accepted, for Mr. Tinner assumed that the probability of dying did increase slightly with age, but in an Invalid Life Table, such as his (Mr. Manly's) own Table 8, the probability of dying decreased with the increase of age up to sixty. Consequently Mr. Tinner's formula for the value of a decreasing assurance based on the proposition that such value varied as the square of a period during which there was a risk, was, for the working part of the table, too large. That was distinctly on the safe side ; but what was the difference in practice ? So far as his own experience went, and it was fairly extensive, the longest period during which there was any risk was probably about three years. that was when c , the contribution, was 5 per-cent, and $\frac{1}{4}$, the rate of pension, was about $1\frac{2}{3}$ per-cent of the total salary. The heaviest risk was during the first year, when it started, say, at 5, and decreased by the end of the first year to $3\frac{1}{3}$; in the second year it decreased from $3\frac{1}{3}$ to $1\frac{2}{3}$, and during the third year from $1\frac{2}{3}$ to nothing. So that roughly there was an assurance for the first year of $4\frac{1}{2}$ per-cent, for the second year $2\frac{1}{2}$ per-cent, and for the last year $5\frac{1}{6}$ ths per-cent of the contributions. That was in the proportion of 417, 215 and 82.

Taking those figures, and assuming that they were numbers at risk, it would be found that, at age 35, according to his Table 8, the deaths should be 67·3, whereas on Mr. Tinner's assumption they would be 71. a difference of about 5 per-cent. Turning to the table at the end of the paper, and taking age 35, and multiplying the value for ($n=1$), .045 by 9, .402 was found for the value of ($n=3$), but the true value was .372, showing that Mr. Tinner's approximate

value was about 8 per-cent too much, but at ages 55, 60 and 65, when the majority of the retirements took place, it would be found that nine times the values for $n=1$ were very little more than the actual values for $n=3$. There were no select tables for invalid lives, but general observation pointed to there being a very heavy mortality in the first year or two, and that it became very much lighter afterwards. If that was the case, then the effect of an aggregate table of invalid lives was the exact opposite of the introduction of select lives into an ordinary aggregate table. The aggregate mortality was improved by the inclusion of the better mortality of the partially or wholly recovered invalids. It was probable, therefore, that the values of the decreasing assurance for three years by Mr. Tinner's formula were really nearer the truth than those obtained from the aggregate table. Having thought the matter over very carefully and thoroughly, he had unhesitatingly accepted Mr. Tinner's proposition, and had approved the adoption of his formula in the valuation of a fund. It might be remembered that in one of his (Mr. Manly's) papers he pointed out that where the contributions were 5 per-cent and the pensions $1\frac{1}{2}$ per-cent of the total salary the risk was covered by an addition of $1\frac{3}{4}$ per-cent of the value of the pensions. It might be said that that was a very small figure. Assuming even the largest error in Mr. Tinner's assumptions it would probably only change the 1·75 into 1·8 or 1·85. After all, the calculations could be no more than approximations—as Mr. Watson said, a scale of salaries was constructed and graduated, and it was assumed that that applied to every case, which was not always correct. Really, it was necessary to base all the calculations on averages, and when it was admitted that all the results were only approximations, whether this particular benefit was taken as 1·7 or 1·8 or 1·85 or even 1·9 per-cent of the value of the pensions did not very much matter, and Mr. Tinner's values might be as true, if not more correct, than more accurate calculations made them out to be.

Mr. Thomas had referred to the formulas, and had pointed out their symmetry and how they fitted in exactly with the formulas which he (Mr. Manly) produced in his paper and Mr. King produced in his. With regard to the application of his assumption to the various conditions of a fund, Mr. Tinner took the case of the return of total contributions without interest and pension based on average salary, and also the case of contributions with interest and pension based on average salary, and similarly contributions with and without interest, and pensions based on final salary. Mr. Tinner suggested a general proposition, that each year's contribution insured the payment of pension of $\frac{1}{2}$ times that year's salary and the corresponding balance (if any) on death after entering upon pension. At first sight that seemed rather a large proposition, but when examined it would be found to be absolutely true in the case of contributions without interest and pension based on average salary; but with regard to contributions with interest and pensions based on average salary it did not seem so obvious. As a matter of fact,

when compound interest was added to the contributions, then as t , the years of service, increased, n , the time during which the decreasing assurance is payable, also increased, but not in the same proportion, and the effect was that the error grew larger as the membership grew longer, and so in the Table of Comparison, which Mr. Tinner had given, the largest error is when t is 35. But, taking into consideration the smallness of that error, he thought the proposition could be accepted without difficulty. The case was different when pensions were based on final salary, for, as the contributions without interest increased, the ratio to the final salary decreased, and thus there was a tendency for the error to appear on the other side, but there again the difference was so small that it could be completely ignored.

With regard to the Schedule at the end of the paper, it looked a somewhat formidable document, and he should have doubted the efficacy of it, had not he seen it in use. Where there was a fixed percentage of final salary as pension, there was evidently a very great saving, and, as Mr. King had pointed out, where a schedule of the values for each age attained had to be produced, there was a still greater saving in time and labour. But he thought Mr. King evidently overlooked the paragraph above the Schedule, in which Mr. Tinner said it was only of advantage when the proportion of terminal salary paid as pension varied directly as the duration. He did not know whether Mr. Tinner had ever tried to apply that form to pension based on average salary, or on a variable percentage of average salary, but he should think it would be practically impossible, and that in such cases it would be found that Mr. King's method gave the shortest way to the results.

THE PRESIDENT, in proposing a vote of thanks to Mr. Tinner for his paper, expressed his own appreciation of it and also his extreme regret that, owing to very great pressure of work, he had not been able to give it the prolonged and careful study which alone could justify him in entering into any detailed criticism either of the first part of the paper, dealing with the methods of the subject, or of the second part, dealing with the separate problem of the Schedules. But he had read the paper through, and could congratulate Mr. Tinner on having dealt with a very difficult subject with a great deal of skill and ingenuity. As to the fact, referred to once or twice, that the results arrived at were only approximate results, he fully sympathised with what had been said by Mr. Watson and Mr. Manly on that subject. Taking the extreme case, which Mr. King had noticed, of a society in which the particular benefit amounted to about 9 per-cent of the total liabilities of the society, and supposing that Mr. Tinner's formulas even produced an error of 5 per-cent in the calculations, a result was arrived at which was within $\frac{1}{2}$ per-cent of the total result of the valuation, and he thought anyone who had had any experience of the valuation of pension funds would feel extremely happy if they felt convinced in their own minds that the result they brought out was within $\frac{1}{2}$ per-cent of the ultimate working out of the fund. He agreed with what had been said as to welcoming

one who had carried the torch of actuarial science outside the limits of the insurance world, and who was endeavouring to bring technical and scientific skill into the work of municipal life.

The motion was unanimously carried.

MR. TINNER, in replying, said that he anticipated considerable criticism of the first part of the paper, because the method outlined therein was obviously only approximate. He was led to consider it in the first place from the point of view of ascertaining the rate of contribution to a fund. Mr. Manly's formula for the benefit, for instance, was not very easy to apply, if he might say so with all deference, and did not give effect to the varying mortality at the different ages at which members might be pensioned : further, they had the additional drawback that the mortality was probably underestimated, because it was assumed to be the mortality of the higher ages, whereas the mortality became lighter as the age of early retirement increased. The formulas got over that difficulty to some extent. They were really devised in the first place for the purpose of ascertaining the rate of contribution, and nothing more ; and he believed they did it more effectively than any system he had so far seen. The section dealing with the application of the formulas to a valuation was written after the first part of the paper was produced, and more for the sake of making the paper complete theoretically than for practical use. Personally, he should be inclined to use some method of approximation rather than apply them to an actual valuation. To ascertain the rates of contribution allowing for this special return was very difficult, and by the time one had made some more or less empirical calculations it would probably be found more satisfactory, and to save time, to use functions somewhat on the lines put forward in the paper.

With regard to the question of the application of the formulas to the case where the proportion of the average salary or final salary for each year of service varied irregularly, he thought that they were in precisely the same position as they were with regard to the valuation of the pensions themselves : it was necessary to produce a separate Valuation Table or Commutation Table for each age at entry. Mr. Thomas had stated that he could get results equally accurate with much less work, but that remark, of course, applied only to the work of valuation, not to ascertaining the rates of contribution. There was a saving in work, but he did not think he should be so convinced that the results were accurate, because, as far as he could see, Mr. Thomas had no means of testing the accuracy of his work. With reference to the schedule for the valuation of pensions based on terminal salaries, Mr. Thomas had referred to the advantage of having valuation factors, rather than applying commutation functions, but he could not help thinking that, if the work was done by reliable men, that advantage was more apparent than real. Mr. King had asked whether a modification of the Schedule could be produced dealing with the case of pensions based on average salary. Where the proportion of average salary taken as pension varied directly as the number of years' service, the problem

was fairly simple, but where the proportion of the average salary salary varied irregularly he did not see how it was to be solved, although he should not like to say that it was insoluble. Mr. King also referred to the comparison of the number of operations involved by his (Mr. King's) method and by that shown in the paper. The saving of work by the use of the schedule was certainly considerable, and it should be remembered that every operation saved meant a risk of error avoided, and from that point of view alone the saving was very desirable. Mr. King rather took exception to his comparison of the number of operations, on the ground that he (Mr. King) did not as a rule work with logarithms but with the arithmometer; but in making the comparison he was guided by the specimen table given at the end of Mr. King's paper on Pension Funds read at the beginning of 1905 (*J.I.A.*, vol. xxxix., p. 188). Mr. Manly had pointed out that in certain cases the error involved in the formulas in the first part of the paper grew larger with the increase of t ; but on the other hand it should be remembered that, in calculating the rate of contribution by means of the formulas, the error involved would not be the largest one, but would be a mean between the largest and the smallest, since all the values of the commutation function were brought into account from the age of entry to the final age. He did not regard the formulas as perfect—far from it; nor did he wish to minimise any disadvantages they had. It was with the object of exhibiting defects that he gave the comparative figures showing in certain cases approximate values of $(\bar{D}A)^{(n)}$ derived from other values of the function in conjunction with the true values, and those showing the error involved by the assumption that each year's contribution purchased a separate pension.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., Barrister-at-Law.

Application for return of statutory deposit of £20,000 on transfer of business. THE question of the circumstances in which the statutory deposit of £20,000 made by a life assurance company under the provisions of the Life Assurance Companies Act, 1870, can be withdrawn upon the

transfer of the business of the depositing company to another company, came before the Courts recently in the case of *In re Life & Health Assurance Association, Limited* (In Liquidation) [1910] 1 Ch. 458. This association was formed under the Companies Acts in 1898 for the purpose, *inter alia*,

of carrying on the business of life assurance, and the statutory deposit of £20,000 was accordingly made by it on 1 March 1898. On 14 May 1909, a resolution was passed for the voluntary winding-up of the association, and by an order of the Court of Session (First Division) in Edinburgh, dated 2 June 1909, the voluntary winding-up was directed to be continued subject to the supervision of the Court. In July 1909, pursuant to a special resolution duly passed, the liquidators entered into a provisional agreement for the transfer of the life assurance business of the association to the General Accident Fire and Life Assurance Corporation. By this agreement all the life assurance contracts of the association current on 10 May 1909, were to be transferred to and taken over by the purchasing company, the latter undertaking all liabilities relating to, and being entitled to all the premiums payable in respect of such assurances as from that day. A sum of £1,000 was also to be paid by the purchasers to the vendors for the goodwill of the business. The vendors were to remain liable for all outstanding claims which had arisen under life assurance contracts prior to 10 May, all of which claims had been discharged when the case came before the Court, and bound themselves, if the proposed transfer was sanctioned by the Court, to join in an application for payment to the purchasers of the sum of £5,500, representing the actuarial value of the policies transferred as at 10 May, with interest at $3\frac{1}{2}$ per-cent from that date, out of the statutory deposit made by the vendors on the formation of the association. On 27 September 1909, the transfer was confirmed by the Scottish Courts under section 14 of the Act of 1870, and on 13 October the £1,000 for goodwill was paid, and the transfer completed. The Life and Health Assurance Association and its liquidators, and the General Accident, Fire and Life Assurance Corporation then joined in a petition to the Court asking that the sum of £5,500 might be paid to the purchasing company as arranged, and that the balance might be paid to the liquidators of the association. The life assurance fund accumulated by the association out of premiums had never amounted to any sum approaching £40,000, but the purchasing corporation had accumulated out of premiums a life assurance fund of upwards of £42,000.

On the petition coming before Eve, J., in February last, he declined to make the order asked for, holding that the association was not entitled to the repayment of any part of the deposit of £20,000 until all its policyholders had released their claims

against the association and the deposit in the manner provided by the Life Assurance Companies Act, 1872. In delivering judgment to this effect, he said :—“ It is urged that inasmuch as the purchasing company will, if an order be made on the petition, receive out of the deposit the full sum which the transferred policyholders would have been entitled to receive thereout in the liquidation of the association, the Court ought not to retain the deposit or any part thereof ; or, putting it in another way, it is argued that the excess of the deposit over and above the sum necessary to satisfy the claims of the policyholders in the liquidation constitutes an asset of the association available for the payment of its debts and the discharge of its liabilities on contracts other than life assurance contracts, and ought now to be paid to the liquidators to be so applied.

“ However convincing these contentions might have been had the liquidation of the association proceeded on other lines—that is to say, on the lines of determining the contracts with the assured and paying them the values of their several policies—I cannot regard them as in anyway apposite to the condition of things which has been brought about by the transfer of the contracts to the purchasing company.

“ In truth and in fact there has been no liquidation properly so-called in regard to the current policies. There has been a transfer of the business, and a transfer, I may add, under

Provisions of
Act of 1870 as to
transfers apply
to a company in
liquidation.

“ section 14 of the Act of 1870, for I think it is hopeless to contend that that section does not apply to a transfer made by a company in liquidation exactly as it applies to a transfer made by a

“ company as a going concern. This being the true nature of the transaction, what is there, apart altogether from any statutory protection for the policyholders such as is to be found in section 7 of the Act of 1872, which entitles the insurers, in the absence of a release or of novation on the part of the assured, to take to themselves any part of the fund which, by the contract of assurance, is appropriated as a security for the policyholder ? The association could not, by transferring the contracts, relieve itself of its liability thereunder, and, on general principles, I should have been prepared to hold that no part of the deposit could be paid to the association until the Court was satisfied that each and all of the assured had released the association from liability, and thereby abandoned their contractual right

" to have the deposit retained to answer the claims under the " policies.

" But the Legislature has not been content to leave the " matter to be dealt with on general principles alone. Fore- " seeing that endless disputes might arise upon the question " whether in fact the assured by conduct had released the " transferring company, Parliament, by section 7 of the Act " of 1872, expressly enacted that where a company has trans- " ferred its business to another company, no policyholder in the " first-named company who shall pay to the other company the " premium accruing due in respect of his policy shall, by reason " of any such payment, or by reason of any other act, be deemed " to have abandoned any claim which he would have had against " the first-mentioned company on due payment of premiums " to that company, or to have accepted, in lieu thereof, the " liability of the other company, unless such abandonment and " acceptance have been signified by some writing signed by " him or by his agent lawfully authorised. No such abandonment " and acceptance by the policyholders of the association are sug- " gested, and accordingly they still retain their full rights against " the association and the deposit, and I have no power to order " any portion of it to be paid to the association or their assignees. " the purchasing company. In so deciding I am adopting in " part the reasoning underlying the decision in *Ex parte Scottish* " *Economic Life Assurance Society* (1890, 45 Ch. D. 220, *J.I.A.*, " vol. xliii, p. 226), and in no way whatever differing from the " judgment in *In re Popular Life Assurance Company* ([1909] " 1 Ch. 80, *J.I.A.*, vol. xliii, p. 224) or the reasons supporting it."

Power of a
contingent or
prospective
creditor to present
a petition for
winding-up.

The case of *In re British Equitable Bond and Mortgage Corporation Limited* [1910] 1 Ch. 574, which deals with the power of a bondholder in a bond investment company to present a petition for winding-up, is of interest for life assurance officials, and incidentally throws some light on the powers of policyholders under the new Assurance Companies Act, 1909. A bondholder in the British Equitable Bond and Investment Corporation held certain bonds under which, in consideration of monthly payments extending over 10 years, a fixed sum was payable at the end of that period. On 18 December 1909 the bondholder presented a petition under section 137 of the Companies (Consolidation) Act, 1908, for an

order that the corporation might be wound-up, on the ground that it was insolvent and unable to pay its debts. At the date of the petition all of the petitioner's bonds were in full force and effect, but none of them had matured. By section 28 of the Companies Act, 1907, which is reproduced partly in section 130 (iv) and partly in section 137, subsection 1 (c) of the Act of 1908, it is provided that any contingent or prospective creditor shall be a creditor entitled to present a petition for winding-up.

Neville, J., before whom the case came, held that the bondholder was a contingent or prospective creditor within the meaning of section 137 of the Act of 1908, and could therefore present a petition for the winding-up of the corporation. In delivering judgment to this effect, he said :—“ No doubt originally the ‘Companies Acts did not allow of the presentation of a petition “for winding-up by a contingent or prospective creditor, and “that continued to be the law until the passing of the Act of “1907, but the Life Assurance Companies Act of 1870, section “21, provided that the Court might order the winding-up of any “assurance company in accordance with the Companies Act, “1862, on the application of one or more of the policyholders “or shareholders, subject to certain conditions, and the section “goes on to say, ‘In determining whether or not the company “‘is insolvent the Court shall take into account its contingent “‘or prospective liability’, and so on. Then in the Companies “Act, 1907, section 28 is introduced, which I think is not without “reference to section 21 of the Life Assurance Companies Act, “1870. That section provides ‘In determining whether a “‘company is unable to pay its debts within the meaning of “‘section 80 of the Companies Act, 1862, the Court shall take “‘into account the contingent and prospective liabilities of the “‘company, and any contingent or prospective creditor shall “‘be a creditor entitled to present a petition for winding-up the “‘company under section 82 of that Act’, and so on. Now “those provisions of section 28 are to be found in the Act of 1908, “but instead of being both contained in one section, they are to “be found in different sections of that Act, and I have no doubt “that the words ‘contingent or prospective creditor’ in the “Act of 1908 bear no other interpretation than that which “would be applied to contingent or prospective creditor in the “Act of 1907; consequently I have to consider what was the “extent of the meaning of the words when they were introduced “into the Act of 1907. It is not denied that such a liability as

" the present is a contingent or prospective liability of the company
" which would have to be taken into consideration under section
" 21 of the Act of 1870, but it is said that the contingent and
" prospective liabilities of the company which have to be taken
" into account in considering insolvency under section 21 are
" different in extent from the claims of a creditor described as
" 'any contingent or prospective creditor' in section 28 of the
" Act of 1907. I cannot think that is so. I think . . . 'con-
" tingent or prospective creditor' includes such a person as a
" policyholder and such a person as the petitioner in the present
" case, whose position does not seem to me distinguishable from
" that of the holder of a policy. It is said that is inconsistent
" with a statutory provision which is made by the Act of 1909
" (Assurance Companies Act), which will not come into force
" until July of that year, which provides (section 15) that the
" Court may order the winding-up of an assurance company
" in accordance with the Companies (Consolidation) Act, 1908,
" 'on the petition of ten or more policyholders owning policies
" of an aggregate value of not less than ten thousand pounds.
" Provided that such a petition shall not be presented except by
" the leave of the Court, and leave shall not be granted until a
" prima facie case has been established to the satisfaction of the
" Court and until security for costs for such amount as the
" Court may think reasonable has been given.' It is said
" that that is an enabling section and not a limitation of the
" general right under the Act of 1870. It seems to me that is
" not so, and that section 15 is in fact a section which limits the
" right of policyholders to the particular case provided for by
" that section. Apart from the question of the Act of 1908,
" they had, undoubtedly, under the Act of 1870 a right to present
" a petition for winding-up, and I think that the operation of this
" section 15 in any event must be to limit the general right given
" under the Act of 1870, that is to say, with regard to companies
" which come within the definition clause (section 1) of the Act
" of 1909 as applied to cases of this kind under section 15. I
" think, therefore, that the contingent or prospective creditors
" mentioned in section 137 of the Act of 1908 includes the present
" petitioner. There is a note to that section in Buckley on
" Companies, 9th edition, page 318, which, referring to the
" words 'contingent or prospective creditor', says 'including
" 'under the Life Assurance Companies Act, 1870, the holder of
" a current policy,' and then there is a reference to the Life

" Assurance Companies Act, 1872. If section 137 does include
 " the holder of a current life policy, then the right to wind-up is
 " not derived under the Life Assurance Companies Act, 1870.
 " If it is only derived under the Life Assurance Companies Act,
 " 1870, then it is not included in this section ; and therefore it
 " seems to me that the brevity of that note somewhat interferes
 " with its lucidity, and I cannot get much assistance from it.
 " Independently of that note, I come to the conclusion that the
 " present petitioner is included in 'any contingent or prospective
 " 'creditor' mentioned in the Act of 1908. The petition must
 " proceed."

Income tax in respect of provision for unexpired risks. The question of the liability of assurance companies to income tax in respect of the provision made for unexpired risks was again before the Courts in the recent case of *Ernest Clark* (Surveyor of Taxes) v. *The Sun Insurance Office*, 1910, 26 T.L.R. 341. The material facts of the case, which came before the King's Bench Division by way of appeal by the Surveyor of Taxes from the decision of the Income Tax Commissioners for the City of London, are as follows :

The respondent company carries on the business of fire insurance and has, since 1888, in addition to other reserves, been in the habit of carrying forward annually 40 per-cent of its yearly premium receipts in respect of its unexpired risks. This provision amounted, at December 1901, to £466,138, and rose, during the triennium on which the assessment appealed against was based, to £522,472. The increase thus amounted to £56,334, and consequently affected the return for the tax year by one-third, or £18,778. This latter sum was the only matter in dispute, the assessment of the office being otherwise agreed at £144,452, which the insurance office claimed to reduce by £18,778, to £125,674. The Commissioners having decided in favour of the insurance office, the Crown appealed. On the appeal coming before the Court, it was contended on behalf of the insurance company, *inter alia* (a) that the premiums are earned, not by being received, but by the office remaining at risk for the term covered by the premium, and that the company is justified in throwing into each year the risk-bearing premium which belongs to it ; (b) that 40 per-cent is in fact, and is recognised by the best informed opinion of the insurance world as being, the irreducible minimum rate for such an allowance, and that the estimate of

40 per-cent for the year of assessment had itself proved greatly insufficient by reason of the losses in respect of the San Francisco fire, and similarly in other years. On behalf of the Crown it was contended, *inter alia*, that no allowance can be made for unearned premiums, but it was admitted that if the insurance company was entitled to such an allowance, then the 40 per-cent claimed was, in this instance, an accurate measurement of the reserve required for the purpose. On the case coming before Bray, J., he dismissed the appeal and gave judgment in favour of the insurance company. In doing so, he said that but for the difficulty raised by the decision in the case of General Accident, Fire, and Life Assurance Corporation *v.* McGowan ([1908] A.C. 207, *J.I.A.*, vol. xlii, p. 407) he would have had no hesitation in holding, from an income tax point of view, that an insurance company was justified in making the usual provision in respect of unexpired risks. "It seemed to him that this was not a "question of law at all, but a question of fact, to be determined "by evidence. But it was said that there had been a certain "practice adopted many years ago (by the Inland Revenue "authorities) and that it was too late to interfere with it now. "Of course if he could find that the House of Lords had decided "that, he would be bound by their decision. It was quite true "that Lord Macnaghten and Lord Collins had said so, but Lord "Loreburn did not, and there was nothing to show that the other "noble lords had agreed with Lord Macnaghten and Lord Collins. "Therefore what they said was mere *dicta*.

"It seemed to him that the facts in each particular case "must be looked at. It was necessary for the respondents to "prove that a certain sum was the proper sum to be deducted "and that it truly represented the liability. The Commissioners "had found as a fact that the respondents were entitled to carry "a percentage of premiums received in one year forward in "reserve in respect of unearned premiums not forming part "of the profits of the year, and that 40 per-cent was a reasonable "and proper allowance." Then, referring to McGowan's case, he said :—"It was quite clear that (in that case) there was no "finding of fact as there was here. There was only the statement "of the practice of the company to deduct $33\frac{1}{3}$ per-cent. It "could fairly be said that the company had not proved, as they "were bound to prove, that their deduction was fair and proper, "and that this was the ground of the decision appeared from the "judgment of Lord Loreburn, at page 211, and at page 212 he

“ said, after stating that the method of assessing fire insurance “ companies adopted by the Commissioners had existed for “ 32 years, ‘ If in any particular case an insurance company “ can show it works hardship, no doubt the rule ought to be “ modified so that the real gains and profits may be ascertained “ as near as may be.’ . . . If evidence was given that it was “ possible to nearly estimate the proper allowance . . . he did “ not think there was anything in McGowan’s case which prevented “ him from saying that the method adopted by the respondents “ was proper. It seemed to him that the respondents had shown “ that they had been asked to pay on £18,000 too much. . . He “ had only one more matter to deal with—namely, the Assurance “ Companies Act, 1909, which obliged these companies to make “ out their accounts in a certain way, and one item was ‘ reserve “ for unexpired risks’, and it was a deduction which must appear “ in the revenue account of the year. So that any inconvenience “ which might be caused by changing the mode of making out “ accounts could not be a matter of importance, for in future this “ mode must be adopted. Of course the Act could have no effect “ on the question before him ; still it did show that what the “ respondents had done was a possible and proper way of making “ out accounts. If the House of Lords in McGowan’s case had “ had the facts before them as he had had them, he thought they “ would have come to a different conclusion. In his opinion in “ this particular case a proper deduction was made, and the appeal “ must be dismissed.”

ACTUARIAL NOTE.

Practical Hints to Students on the application of the formula for Integration by Parts to Life Contingency problems.

THE well-known formula for integration by parts, which is so frequently employed in mathematical investigations of all kinds, has been specially prolific in its application to life contingency problems, for reasons which will be discussed later. A number of the most important standard formulæ are, in fact, deduced by this method, as every reader of the *Text-Book*, Part II, knows. Nevertheless, there is reason to think that many students do not properly appreciate the ease and certainty with which the formula

can be applied, and that, owing to a lack of systematic ideas on the subject, they are rather inclined to view each individual application as a separate conjuring trick which must be learnt by heart. The object of this note is to show how a more systematic knowledge of the subject may be obtained, and in particular:

1. To replace the ordinary expression for integration by parts by a working formula, or rather, two intimately related formulæ, which can be applied with the greatest ease.
2. To show how these formulæ can be deduced in a most elementary way by means of finite difference formulæ which pass into formulæ of the calculus when the interval of differencing is made indefinitely small.
3. To give some applications of the working formulæ, showing that by their means many standard results and also other interesting forms may be readily obtained.
4. To give some hints to the student as to the most advantageous shape in which to arrange the expressions which it is desired to integrate.

Demonstration of the Working Formulae.

(2). Consider first the following series in which the variable takes all values from a to b , proceeding by intervals of h :

$$h^2[U_a V_a + U_{a+h}(V_a + V_{a+h}) + \dots + U_b(V_a + V_{a+h} + \dots + V_b)] \\ \text{i.e., } \sum_a^b U_t(\sum_a^t V_t, h)h$$

$$= h^2[V_a(U_a + U_{a+h} + \dots + U_b) + \dots + V_{b-h}(U_{b-h} + U_b) + V_b U_b] \\ = \sum_a^b V_t(\sum_a^b U_t, h)h$$

Hence,

$$\sum_a^b U_t(\sum_a^t V_t, h) \cdot h = \sum_a^b V_t(\sum_a^b U_t, h)h \quad \dots \quad (1)$$

Now make h infinitely small, when $\sum \dots h$ becomes $\int \dots dt$, and we have

$$\int_a^b U_t \left(\int_a^t V_k \cdot dk \right) \cdot dt = \int_a^b V_t \left(\int_t^b U_k \cdot dk \right) dt \quad \dots \quad (2)$$

(3.) Now put v_t for V_t and $\frac{du_t}{dt}$ for U_t , whence, by definition

$$\int_t^b U_k dk = \int_t^b \frac{du_k}{dk} dk = u_b - u_t, \text{ and we have}$$

$$\begin{aligned} \int_a^b \frac{du_t}{dt} \left(\int_a^t v_k dk \right) dt &= \int_a^b v_t / (u_b - u_t) dt \\ &= u_b \int_a^b v_t dt - \int_a^b u_t v_t dt \end{aligned}$$

$$\therefore \int_a^b u_t v_t dt = u_b \int_a^b v_t dt - \int_a^b \frac{du_t}{dt} \left(\int_a^t v_k dk \right) dt \quad . \quad [A] *$$

(4.) Again, put v_t for U_t and $\frac{du_t}{dt}$ for V_t , so that

$$\int_a^t V_k dk = \int_a^t \frac{du_k}{dk} dk = u_t - u_a$$

and we have

$$\int_a^b v_t (u_t - u_a) dt = \int_a^b \frac{du_t}{dt} \left(\int_a^b v_k dk \right) dt$$

or

$$\int_a^b u_t v_t dt - u_a \int_a^b v_t dt = \int_a^b \frac{du_t}{dt} \left(\int_a^b v_k dk \right) dt$$

or

$$\int_a^b u_t v_t dt = u_a \int_a^b v_t dt + \int_a^b \frac{du_t}{dt} \left(\int_a^b v_k dk \right) dt \quad . \quad [B]$$

(5.) Formulae A and B are the fundamental working formulae. They may also be obtained :

(i) From the ordinary formula for integration by parts,

namely, $\int U \frac{dV}{dt} dt = UV - \int V \frac{dU}{dt} dt$, by putting

$U = u_t$ and $V = \int_a^t v_k dk$ and $-\int_t^b v_k dk$ respectively.

* This formula was given, *J.I.A.*, xxxiv, 249, by Mr. G. J. Lidstone.

(ii) As the limit of the finite sum

$$h[u_ar_a + u_{a+h}r_{a+h} + \dots + u_b r_b]$$

substituting $u_a + \delta u_a + \delta u_{a+h} + \dots + \delta u_{t-h}$ for u_t , or

alternatively substituting $u_b - \delta u_{b-h} - \delta u_{b-2h} - \dots - \delta u_t$ for u_t ;

and then rearranging the terms, expressing them in finite sums (Σ) and passing to the corresponding integrals (\int).

(6.) The student is recommended to commit to memory not only the formulae A and B, but also the differential co-efficients and indefinite integrals of the principal elementary functions which are likely to occur in the application of the formulae. These are collected in the following statement. It will be an assistance in remembering A and B to notice that when the first term on the right-hand side involves u_b (where b is the higher limit of the principal integration), the integral in brackets is taken negatively from a (the lower limit) to t ; and when the first term involves u_a , the integral in brackets is taken positively from t to b .

Reference Table of Standard Forms for the application of the method of Integration by Parts to Life Contingencies.

Function of Independent Variable t	Differential Co-efficient	Integral
u_t	$\frac{du_t}{dt}$	$\int u_t dt$
1	0	t
r^{x+t}	$-\delta, r^{x+t}$	$-r^{x+t}, \delta$
l_{x+t}	$-\mu_{x+t}, l_{x+t}$	$-l_{x+t}, r_{x+t}$
$D_{x+t} = r^{x+t}, l_{x+t}$	$-(\mu_{x+t} + \delta) D_{x+t}$	$-\bar{N}_{x+t} = -D_{x+t}, \bar{d}_{x+t}$
$r^{x+t}, \mu_{x+t} l_{x+t}$...	$-\bar{M}_{x+t} = -D_{x+t} \bar{N}_{x+t}$

$$\begin{aligned} \int_a^b u_{x+t} \cdot r_{x+t} dt &= u_{x+b} \int_a^b r_{x+t} dt - \int_a^b \frac{du_{x+t}}{dt} \left(\int_a^t r_{x+k} dk \right) dt \\ &= u_{x+a} \int_a^b r_{x+t} dt + \int_a^b \frac{du_{x+t}}{dt} \left(\int_t^b r_{x+k} dk \right) dt \end{aligned}$$

Note that the integrals in brackets ($\int \dots dt$) are functions of t in respect only of one of the limits of integration.

(7). If the function to be integrated consists of only two factors, either of these can be treated as u and the other as v ; and then either Formula A or B can be applied, giving four different results in all. If there are more than two factors, they must be separated into two groups, and the product of one group treated as u and the product of the other as v ; and then there are many more possible results. Some of these will usually be standard forms, others interesting if not useful, others curious and instructive, and some impracticable. The fact that many of these alternative results lead to practicable integrations is due to the principal elementary functions that appear under the integral sign being themselves capable both of differentiation and of integration in a workable form, as will be seen from the table just given; and it is for this reason that integration by parts has proved so powerful an instrument in actuarial research. Some examples may now be given, and on pages 410 and 411 is given a reference table of some of the principal formulæ which may be similarly deduced.

$$\text{Take } \tilde{\sigma}_x = \frac{1}{l_x} \int_0^\infty v^t \cdot l_{x+t} \cdot dt.$$

(1) Put $l_{x+t} = u_t$ and $v^t = r_t$.

From (A) we get

$$\tilde{\sigma}_x = \frac{1}{l_x} \int_0^x l_{x+t} \cdot \mu_{x+t} \cdot \tilde{\sigma}_t \cdot dt = \frac{1}{l_x} \int_0^\infty v^t l_{x+t} \cdot \mu_{x+t} \cdot \tilde{s}_t \cdot dt$$

From (B) we get

$$\begin{aligned} \tilde{\sigma}_x &= \tilde{\sigma}_x - \int_0^x v^t \frac{l_{x+t}}{l_x} \mu_{x+t} \tilde{\sigma}_x dt \\ &= \frac{1 - \bar{A}_x}{\delta} \end{aligned}$$

(2) Put $v^t = u_t$ and $l_{x+t} = r_t$.

From (A) we get

$$\begin{aligned} \tilde{\sigma}_x &= \frac{1}{l_x} \int_0^\infty v^t \delta \left(\int_0^t l_{x+k} dk \right) dt \\ &= \delta \int_0^\infty v^t \hat{e}_{kt} dt \end{aligned}$$

From (B) we get

$$\begin{aligned}\bar{a}_x &= \frac{1}{l_x} \int_0^x l_{x+t} dt - \frac{1}{l_x} \int_0^x \delta v^t \left(\int_t^x l_{x+k} dk \right) dt \\ &= \bar{e}_x - \delta \int_0^x v^t \cdot {}_t p_x \cdot \bar{e}_{x+t} dt\end{aligned}$$

(8). It is impracticable to give any universal rules for the most advantageous arrangements of the factors under the integral sign before applying the formulæ. The following hints may, however, be of service to students, who will appreciate them better on working through the foregoing examples for the second time.

- (i) Factors of the form $\mu_{x+t} l_{x+t}$ generally suggest the substitution of $-\frac{dl_{x+t}}{dt}$, which may then be treated as v_t , which is easily integrable, giving $\int_a^t = l_{x+a} - l_{x+t}$ (formula A) and $\int_t^b = l_{x+t} - l_{x+b}$ (formula B).
- (ii) Conversely, a factor of the form $(l_{x+a} - l_{x+t})$ may usually be treated as u_t , which is easily differentiated, giving $\frac{du_t}{dt} = \mu_{x+t} l_{x+t}$. In this case formula B is usually the more advantageous because u_a is zero, and thus the first term disappears.
- (iii) v^t , where it can be treated as a separate factor, may be treated either as u_t or v_t , as it is easily differentiated or integrated in a convenient form.
- (iv) $v^t l_{x+t}$ may be replaced by $v^{x+t} l_{x+t} = D_{x+t}$, subject to the result being divided by the constant factor v^x . For example :

$$\begin{aligned}\frac{1}{l_x l_y \dots} \int v^t l_{x+t} l_{y+t} \dots dt &= \frac{1}{v^x l_x l_y \dots} \int v^{x+t} l_{x+t} l_{y+t} \dots dt \\ &= \frac{1}{D_x l_y \dots} \int D_{x+t} l_{y+t} \dots dt\end{aligned}$$

D_{x+t} may then be treated as u_t , giving

$$\frac{d}{dt} u_t = -(\mu_{x+t} + \delta) D_{x+t}$$

or as v_t , giving $\int v_t dt = -\bar{N}_{x+t} = -D_{x+t} \bar{\rho}_{x+t}$.

(v.) Similarly, $v' \mu_{x+t} \cdot l_{x+t}$ may be replaced by $v^{x+t} \mu_{x+t} l_{x+t}$ and treated as v_t , giving

$$\int v_t dt = -\bar{M}_{x+t} = -v^{x+t} l_{x+t} \bar{\Lambda}_{x+t}.$$

(9). It may be worth while to point out that formula (1), or formula (2) when continuous functions are used, will be of considerable assistance in transforming the complicated expressions which often occur in pension fund and other problems. As an example the problem discussed by Mr. E. C. Thomas, *J.I.A.*, xxxviii, 276, may be taken. There the expression to be summed is :

$$\begin{aligned} & [s_x(d_x \times v^{\frac{1}{2}} \times \bar{1} + j^{\frac{1}{2}})] + [s_x(d_{x+1} \times v^{1\frac{1}{2}} \times \bar{1} + j^{1\frac{1}{2}}) + s_{x+1}(d_{x+1} \times v^{1\frac{1}{2}} \times \bar{1} + j^{\frac{1}{2}})] + \dots \\ & = v^1(1+j)^{\frac{1}{2}} d_x s_x + v^{1\frac{1}{2}}(1+j)^{1\frac{1}{2}} d_{x+1} [s_x + s_{x+1}(1+j)^{-1}] + \dots \\ & = \sum_v v^{t+\frac{1}{2}} (1+j)^{t+\frac{1}{2}} d_{x+t} \sum_k s_{x+k} (1+j)^{-k} \\ & = \sum_v s_{x+t} (1+j)^{-t} \sum_k v^{k+\frac{1}{2}} (1+j)^{k+\frac{1}{2}} d_{x+k} \text{ by formula (1)} \end{aligned}$$

leading at once to Mr. Thomas' formula (8), putting

$$v^{x+k+\frac{1}{2}} / (1+j)^{x+k+\frac{1}{2}} \cdot d_{x+k} = C_{x+k}$$

$$\sum C_{x+k} = M_{x+k}$$

$$\sum s_{x+t} \cdot (1+j)^{-x-t} M_{x+t} = M''_{x+t}$$

$$\sum M''_{x+t} = R''_x$$

(10). Again, take the well-known formula (*Text-Book*, Part II, p. 249, par. 18) :

$$\begin{aligned}
 a_{y,x} &= \frac{1}{D_x l_y} \sum_1^{\infty} d_{y+t+1} \cdot D_{x+t} \\
 &= \frac{1}{D_x l_y} \sum_1^{\infty} d_{y+t+1} \cdot \sum_1^{\infty} D_{x+t} \\
 &= \frac{1}{D_x l_y} \sum_1^{\infty} D_{x+t} \cdot (l_y - l_{y+t}) \\
 &= \frac{\sum_1^{\infty} D_{x+t}}{D_x} - \frac{\sum_1^{\infty} D_{x+t} \cdot l_{y+t}}{D_x l_y} \\
 &= a_x - a_{xy}
 \end{aligned}$$

Many other standard formulae can be easily obtained in the same way, by the use of formula (1).

G. J. L.

S. E. M.

Reference Table of formulae derivable directly

Page	Formula	Function	Function expressed as integral	Whether A or B	REFERENCE TO
					Function to be substituted for u_t
181	7	\bar{a}_x	$\frac{1}{l_x} \int_0^x r^t l_{x+t} dt$	B	l_{x+t}
181	7	$\bar{\Lambda}_x$	$\frac{1}{l_x} \int_0^x r^t l_{x+t} \mu_{x+t} dt$	A or B	r^t
258	24	$\bar{a}_{y/x}$	$\frac{1}{l_x l_y} \int_0^x r^t l_{x+t} (l_y - l_{y+t}) dt$	B	$l_y - l_{y+t}$
258	25	$\bar{a}_z _{xy}$	$\frac{1}{l_x l_y l_z} \int_0^x r^t l_{x+t} l_{y-t} (l_z - l_{z-t}) dt$	B	$l_z - l_{z+t}$
258	26	$\bar{a}_{yz/x}$	$\frac{1}{l_x l_y l_z} \int_0^x r^t l_{x+t} (l_y l_z - l_{y+t} l_{z-t}) dt$	B	$l_y l_z - l_{y+t} l_{z+t}$
258	28	$\bar{a}_{\bar{yz}} _x$	$\frac{1}{l_x l_y l_z} \int_0^x r^t l_{x+t} (l_y - l_{y+t}) (l_z - l_{z-t}) dt$	B	$(l_y - l_{y+t})(l_z - l_{z+t})$
265	11	$\bar{a}_{yz} _x$	$\frac{1}{l_x} \int_0^x r^t l_{x+t} r^1 l_{z-y} dt$	B	$t^1 l_{z-y}$
270	24	$\bar{\Lambda}_{w/x/y/z}^{(2)}$	$\frac{1}{l_w l_x l_y l_z} \int_0^x r^t l_w t^1 l_x t^1 l_y (l_z - t^1 l_z)^t \mu_y^t \bar{\Lambda}_{w,x}^1 dt$	B	$l_z - l_{z+t}$
193	...	$(I\bar{a})_1$	$\int_0^1 t r^t dt$	A or B	t
		$(I\bar{\Lambda})_x$	$\frac{1}{l_x} \int_0^x t r^t l_{x+t} \mu_{x+t} dt$	B	$t r^t$
		Y_x	$\int_0^x t l_{x+t} dt$	B	t

from the Integration Formulae A and B.

WORKING FORMULÆ

Function to be substituted for
 v_t

Resulting Formulae

v^t

$1 - \frac{\bar{A}_x}{\delta}$

$l_{x+t} \mu_{x+t}$

$1 - \delta \tilde{\mu}_x$

$v^t l_{x+t}$

$\frac{1}{l_x l_y} \int_0^x v^t l_{x+t} l_{y+t} \mu_{y+t} \tilde{\mu}_{x-t} dt$

$v^t l_{x+t} l_{y+t}$

$\frac{1}{l_x l_y l_z} \int_0^x v^t l_{x+t} l_{y+t} l_{z+t} (\mu_{y+t} + \mu_{z+t}) \tilde{\mu}_{x+t} dt$

$v^t l_{x+t}$

$\frac{1}{l_x l_y l_z} \int_0^x v^t l_{x+t} l_{y+t} l_{z+t} (\mu_{y+t} + \mu_{z+t}) \tilde{\mu}_{x+t} dt$

$v^t l_{x+t}$

$\frac{1}{l_x l_y l_z} \int_0^x v^t l_{x+t} l_{y+t} l_{z+t} \mu_{z+t} \tilde{\mu}_{x+t} dt$

$v^t l_w l_x l_y t \mu_y \tilde{A}_{wx}^1$

$\frac{1}{l_w l_x l_y l_z} \int_0^x v^t l_w l_x l_y l_z \mu_z \tilde{A}_{wxy}^2 dt$

v^t

$\frac{v^t (i - \delta)}{\delta^2}$

$l_{x+t} \mu_{x+t}$

$\tilde{\mu}_x - \delta (\ln \tilde{\mu})_x$

l_{x+t}

$\int_0^x T_{x-t} dt$

REVIEW.

*The German Insurance Lexicon.**

IN comparison with many of the other great sciences, that of insurance has long been singularly neglected in the matter of encyclopædic works of reference. The late Cornelius Walford's, in spite of its incompleteness, has remained in absolute possession of the field until very recently. There had been mention some time since of an American scheme on a larger scale designed to supply the deficiency, but again, as in numerous other branches of scientific endeavour, the Germans have been the first to achieve success, by means of their recently published Insurance Lexicon.

The "Versicherungs-Lexikon", although mainly restricted to insurance in Germany, Austria and Switzerland, represents at any rate the first attempt which has been brought to fruition to provide a reference book covering all branches of science affecting private and social insurance. For this the insurance world is indebted to Professor Dr. Alfred Manes of Berlin, assisted by the following distinguished collaborateurs :—

- Professor Dr. v. d. Borght (Berlin).
- Dr. Domizlaff (Hanover).
- Professor Dr. Ehrenberg (Göttingen).
- H. Ehrlich (Cologne).
- Professor Dr. Emminghaus (Gotha).
- Dr. Feilchenfeld (Berlin).
- Professor Dr. Florschütz (Gotha).
- O. Hagen (Berlin).
- H. v. Knebel Doeberitz (Berlin).
- Professor Dr. Lexis (Göttingen).
- Professor Dr. Loewy (Freiburg).
- Professor Dr. Moldenhauer (Cologne).
- The Ritter v. Rasp (Munich).
- Dr. Reuss (Berlin).
- Dr. Samwer (Gotha).
- K. Schneider (Stettin).

Professor Dr. Manes, who is General Secretary of the German Federation for Insurance Science, Editor of the Journal of that body, and author of numerous works on insurance, will be best known to British actuaries for the thorough success which attended his efforts as organising Secretary of the Fifth International Actuarial Congress, held at Berlin in 1906. In his Preface to the present Encyclopædia, he ascribes the ill-success of previous attempts in the same direction

* [Versicherungs-Lexikon, ein Nachschlagewerk für alle Wissenschaftsbereiche der Privat und der Sozial-Versicherung insbesondere in Deutschland, Oesterreich und der Schweiz, herausgegeben von Professor Dr. Alfred Manes. Published by J. C. B. Mohr (Paul Siebeck) at Tübingen, Germany, price 28 marks, in paper; or, 30 marks bound.]

to the fact that they covered too wide a range. Warned by this, he sought, in association with prominent theoretical and practical insurance men, to bring the whole of insurance science within the scope of a single volume. The object was to produce a reliable work of reference, containing everything important relative to insurance, firstly in the German Empire, and secondly in Austria and Switzerland; other countries being considered when it seemed advisable to do so. The description of present day circumstances received primary attention, historical matters being noticed but shortly, and biographical notices restricted to persons no longer living.

The work is published in two half-volumes of some 840 pages in all, consisting of about 300 principal articles, each prepared by the individual writers as far as possible according to the branch of insurance, each article having the author's initials appended. The scheme of the work was the following:—

1. General.
2. Contract Law.
3. Law of Supervision (including Taxation).
4. Life Assurance (including Insurance Mathematics, Sicknes and Invalidity Insurance).
5. Accident and Liability Insurance.
6. Transport ..
7. Fire ..
8. Hail and Cattle ..
9. Smaller branches of ..
10. Reinsurance.
11. Insurance Medicine.
12. Social Insurance (all branches of Workmen's Insurance, etc.).

There is given a Systematic Register, showing how the articles fall into the above scheme, and at the end there are very full Indexes to Persons and Things, including not only the articles themselves but all the references contained in them. The latter Index takes 36 pages, and it is recommended to make use of it first of all in consulting the work. The printing is throughout in roman type, and contractions of frequently-recurring long words are freely employed to save space. The style is for the most part simple and clear, starting in each article with a definition of the subject dealt with, so that a perusal of it does not present so many difficulties to the foreigner as would be the case were a more florid literary style employed. For those desirous of pursuing the study of any question, a selected bibliography is attached to many of the articles, most of the books mentioned being already in the library of the German Federation for Insurance Science.

As to Encyclopædias in general, there would as yet appear to be no uniformity of opinion as to what should be expected of them. Logically considered, such works seem scarcely fitted to supply an educational course, however complete they may be. On the other hand, a mere Index, even the most elaborate, if devoid of

explanatory matter, would not fulfil requirements, and it would appear that the ideal conditions should be between the two extremes. Probably the best for practical purposes would be to have, not a series of articles of uniform length, but a greater number, of which the bulk would be of less compass, only important matters having much space devoted to them. In other words, space should be in proportion to the importance of the subject. Each article should so far as feasible, assume ignorance on the part of the reader—that is, the introduction should be especially careful, while the more advanced aspects, less likely to be sought by the novice, might be treated at less length, references being inserted instead. In this way an Encyclopædia can be of great value, not only as a work of reference, but also as an up-to-date codification of existing knowledge.

The “Versicherungs-Lexikon” does not precisely conform to these canons, for there are, as already mentioned, comparatively few articles; but the effect of the many entries is achieved to a large degree by the very full Indexes. Considering the Lexicon as a whole, one finds it to possess very much the characteristics that might have been expected. Thus, Insurance Legislation has of late years been so much to the fore in Germany, that it is not surprising to find that much space is devoted to the question. There is, of course, general insurance law, insurance contract law, laws of supervision and taxation, as well as that applicable to particular branches of insurance. These are very good, and worth the study of all interested in insurance legislation. As against this, the treatment of financial questions would, from an English actuarial standpoint, appear to be rather slight. A great deal, however, can be learned in small compass about State Insurance, which in Germany has attained so remarkable a development, and this is a topic likely to demand attention in Great Britain in the near future. There is also a long article on Unemployment Insurance, a subject which is at present closely engaging the attention of our leading politicians of all parties. Mathematics does not receive over much attention, but many business topics are discussed at fair length.

In conclusion, it can be said that the prompt completion of so extensive and important an undertaking reflects every credit upon Professor Manes and his fellow workers. They have shown how much can be done, and what yet remains undone, while indicating the lines upon which further development may be attempted. It is to be hoped that we shall yet see an International Insurance Encyclopædia, which can only be produced by the combined efforts of the best insurance minds in all civilised countries. This in itself would be a great aid to and proof of solidarity and comradeship in all insurance circles, irrespective of nationality.

G. W. R.

JOURNAL OF THE INSTITUTE OF ACTUARIES.

Analysis and Apportionment of the Expenses of Management of a Life Office with a view to ascertaining the Office Premium Loadings. By H. J. RIETSCHEL, F.I.A., of the Sun Life Assurance Society.

[Read before the Institute, 25 April 1910.]

IN 1907 I had the honour of reading before the Institute a paper dealing with the Comparative Bonuses under Whole-Life and Endowment Assurances, and therein I pointed out that the method of apportioning the expenses between the two classes had a considerable effect upon the rates of bonus, and in the discussion more than one speaker referred to the relative premiums as vitally affecting the conclusions to be drawn from any investigation. The solution of the first problem will, mortality apart, also give us the key to the rates of premium which an office should charge for Whole-Life and Endowment Assurances respectively. As the premiums are

the foundation of our business it is of the utmost importance that an analysis of the expenses should be made periodically in order to ascertain the adequacy of the remuneration the office receives for the risks it undertakes. The question of the office premiums has been frequently discussed, but I do not think any paper has been submitted to this Institute in which an agreement has been shown between the loadings adopted in obtaining the office premiums and the actual expenses of the office. The object of the paper is to elicit the views of the profession upon this subject.

The commission paid by the majority of offices is, for the principal classes of Assurance, usually a percentage of the sum assured on payment of the first annual premium and a percentage of the office renewal premiums. The loading required to provide for the commissions is therefore easily ascertained and requires no discussion.

The expenses of management divide themselves broadly into two groups :

- (a) the initial expenses incurred in obtaining the new business, and
- (b) the renewal expenses, *i.e.*, the expenses incurred in maintaining the existing business on the books.

Such items as "Advertising," "Medical Fees" and "Travelling Expenses" come under the heading of initial expenses, the Cost of Valuation is a renewal expense, and so, in the main, are the Legal Costs, but many of the items which go to form the expenses of management belong partly to the initial expenses of obtaining business and partly to the expenses of renewal. The allocation of these items is a matter of considerable difficulty and much depends upon the personal judgment of the actuary. The apportionment suggested in Table A is therefore to some extent empirical and the discussion will probably show that considerable difference of opinion exists on this point.

It has been held that the initial expenditure incurred in placing a policy on the books does not vary with the sum assured and is the same in amount whether the policy be one for £100 or £10,000, but, for the reasons hereafter stated, I have dismissed this plan for allocating the expenses as impracticable and have arrived at the conclusion that an assured

TABLE A.
Apportionment of Expenses of Management.

Main items comprised under "Expenses of Management"	PROPORTION ALLOCATED TO																						
	(a) New Business	(b) Renewals																					
(1) Rents, Rates, Taxes and expenses of maintenance of offices (fuel, lighting, furniture, lifts, repairs, &c.) .	$\frac{1}{2}$	$\frac{1}{2}$																					
(2) Salaries :																							
(a) Say $\frac{3}{4}$ of Total Salary List .	<table style="margin-left: 20px;"> <tr> <td>Principal Officers . . .</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td></tr> <tr> <td>Secretarial Department . . .</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td></tr> <tr> <td>New Business , , .</td><td>1</td><td>Nil</td></tr> <tr> <td>Claims Settlement , , .</td><td>Nil</td><td>1</td></tr> <tr> <td>Actuarial , , .</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td></tr> <tr> <td>Accounts and Investment Department . . .</td><td>$\frac{1}{4}$</td><td>$\frac{3}{4}$</td></tr> <tr> <td>Agency Extension Department . . .</td><td>1</td><td>Nil</td></tr> </table>	Principal Officers . . .	$\frac{1}{2}$	$\frac{1}{2}$	Secretarial Department . . .	$\frac{1}{2}$	$\frac{1}{2}$	New Business , , .	1	Nil	Claims Settlement , , .	Nil	1	Actuarial , , .	$\frac{1}{2}$	$\frac{1}{2}$	Accounts and Investment Department . . .	$\frac{1}{4}$	$\frac{3}{4}$	Agency Extension Department . . .	1	Nil	
Principal Officers . . .	$\frac{1}{2}$	$\frac{1}{2}$																					
Secretarial Department . . .	$\frac{1}{2}$	$\frac{1}{2}$																					
New Business , , .	1	Nil																					
Claims Settlement , , .	Nil	1																					
Actuarial , , .	$\frac{1}{2}$	$\frac{1}{2}$																					
Accounts and Investment Department . . .	$\frac{1}{4}$	$\frac{3}{4}$																					
Agency Extension Department . . .	1	Nil																					
The above proportions added together will be found in practice to be very nearly equal to	$\frac{1}{2}$	$\frac{1}{2}$																					
(b) Say $\frac{1}{4}$ of Total Salary List .	<table style="margin-left: 20px;"> <tr> <td>Inspectors and Salaried Agents . . .</td><td>1</td><td>Nil</td></tr> <tr> <td>Branch Managers . . .</td><td>$\frac{3}{4}$</td><td>$\frac{1}{4}$</td></tr> </table>	Inspectors and Salaried Agents . . .	1	Nil	Branch Managers . . .	$\frac{3}{4}$	$\frac{1}{4}$																
Inspectors and Salaried Agents . . .	1	Nil																					
Branch Managers . . .	$\frac{3}{4}$	$\frac{1}{4}$																					
If the Total Branch Managers' Salaries are assumed equal to the Total Salaries of the Inspectors the proportion for the last-mentioned two items will be	$\frac{7}{8}$	$\frac{1}{8}$																					
The proportion of Total Salaries applicable to New Business will, therefore, be $\frac{3}{4} (\frac{1}{2}) + \frac{1}{4} (\frac{7}{8}) =$ and the proportion applicable to Renewals	$\frac{11}{16}$ =	$\frac{3}{16}$ $\frac{3}{16}$																					
(3) Superannuation (treated as Deferred Salary)	$\frac{1}{2}$	$\frac{1}{2}$																					
(4) Directors Fees	$\frac{1}{2}$	$\frac{1}{2}$																					
(5) Auditors Fees	$\frac{1}{4}$	$\frac{3}{4}$																					
(6) Printing and Stationery and Books	$\frac{1}{2}$	$\frac{1}{2}$																					
(7) Advertising	1	Nil																					
(8) Travelling Expenses	1	Nil																					
(9) Medical Fees	1	Nil																					
(10) Policy, Receipt and other Stamps	$\frac{1}{2}$	$\frac{1}{2}$																					
(11) Postages	$\frac{1}{2}$	$\frac{1}{2}$																					
(12) Legal Expenses	Nil	1																					
(13) Valuation Expenses	Nil	1																					
(14) Other General Expenses	$\frac{1}{2}$	$\frac{1}{2}$																					

must contribute to the initial expenses in the proportion in which he benefits by the assurance, *i.e.*, in proportion to the sum assured.

Turning now to the renewal expenditure a little consideration will show that this may be divided into

- (1) the *minimum* expense, at which the existing business can be conducted, and this depends not upon the amount of the premium income but upon the number of contracts in force, or (on the arguments set out on a subsequent page) upon the sums assured in force ; and
- (2) a portion of the further expense, which (although not absolutely necessary to the conduct of the business) is incurred in order to increase the efficiency and reputation of the office. The motive of this expenditure is, of course, the procuration of new business, but it is only fair that the existing business should bear a proportion of such expenditure, as it participates in the benefits derived therefrom. Into Renewal Expenditure would therefore fall that part of the following items which has not already been allocated to Initial Expenses :

The additional cost of providing expensive offices in central positions rather than cheaper premises in less favourable situations.

The salaries of the additional staff employed to ensure the rapid handling of the correspondence and prompt attention to policyholders' requirements,

Attractive stationery, &c.

As the public measures the extravagance, or otherwise, of an office mainly by the ratio of the expenses to the office premium income, the aim of the management is to keep this ratio down to a certain percentage, and, as a consequence, the amount of these further expenses will be regulated by the size of the premium income and will take the form of a percentage of such income.

The renewal expenses will, therefore, depend partly on the

sums assured in force and partly on the premium income. I propose to divide them as follows :

TABLE B.

Apportionment of Renewal Expenses.

	Proportion of item included in Revenue Account charged to Renewals (See Table A)	Proportion treated as a Percentage of the Sums Assured in force	Proportion treated as a Percentage of the Premium Income
(1) Rent, &c.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
(2) Salaries	$\frac{1\frac{3}{4}}{5\frac{1}{2}}$	$\frac{1\frac{3}{4}}{6\frac{1}{4}}$	$\frac{1\frac{3}{4}}{6\frac{1}{4}}$
(3) Superannuation.	$\frac{1\frac{1}{2}}{5\frac{1}{2}}$	$\frac{1\frac{1}{2}}{6\frac{1}{4}}$	$\frac{1\frac{1}{2}}{6\frac{1}{4}}$
(4) Directors' Fees	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
(5) Auditors' Fees	$\frac{3}{4}$	$\frac{3}{4}$	Nil
(6) Printing, Stationery and Books	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
(10) Policy, Receipt and other Stamps	$\frac{1}{2}$	$\frac{1}{2}$	Nil
(11) Postages	$\frac{1}{2}$	$\frac{1}{2}$	Nil
(12) Legal Expenses	1	1	Nil
(13) Valuation Expenses	1	$\frac{1}{2}$	$\frac{1}{2}$
(14) Other General Expenses	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

In order to illustrate the above analysis and also to institute a comparison with other methods of allocating the expenses, I have set out in the Appendix the results obtained from the revenue accounts of three offices, which publish their expenses of management in detail, and whose general expense ratios (Commission and Expenses to Office Premiums) are 14, 15 and 10 per-cent respectively. The actual figures have been disguised so that the offices shall not be recognizable. As there are certain special features attaching to office No. 1, and as the general expense ratio of office No. 3 is below the average, I have in this paper adopted the loadings obtained from No. 2, and these are shown in Table C.

Method D, which allocates the same proportion of the expense to each policy irrespective of the sum assured, produces office premiums which are high for the smaller sums assured and low for the larger policies. Thus, for a policy of £100 the office premium is $107.9\pi + .373$ (or assuming a net premium of £3 per-cent the loading would be 12s. 3d. or over 20 per-cent),

TABLE C.

Method employed in allocating expenses	INITIAL EXPENSES		RENEWAL EXPENSES	
	Percentage of New Business Sum Assured		Proportion treated as a percentage of the Sums Assured in force, Percentage	Proportion treated as a percentage of the Premium Income, Percentage
(A) Method, as described above :				
Expenses of Management	3·30		·068	2·24
Commission	1·00		...	2·50
Total	4·30		·068	4·74
(B) Method, as described above, except that Renewal Expenses are treated entirely as a percentage of Premium Income :				
Expenses of Management	3·30		...	4·93
Commission	1·00		...	2·50
Total	4·30		...	7·43
(C) Method, as described above, except that Renewal Expenses are treated entirely as a percentage of the Sums Assured in Force—				
Expenses of Management	3·30		·125	...
Commission	1·00		...	2·50
Total	4·30		·125	2·50
(D) Expenses treated as a fixed amount per contract		Per contract	Per contract (excluding Commission)	
		4·42	·189	
The Office Premiums, assuming Net Premiums calculated on the basis of $3\frac{1}{2}\%$ interest, will be as follows:				
Method (A)	$\pi + \frac{0.43}{a} + 0.0068 + 0.0474\pi = 1.090\pi + 0.00214$			
Method (B)	$\pi + \frac{0.43}{a} + 0.743\pi = 1.117\pi + 0.00146$			
Method (C)	$\pi + \frac{0.43}{a} + 0.00125 + 0.025\pi = 1.068\pi + 0.00271$			
Method (D)	For a Contract of £100, Sum Assured			
	$160\pi + \frac{4.42}{a} + 1.89 + \frac{1.00}{a} + \underbrace{0.025 \times 100\pi}_{\text{Commission}} = 107.9\pi + 3.73$			
	For a Contract of £10,000, Sum Assured			
	$10,000\pi + \frac{4.42}{a} + 1.89 + \frac{100}{a} + \underbrace{0.025 \times 10,000\pi}_{\text{Commission}} = 10,354\pi + 3.7$			

whereas for a sum assured of £10,000 the premium would be $10.354\pi + 3.7$, which would include a loading for commission of $350\pi + 3.4$ and only $4\pi + 3$, or 9s. 5d. for expenses.

Office No. 2, from whose figures the above loadings are obtained, has, however, an average sum assured which is much below that of the majority of ordinary offices, its average policy being only slightly in excess of £150 and this fact tends to obscure the full effect of the above method upon the premiums for the smaller sums assured. In the case of office No. 3, with an average sum assured approaching £400, the office premium for a policy of £100 would be $117.8\pi + .824$ (or assuming a net premium of £3 per-cent the loading would be as much as £1. 7s. 2d., or nearly 50 per-cent of the net premium).

It is at present the universal practice to charge definite rates of premium per-cent of the sum assured regardless of the amount of the policy ; that is to say, a policy for £10,000 pays a premium ten times as large as that charged for a policy of £1000, and, as a result, the former policy will contribute ten times as much to the expenses as the latter. If offices generally adopted Method D for the apportionment of expenses the high rates of premium for the small sums assured would greatly reduce the number of policies effected for small amounts and, unless in their place an equal number of larger assurances could be secured, the rates of expense per policy would rise. It will readily be admitted that the area from which the larger policies can be drawn is very much less than that from which the smaller policies are obtained, and it is difficult to see how the loss of the latter could possibly be made good, especially as the consequent restriction of the sources, from which life assurance business would be secured, would increase the severity of the competition therefor. If the same number of policies are not obtained as formerly, then the higher contributions to expenses required from each policy would necessitate increases in the premiums, which would further tend to reduce the number of policies effected for small amounts, thereby placing yet more difficulties in the way of procuring an adequate number of policies. Under all the circumstances Method D must I think be dismissed as impracticable.

Method B assumes that the renewal expenses are entirely a percentage of the premiums, and Method C that they depend solely upon the total sums assured in force. The considerations mentioned in the paragraph relating to renewal

expenses lead me to think that neither Method B nor C is scientific, but that a combination of the two Methods as adopted in Method A would more nearly accord with practice. Method B would throw too great an expense upon the more highly rated Policies, and on the other hand Method C too little. Generally speaking the differences between the premiums produced by Methods A, B and C are inconsiderable. The premiums hereafter given have been based upon the results obtained by Method A.

Before closing the discussion on the various methods of apportioning the expenses reference should perhaps be made to yet another alternative, namely, the plan of loading the premiums with a uniform percentage based upon the ratio of the expenses of management and commission to the premium income. The proper application to the method would call for an increase in the office premiums whenever the general expense ratio exceeded the percentage loading, and this would frequently be indefensible; for example, the increased ratio might be due to a large accession of new business, which had been obtained at as cheap, or a cheaper, rate than the former smaller new business, yet, in spite of this, the method would demand larger office premiums. To take another example, the Initial Commission on a £100 Endowment Assurance, subject to an Annual Premium of £10, and a £100 Whole-Life Assurance, subject to an Annual Premium of £2, is, say, £1 in each case.

An annual contribution of $\frac{1}{a_{x^{10}}}$, or about .125, from the

Endowment Assurance Policy and $\frac{1}{a_x}$, or about .070, from the Whole Life Policy would cover the Initial Commission: the uniform percentage loading plan would, however, charge the Endowment Assurance Policy yearly five times as much as the Whole-Life Policy. The method cannot, therefore, be adopted if an equitable apportionment is desired.

WHOLE-LIFE AND ENDOWMENT ASSURANCES, WITHOUT PROFITS.

It is now possible to frame a table of minimum "Without-Profit" Premiums. The Mortality has been taken on the O^M Select Table and the rate of interest as $3\frac{1}{2}$ per-cent, which is about $\frac{1}{4}$ per-cent less than that earned by the majority of first-class offices.

TABLE D.

Without-Profit Office Premiums per £100 Sum Assured.

	WHOLE-LIFE ASSURANCES						ENDOWMENT ASSURANCES MATURING IN THE																	
	Age at entry			Age at entry			Age at entry			60th year of age		50th year of age												
	30	40	50	30	40	50	30	40	50	30	40	30	40											
Formula :	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.									
P = 1.090	$\frac{[OM]^{3\frac{1}{2}\%}}{\pi_{[x]}}$	+ .00214						2	0	8	2	15	1	3	18	11								
		2	17	7	4	12	8	4	8	0	9	12	4											
Premiums of 8 first-class offices :									
Office—(a)	.	.	.	2	0	9	2	15	0	3	19	9	2	17	7	4	12	4	4	7	1	9	10	4
(b)	.	.	.	1	18	10	2	13	5	3	17	0	2	15	9	4	11	6	4	6	8	...		
(c)	.	.	.	1	18	6	2	12	5	3	15	10	2	13	5	4	7	4	4	2	9	...		
(d)	.	.	.	1	18	0	2	12	0	3	15	6	2	17	3	4	11	6	4	7	4	9	10	0
(e)	.	.	.	2	1	7	2	15	11	3	19	9	2	18	9	4	13	6	4	8	5	...		
(f)	.	.	.	2	0	0	2	14	6	3	18	8	2	17	0	4	11	6	4	6	6	...		
(g)	.	.	.	1	19	11	2	14	8	3	19	8	2	14	3	4	9	10	4	5	2	...		
(h)	.	.	.	2	2	2	2	16	2	3	18	1	2	18	11	4	13	10	4	9	3	9	13	7

From Table D it will be seen that several of the offices charge premiums which closely follow my specimen premiums, but in the case of some of them the tabular rates are considerably lower than mine.

WITH-PROFIT PREMIUMS.

The question of "With-Profit" Loadings is somewhat outside the scope of the paper, which professes to deal with the "Expense" Loadings only, and I do not propose to devote much space to it.

In order to obtain the With-Profit Rates of Premium the additional loading must be ascertained which should be charged for the privilege of participating in the profits, and, as has been demonstrated by several contributors to the *Journal*, this loading should approximately provide for a bonus of the *form* to be declared. If a Uniform Simple Reversionary Bonus is desired then the "With-Profit" Loading should at all ages at entry and for all classes secure approximately such a bonus. The

loading need not, of course, be sufficient to produce the actual rate of bonus which it is hoped to allot, as in most offices the bonus loading will be supplemented by profit from other sources.

The following are premiums loaded to provide for a uniform simple reversionary bonus of 20s. per-cent per annum of the sum assured :

TABLE E.

With-Profit Office Annual Premiums per £100 Sum Assured.

WHOLE-LIFE ASSURANCES						ENDOWMENT ASSURANCES MATURING IN THE							
						60th year of age				50th year of age			
Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry	Age at entry
30	40	50	30	40	50	30	40	30	40	30	40	30	40
Formula : $P = 1.090\pi_{[x]}^{O[M]2} + .00214 + .01\pi(1A)$	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.					
	2 10	1 3	6	0 4	11 5	3 9	4 5	6	7 5	1 10	10	8	10

Premiums of 4 first-class offices declaring Uniform Simple Reversionary Bonuses :

Office—	(a) . . .	2	9	9	3	4	1	4	8	3	3	8	0	5	6	0	5	0	6	...	
	(b) . . .	2	9	10	3	5	9	4	11	3	3	9	2	5	7	0	5	1	10	10	11
	(c) . . .	2	9	0	3	4	8	4	10	10	3	7	4	5	4	1	4	19	5	10	7
	(d) . . .	2	8	11	3	4	7	4	10	8	3	8	0	5	4	8	5	0	1	10	9

The premiums of the offices given in Table E are on the whole in agreement with my specimen premiums. I have not calculated premiums to provide for other forms of bonus, as the formula can be adapted to meet any particular case.

JOINT-LIVES ASSURANCES.

In the following table are a few specimen Joint-Life Non-Participating Office Premiums obtained from the formula

$$P'_{xy} = 1.090\pi_{[xy]}^{O[M]2} + .00214$$

and underneath are set out the corresponding published rates of five offices. I have not given the "With-Profit"

Premiums as (1) the calculation will involve no departure from the principles underlying the Whole-Life With-Profit Premium formula, and (2) the premiums will vary with each office according to the form of the bonus to be declared.

TABLE F.

Annual Premiums for Joint-Life Assurances of £100, Without Profits.

—	30 & 40	30 & 50	30 & 60	40 & 40	40 & 50	40 & 60	50 & 60	60 & 60
$P'_{xy} = 1.000 \frac{0.033}{\pi} + 0.0214$	£ s. d.	3 10 9 4 12 0 6 10 7 4 0 1 4 19 4 6 16 3 7 9 1 8 17 7						
Office (a) . .	3 14 10 4 17 7 7 1 7 4 4 7 5 5 5 7 7 9 8 2 7 9 17 3							
(b) . .	3 10 10 4 11 3 6 10 5 3 19 7 4 19 0 6 14 6 7 8 2 8 15 2							
(c) . .	3 15 0 4 17 4 7 0 4 4 4 9 5 5 2 7 6 7 8 1 4 9 15 2							
(d) . .	3 15 4 4 17 8 ... 4 5 3 5 5 5							
(e) . .	3 13 7 4 17 1 ... 4 3 10 5 4 10							

I will now deal with a few cases in which the method of apportionment of the expenses adopted by me might be held to break down.

TEMPORARY ASSURANCES.

If these assurances could bear their full share of the expenses of management the office premium would (assuming the agent's commission to be 10 per-cent of the first annual premium and $2\frac{1}{2}$ per-cent of the renewal premiums) be

$$\pi_{[x]n}^1 + \frac{.033}{a_{[x]n}} + \underbrace{.00068 + .0224\pi_{[x]n}^1}_{\text{Initial Renewal Expenses.}} + \underbrace{.10\pi_{[x]n}^1 + .025\pi_{[x]n}^1}_{\text{Commission.}}$$

$$= 1.0474\pi_{[x]n}^1 + \frac{.10\pi_{[x]n}^1}{a_{[x]n}} + .00068 + \frac{.033}{a_{[x]n}}$$

It is obvious that for the shorter terms, which are the only terms of real importance, the initial expenditure of £3.30 per-cent of the sum assured cannot possibly be secured. The only way to regard these assurances is that they are supplemental to the general business of the office and assist to bear such a proportion of the initial expenses as competition will allow.

I have employed the O^{NM} Select Table, with $3\frac{1}{2}$ per-cent

interest, for the purpose of obtaining my office premiums. These are given in Table G together with a statement of the amount of the initial expenditure which they will cover. It will be seen from Table H that the premiums in Table G are fair average rates and that for terms under 16 years the office must, therefore, be satisfied with premiums which will provide only a portion of the ordinary initial expenditure. For a term of 1 year the initial expenditure, apart from Commission, borne by the premium is 6*s.* per-cent of the sum assured, for term 2 years 10*s.* per-cent, term 3 years 14*s.* per-cent, the contribution increasing with the term of the Assurance by 4*s.* per-cent for each additional year until the term 16 years is reached, when 66*s.* per-cent is obtained, which is the full initial expenditure incurred by the office in procuring its ordinary business. These percentages are for terms one and two years, where the sum assured is £100, insufficient to meet even a 10*s. 6d.* medical fee, and it might therefore be well to stipulate that for policies of sums assured of less than, say, £250 the medical fee should be paid by the proposer unless he can be examined by a salaried medical officer without special expense to the office. As these contributions to the initial expenditure are spread over the whole term of each policy the office will lose a portion of the above contributions in the case of each assurance which lapses before the date of expiry. If a Whole-Life or Endowment Assurance Policy lapses or is surrendered the whole, or at any rate a considerable part, of the unpaid contributions to the initial expenditure can be deducted from the policy value. Short Term Assurances, however, acquire practically no reserve value, so that even the smaller contributions obtained from them cannot be regarded as in any way so well secured as the adequate contributions obtained from Whole-Life or Endowment Assurances. Before leaving the subject of the initial expenditure I may perhaps draw attention to the fact that for ages at entry 50 and 60, terms 1-3 years, the premiums charged by the offices are usually smaller than my specimen rates and provide for practically no initial expenditure.

I have not given the premiums for ages below 30, but, when these are calculated, it should be borne in mind that there is a considerable dip in the mortality curve somewhere about ages 25 to 30 and that, therefore, for short terms the rates of premium should probably be uniform for all ages below 30.

Another point to be remembered is that as the initial expenditure is constant at 66s. per-cent of the sum assured for all terms beyond 15 years the annual loading required to meet the initial expenditure becomes smaller the longer the term of the assurance, and, as a result of this, it will be found that for certain terms for the lower ages at entry the office premiums obtained by the ordinary formula would decrease as the term increased. Such is the case for age at entry 30, term 18 years, for which the office premium is 1d. per-cent less than that for term 17 years. Again for age at entry 20 there is practically no increase in premium between terms 15 and 25 years, the annual premium for 15 years being £1. 1s. 10d. per-cent and for term 25 years £1. 2s. 5d. per-cent. It is, of course, impracticable to publish rates which decrease with the term, and slight adjustments should therefore be made in the premiums in order to secure regular progression.

TABLE G.

Annual Premiums for Temporary Assurances of £100.

$$\text{Formula : } P_{x^n}^1 = 1.0474 \pi_{(x)}^1 + \frac{10\pi_{(x)}^1}{a_{(x)}^n} + .00068 + \frac{\alpha}{a_{(x)}^n}, \text{ where } \alpha$$

is the initial expenditure, apart from commission, provided for in the office premium.

Term of Assurance	Initial Expenditure provided for in Office Premium, per £100 Sum Assured	Age at Entry	Age at Entry	Age at Entry	Age at Entry		
						100α	30
1	6s.	£ s. d.	£ s. d.	£ s. d.	£ s. d.		
2	10s.	0 18 2	1 1 10	1 10 9	2 12 6		
3	14s.	0 19 7	1 2 5	1 11 9	2 14 6		
4	18s.	1 0 4	1 5 0	1 16 3	3 3 6		
5	22s.	1 1 1	1 6 2	1 18 6	3 5 2		
6	26s.	1 1 8	1 7 1	2 0 5	3 12 4		
7	30s.	1 2 2	1 8 0	2 2 2	3 16 1		
8	34s.	1 2 8	1 8 10	2 3 11	3 19 8		
9	38s.	1 3 1	1 9 7	2 5 6	4 3 1		
10	42s.	1 3 6	1 10 5	2 7 2	4 6 5		
15	62s.	1 5 7	1 14 6	2 15 3	5 2 6		
20	66s.	1 6 8	1 17 8	3 2 11	...		
25	66s.	1 8 2	2 1 5	3 10 3	...		

TABLE H.

Comparison of the Annual Premiums in Table G with those quoted by five first-class offices.

—	Term of Assurance Years	Age at Entry 30			Age at Entry 40			Age at Entry 50			Age at Entry 60			
		£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
Premium, Table G	.	1	0	18	2	1	1	10	1	10	9	2	12	6
Office (a)	0	18	7	1	0	8	1	5	9	...		
„ (b)	0	18	1	1	0	1	1	5	1	1	18	8
„ (c)	0	17	6	0	19	9	1	5	2	..		
„ (d)	0	18	10	1	0	3	1	5	9	2	0	9
„ (e)	0	19	3	1	2	2	1	9	4	2	9	2
Premium, Table G	.	3	0	19	7	1	3	8	1	13	11	2	18	9
Office (a)	1	0	6	1	3	2	1	11	1	...		
„ (b)	1	1	6	1	4	1	1	11	11	2	15	6
„ (c)	1	0	0	1	3	0	1	11	6	...		
„ (d)	0	19	7	1	2	7	1	11	3	2	17	5
„ (e)	1	0	2	1	4	5	1	13	2	2	17	3
Premium, Table G	.	5	1	1	1	1	6	2	1	18	6	3	8	2
Office (a)	1	2	0	1	5	7	1	16	5	...		
„ (b)	1	2	6	1	6	0	1	16	7	3	6	10
„ (c)	1	1	0	1	5	6	1	16	6	...		
„ (d)	1	1	4	1	5	3	1	17	0	3	10	7
„ (e)	1	1	3	1	6	2	1	17	0	3	6	0
Premium, Table G	.	7	1	2	2	1	8	0	2	2	2	3	16	1
Office (a)	1	3	2	1	7	6	2	0	6	...		
„ (b)	1	2	9	1	6	11	1	19	9	3	14	1
„ (c)	1	1	6	1	7	3	2	0	4	...		
„ (d)	1	2	6	1	7	2	2	1	6	3	19	7
„ (e)	1	2	7	1	8	1	2	0	9	3	13	11
Premium, Table G	.	10	1	3	6	1	10	5	2	7	2	4	6	5
Office (a)		
„ (b)	1	4	2	1	9	6	2	4	10	4	3	10
„ (c)	1	2	9	1	9	0	2	5	6	...		
„ (d)	1	3	10	1	9	9	2	6	9	4	10	2
„ (e)	1	4	7	1	10	5	2	5	5	4	4	0
Premium, Table G	.	15	1	5	7	1	14	6	2	15	3	5	2	6
Three other offices	{	...	1	5	8	1	13	9	2	13	11	...		
Three other offices	{	...	1	5	3	1	13	10	2	15	7	5	6	11
Three other offices	{	...	1	5	6	1	14	0	2	14	0	...		
Premium, Table G	.	20	1	6	8	1	17	8	3	2	11	...		
Three other offices	{	...	1	7	3	1	17	8			
Three other offices	{	...	1	7	0	1	17	11	3	4	2	...		
Three other offices	{	...	1	7	6	1	18	0	3	3	0	...		

MORTALITY TABLES EMPLOYED.

I should now like to make a few remarks in order to forestall criticism on account of the choice of the O^MSelect Table for Whole-

Life Non-Participating Premiums and the O^{NM} Select Table for Temporary Assurance Premiums. The O^{NM} Table is derived from the Whole-Life Non-Participating Experience of the British Offices and shows a mortality considerably higher than that of the O^M Select Table, which is the experience amongst the Participating policy holders and the question occurs whether in any one office the mortality in the Without-Profit Whole-Life Section will be so much higher than that in the With-Profit Section, as is shown by the relation of the O^{NM} to the O^M Mortality. Amongst the offices contributing to the O^{NM} Experience there are several which issue a very large number of Without-Profit Life Policies in connection with financial transactions; it is generally admitted that amongst such policies a high rate of mortality has been experienced, and an analysis of the O^{NM} Experience might possibly show that the higher mortality is mainly due to these financial policies. There will, of course, be a certain amount of adverse selection against every office, but the extent thereof will vary with each office and cannot be ascertained without a separate investigation. In an average office the mortality will probably be nearly the same in both classes, whereas in another transacting a large loan business the mortality in the Without-Profit Section will be much above that in the With-Profit Section. On account of the impossibility of measuring the extent of the selection against any particular company, and of the surmise that the "loan" policies have been one of the principal causes of the higher O^{NM} Mortality, I have employed the O^M Table for both With and Without-Profit Whole-Life Premiums. Of course the choice of the table used must depend upon the source from which the business of the office is drawn.

Temporary Assurances are mostly effected for financial rather than thrift purposes, and it is reasonable to suppose that the mortality will be higher in this class than in the Whole-Life Class. I have, therefore, adopted the O^{NM} Select Tables, although it is likely that in some offices a table showing even higher rates of mortality might be chosen. The British Offices Temporary Assurance Experience is not sufficiently extensive for practical use, and, as Mr. Ackland stated in the discussion on Mr. Elderton's paper on Temporary Assurances, the tables setting forth the experience of the 60 offices in this class are rather disappointing, and the more one probed it the less one was satisfied with the results brought out by that experience.

Pure Endowments payable at the end of a fixed term if a life shall live so long. With Return of all premiums paid in the event of previous death.

In Table J will be found the annual office premiums per £100 of Assurance obtained by the use of the undernoted formula, which provides merely for the renewal expenditure and a commission of 10s. per-cent of the sum assured on payment of the first year's premium and $2\frac{1}{2}$ per-cent of the renewal premiums. On comparing these premiums with those charged by first-class offices it will be seen that the published rates of the majority not only provide for no contribution towards the initial expenses but do not even cover the whole renewal expenditure.

TABLE J.

		Term 10 years		Term 15 years		Term 20 years
Formula :						
$P' \frac{1}{x^n} = \pi \frac{3\frac{1}{n}}{n} + .00068 + .0224\pi \frac{3\frac{1}{n}}{n} + .005 + a \frac{3\frac{1}{n}}{n} + .025\pi \frac{3\frac{1}{n}}{n}$						
$\left(\pi \frac{1}{x^n} \text{ (With Return) is taken to be } = \pi_n = \frac{1}{s_{n+1} - 1} \right)$		8 15 0		5 7 1		3 13 7
$= 1.0524\pi \frac{3\frac{1}{n}}{n} + .00083$						
Office—(1)	.	8 13	5	5 5 5	3 12 0	
(2)	.	8 9	5	5 4 5	3 12 4	
(3)	.	8 13	1	5 5 5	3 12 3	
(4)	.	8 13	8	5 7 1	3 14 2	
(5)	.	8 17	5	5 8 11	3 14 11	
(6)	.	8 17	4	5 8 8	3 10 4	
(7)	.	8 9	5	5 4 5	3 12 4	
(8)	.	8 15	10	5 7 8	3 14 0	

SURVIVORSHIP ASSURANCES.

I have experienced considerable difficulty in formulating a suitable method for loading Survivorship Premiums by reason

of the wide range of the risks, which vary from merely nominal risks to practically Whole-Life Assurances—thus, 20 versus 80 is a small risk, whereas 80 versus 20 approaches a Whole-Life Assurance payable on the death of 80. The premium for the former can only bear a comparatively small loading to provide for the initial expenses, whereas the latter should include the loading for the full proportion of all the expenses in the same way as a Whole-Life Premium since the office premium for the latter must not be much less than that for a Whole-Life Assurance on 80; *i.e.*, for these extreme cases the loaded premium must be equal to $1.090\pi_{xy}^1 + .00214$.

Regarding first the initial expenses—if the full proportion of these could be charged to each assurance the loading in respect of each policy would be

$$\frac{.033}{a_{xy}} = .033(\pi_{xy} + d_{.033}) = .033(\pi_{xy}^1 + \pi_{xy}^1) + .00111.$$

When the net annual premium is comparatively small it is evidently impracticable to attempt to obtain the full contribution in respect of initial expenses, as the resulting office premium would be unduly heavy.* If the survivorship premiums are loaded $.033\pi_{xy}^1 + .00111$, we shall then assume that the two survivorship assurances, which together form a joint-lives assurance, jointly make a somewhat larger contribution to the initial expenses than would be contributed by the office premium for an ordinary Whole-Life Assurance.†

Adding the full loading for renewal expenses and, allowing for an initial commission of 10 per-cent of the first annual

* $\pi_{[20][80]}^1$ O[NM] and O[af] 3% = .0604. $\frac{.033}{a_{[xy]}} = .00558$, which is equivalent to a loading of 92 per-cent of the net premium.

$$\begin{aligned} \dagger .033\pi_{xy}^1 + .00111 + .033\pi_{xy}^1 + .00111 &= .033(\pi_{xy}^1 + d) + .033(\pi_{xy}^1 + d) \\ &= .033(\pi_{xy} + d) + d \\ &= \frac{.033}{a_{xy}} + d \end{aligned}$$

i.e., contribution by both survivorship policies = $\left(\frac{.033}{a_{xy}} + d \right) a_{xy} = .033 + d a_{xy}$

premium and $2\frac{1}{2}$ per-cent of renewal premiums, the following formula results :

$$P_{xy}^1 = \underbrace{\pi_{xy}^1 + 0.033(\pi_{xy}^1) + 0.00111 + 0.00068}_{\text{Initial expenses.}} + \underbrace{0.0224\pi_{xy}^1}_{\text{Renewal expenses.}} + \underbrace{\frac{1\pi_{xy}^1}{a_{xy}} + 0.025\pi_{xy}^1}_{\text{Commission.}}$$

$\frac{1\pi_{xy}^1}{a_{xy}}$ is small, and we may assume a_{xy} on the average equal to 10,

in which case

$$P_{xy}^1 = \pi_{xy}^1 + 0.033\pi_{xy}^1 + 0.00111 + 0.00068 + 0.0224\pi_{xy}^1 + \frac{1\pi_{xy}^1}{10} + 0.025\pi_{xy}^1$$

$= 1.090\pi_{xy}^1 + 0.00179$, which is only .00035 (8d. % of the sum assured) less than that produced by the Whole-Life Formula of $1.090\pi + 0.00214$. I propose to adopt the latter formula, which would result in the office obtaining a further contribution to the initial expenses of $.00035a_{xy}$.

To summarise the effect of the adoption of the last mentioned formula :

- (1) The *two* survivorship policies, which together build up a joint-lives assurance, jointly contribute somewhat more than the proportion of the office initial expenses charged to an ordinary Whole-Life or Endowment Assurance Policy, and the formula in apportioning this contribution between the two survivorship assurances has some regard to the rate of premium, thus avoiding prohibitive loadings on account of initial expenses in the case of those combinations of ages, which produce very low net premiums.
- (2) Whenever the survivorship risk does not differ appreciably from that incurred under a Whole-Life Assurance the loading charged to the Survivorship Policy will be consistent with that contributed by a Whole-Life Policy.
- (3) Each Survivorship Policy bears its full share of the renewal expenses.

The following are some specimen premiums and, for comparison, those published by four first-class offices are also given ; the

tables of mortality employed are the O^{NM} Select Table for the Assured Lives and the O^{M} Select Table for the Counter Lives, and the rate of interest $3\frac{1}{2}$ per-cent, as before :

TABLE K.

Survivorship Assurance Premiums per £100 Sum Assured.

—	20 versus 60	40 versus 60	60 versus 60	20 versus 50	40 versus 50
Formula:	£ s. d.				
$1.090\pi_{(xy)}^1 + .00214$	1 0 2	1 16 3	5 8 5	1 1 9	2 2 2
Office (a) . . .	1 0 5	1 14 5	5 0 2	1 2 0	1 19 10
„ (b) . . .	1 0 0	1 14 6	5 2 10	1 1 2	1 19 4
„ (c) . . .	1 0 1	1 14 11	...	1 1 6	2 0 4
„ (d) . . .	0 18 8	1 14 11	5 5 2	1 0 5	2 0 2

It may perhaps be well to mention that the office should stipulate for payment of the medical fee in all those cases where the premium is a small one, unless the proposed assured can be examined by the ordinary medical officer without special expense.

SINGLE AND LIMITED PAYMENT PREMIUM.

In the case of policies effected by single premiums the office saves (1) the actual cost of collection of renewal premiums, and (2) the commission on renewal premiums.

The first item is incapable of assessment, but, in any case, it cannot be considerable and may be ignored.

The Whole-Life Single Premiums will, therefore, be obtained from the formula $A'_x = \{(1.090 - .025)\pi_{(x)}^{O^{\text{NM}} 3\frac{1}{2}} + .00214\} a_{(x)}^{O^{\text{NM}} 3\frac{1}{2}}$, the deduction of .025 from the percentage loading is made on account of the saving in the renewal commission. If, as is frequently the case, this saving is counterbalanced by a large initial commission allowance must be made therefor. The formula for the other classes of assurance would be similar in form.

As renewal commission is payable on limited payment premiums the rates will be obtained by commuting the ordinary office annual premiums by the appropriate annuity-values.

CONCLUSION.

I submit this paper to the Institute in the hope that it may be the means of drawing expressions of opinion from the more eminent members of the profession, first, upon my contention that no scheme of loadings can be defended, which has not for its basis an agreement between the actual expenses of management and the contributions to expenses by individual policyholders, and, secondly, upon the methods which should be employed to apportion equitably the expenses between the various policies comprising the business of an office. Although I have approached the question from the point of view of the office premiums it must not be forgotten that the methods adopted will have an important effect upon the relative rates of bonus which should be allotted to Whole-Life and Endowment Assurances.

Finally, I wish to express my indebtedness to the very valuable papers on Premiums written by Messrs. Moir & Chatham, to Mr. Ackland for his notes on the British Offices Temporary Assurance Experience and to Mr. Elderton for his paper on Temporary Assurance Premiums.

APPENDIX.

OFFICE NO. 1.—*Expenses of Management.*

Items	Mean of the Expenses in the year preceding and the year succeeding the date of the Valuation	PORTION ALLOCATED TO			
		(b) Renewals			
		(a) New Business	As a percentage of the Sums Assured in force	As a percentage of the Net Premiums	
Rents, &c.	£1,405	£703	£351	£351	
Improvements	554	277	139	138	
Salaries	12,506	7,425	2,541	2,540	
Directors' Fees	904	452	226	226	
Auditors' Fees	219	55	164	...	
Printing	962	481	240	241	
General Expenses	1,961	980	490	491	
Travelling Expenses	327	327	
Advertising	518	518	
Medical Fees	3,604	3,604	
Stamps	1,452	726	726	...	
Legal	171	...	171	...	
	£24,583	£15,548	£5,048	£3,987	£9,035
 Mean of the New Business effected in year preceding and year succeeding date of Valuation		Number of Policies and Sums Assured in force at date of Valuation		Mean of the Net Premiums corresponding to the Office Premiums in the Revenue Accounts in the years preceding and succeeding the date of Valuation	
Number of Policies 2,165		Sums Assured £667,099		£28,727 Policies for Sums Assured of £8,044,663 *	
 Per £100 of New Business Sum Assured		Per £100 of Sum Assured in force		Per £100 of Net Premium	
Rates of Expenses:		2·33		·063 1·85	
(Method A)		
(Method B)		2·33		Per £100 of Net Premium 4·18	
(Method C)		2·33		Per £100 of Sum Assured in force ·112	
(Method D)		Per Policy 6·31		Per Policy in force ·314	

* The number of Policies and Sums Assured are the Totals, excluding Temporary Assurances, Endowments, Deferred and Survivorship Assurances, &c., which are unable to bear their full share of the expenses of management. The total of these special classes, which have been excluded, is small.

† Net Premiums taken as Revenue Account Office Premium Total

$$\times \frac{\text{Total Net Premiums per Valuation Returns}}{\text{Total Office Premiums per Valuation Returns}}$$

Office Premiums required by Office No. 1 in order to meet its Expenses of Management.

*Commission assumed £1 per-cent of Sum Assured on payment of First Year's Premium and $2\frac{1}{2}$ per-cent of Renewal Premiums.
Interest $3\frac{1}{2}$ per-cent.*

Method A :

$$\pi + \frac{.0233}{a} + .00063 + .0185\pi + \underbrace{\frac{.01}{a} + .025\pi}_{\text{Commission.}} = 1.077\pi + .00176$$

Method B :

$$\pi + \frac{.0233}{a} + .0418\pi + \frac{.01}{a} + .025\pi = 1.100\pi + .00113$$

Method C :

$$\pi + \frac{.0233}{a} + .00112 + \frac{.01}{a} + .025\pi = 1.058\pi + .00225$$

Method D :

For a Contract of £100 Sum Assured.

$$100\pi + \frac{6.31}{a} + .314 + \underbrace{\frac{1.00}{a} + .025 \times 100\pi}_{\text{Commission.}} = 109.8\pi + .561$$

For a Contract of £10,000 Sum Assured.

$$10,000\pi + \frac{6.31}{a} + .314 + \underbrace{\frac{100}{a} + .025 \times 10,000\pi}_{\text{Commission.}} = 10,356\pi + 3.9$$

OFFICE No. 2.—*Expenses of Management.*

Items	Mean of the Expenses in the year preceding and the year succeeding the date of the Valuation	PORTION ALLOCATED TO			
		(a) New Business	(b) Renewals		
			As a percentage of the Sums Assured in force	As a percentage of the Net Premiums	£
Rents, &c. . .	3,041	1,521	760	760	
Salaries . . .	14,565	8,648	2,959	2,958	
Directors' Fees . . .	5,804	2,902	1,451	1,451	
Auditors' Fees . . .	488	122	366	...	
Actuary's Fees . . .	123	...	123	...	
Printing . . .	2,168	1,084	542	542	
Furniture . . .	290	145	72	73	
Agents' Salaries and Travelling Expenses	4,348	4,348	
Agency Extension . . .	1,504	1,504	
Medical Fees . . .	2,419	2,419	
Policy and other Stamps	1,352	676	676	...	
Legal Expenses . . .	295	...	295	...	
Messengers' and Petty Expenses . . .	2,558	1,279	630	640	Total Renewal Expenses
Valuation Expenses . . .	1,973	...	987	986	
Total . . .	£40,928	£24,648	£8,870	£7,410	£16,280

	Mean of the New Business effected in year preceding and in year succeeding date of Valuation	Number of Policies and Sums Assured in force at date of Valuation		Mean of the Net Premiums corresponding to the Office Premiums in the Revenue Accounts for the years preceding and succeeding the date of Valuation
		Number of Policies	Sums Assured	
		5,576	£747,198	£330,038 †
Rates of Expense (Method A)			Per £100 of New Business Sum Assured 3·30	Per £100 of Sum Assured in force ·065 Per £100 of Net Premium 2·24
Rates of Expense (Method B): Renewal Expenditure treated entirely as a percentage of the premium income . . .			3·30	Per £100 of Net Premium 4·93
Rates of Expense (Method C): Renewal Expenditure treated entirely as a percentage of the Sums Assured in force . . .			3·30	Per £100 of Sum Assured in force ·125
Rates of Expense (Method D)			Per Policy 4·42	Per Policy in force ·189

* The number of Policies and Sums Assured are the totals, excluding Temporary Assurances, Endowments, Deferred and Survivorship Assurances, &c., which are unable to bear their full share of the expenses of management. The total of these special classes, which have been excluded, is small.

† Net Premiums taken as Revenue Account Office Premium Total

 Total Net Premiums per Valuation Returns.

 × Total Office Premiums per Valuation Returns.

Office Premiums required by Office No. 2 in order to meet its Expenses of Management.

*Commission assumed £1 per-cent of Sum Assured on payment of First Year's Premium and $2\frac{1}{2}$ per-cent of Renewal Premiums.
Interest $3\frac{1}{2}$ per-cent.*

Method A:

$$\pi + \frac{.0330}{a} + .00068 + .0224\pi + \underbrace{\frac{.01}{a} + .025\pi}_{\text{Commission}} = 1.090\pi + .00214$$

Method B:

$$\pi + \frac{.0330}{a} + .0493\pi + \frac{.01}{a} + .025\pi = 1.117\pi + .00146$$

Method C:

$$\pi + \frac{.0330}{a} + .00125 + \frac{.01}{a} + .025\pi = 1.068\pi + .00271$$

Method D:

For a Contract of £100 Sum Assured.

$$100\pi + \frac{4.42}{a} + .189 + \underbrace{\frac{1.00}{a} + .025 \times 100\pi}_{\text{Commission}} = 107.9\pi + .373$$

For a Contract of £10,000 Sum Assured.

$$10,000\pi + \frac{4.42}{a} + .189 + \underbrace{\frac{100}{a} + .025 \times 10,000\pi}_{\text{Commission}} = 10,354\pi + 3.7$$

OFFICE NO. 3.—*Expenses of Management.*

Items	Mean of the Expenses in the year preceding and the year succeeding the date of the Valuation	PORTION ALLOCATED TO			
		(a) New Business	As a percentage of the Sums Assured in force	(b) Renewals	As a percentage of the Net Premiums
Rents, &c. . . .	1,456	728	364	364	
Furniture	296	148	74	74	
Incidental Expenses	495	247	124	124	
Salaries	4,727	2,807	960	960	
Directors' Fees	2,171	1,086	542	543	
Auditors' Fees	218	54	164	...	
Printing	899	450	225	224	
Agency Expenses and Advertising	5,227	5,227	
Medical Fees	1,126	1,126	
Stamps	697	348	349	...	
Legal Expenses	180	...	180	...	
Valuation Expenses	327	...	161	163	Total Renewal Expenses
	£17,819	£12,221	£3,146	£2,452	£5,598

	Mean of the New Business effected in year preceding and in year succeeding date of Valuation	Number of Policies and Sums Assured in force at date of Valuation	Mean of the Net Premiums corresponding to the Office Premiums in the Revenue Accounts for the years preceding and succeeding the date of Valuation
	Number of Policies 854	Sums Assured £326,345	18,203 Policies for Sums Assured of £6,915,533 *
		Per £100 of New Business Sum Assured	Per £100 of Sum Assured in force
Rates of Expense:		3·74	·045
(Method A)	...	3·74	1·21
(Method B)	...	3·74	2·76
(Method C)	...	3·74	·081
(Method D)	...	Per Policy 14·30	Per Policy in force. ·307

* The number of Policies and Sums Assured are the totals, excluding Temporary Assurances, Endowments, Deferred and Survivorship Assurances, &c., which are unable to bear their full share of the expenses of management. The total of these special classes, which have been excluded is small.

† Net Premiums taken as Revenue Account Office Premium Total

Total Net Premiums per Valuation Returns

× Total Office Premiums per Valuation Returns

Office Premiums required by Office No. 3 in order to meet its Expenses of Management.

Commission assumed £1 per-cent of Sum Assured on payment of First Year's Premium, and $2\frac{1}{2}$ per-cent of Renewal Premiums. Interest $3\frac{1}{2}$ per-cent.

Method A :

$$\pi + \frac{.0374}{a} + .00045 + .0121\pi + \underbrace{\frac{.01}{a} + .025\pi}_{\text{Commission.}} = 1.085\pi + .00205$$

Method B :

$$\pi + \frac{.0374}{a} + .0276\pi + \frac{.01}{a} + .025\pi = 1.100\pi + .00160$$

Method C :

$$\pi + \frac{.0374}{a} + .00081 + \frac{.01}{a} + .025\pi = 1.072\pi + .00241$$

Method D :

For a Contract of £100 Sum Assured,

$$100\pi + \frac{14.30}{a} + .307 + \underbrace{\frac{1.00}{a} + .025 \times 100\pi}_{\text{Commission.}} = 117.8\pi + .824$$

For a Contract of £10,000 Sum Assured,

$$10,000\pi + \frac{14.30}{a} + .307 + \underbrace{\frac{100}{a} + .025 \times 10,000\pi}_{\text{Commission.}} = 10,364\pi + 4.2$$

ABSTRACT OF THE DISCUSSION.

Mr. H. J. P. OAKLEY said that the subject of the paper presented that evening was one of very practical interest, and appeared to be the outcome of a former paper by the same author on "Comparative Bonuses under Whole-Life and Endowment Assurances," read to the members about three years ago (*J.I.A.*.,

vol. xli, p. 273). Therein he dealt with the allocation of profits from the three sources—interest, mortality and loadings; and in connection with the latter item made a short investigation into the effect of various changes in the method of loading the premiums of the respective classes. In the present paper, the author took up the important question of expenses, in order to ascertain how the loadings should in equity be made, and with that object in view, had sought to evolve a method of apportionment of expenses, which might be applied to all the principal classes of policies. Having made an analysis of the expenditure of different existing offices, he gave four different methods of fixing the necessary loadings. The particular method favoured by the author (method A) was the division of expenses into initial and renewal, the former being charged on new business sums assured, while the latter (*i.e.*, the renewal expenses) were further divided into two portions, one portion being charged in the same manner as the initial expenses, the second being charged as a percentage of the premium income. He did not propose to discuss the fractional divisions themselves, as set out in Tables A and B, for they were, on the author's admission, empirical, and would doubtless vary in almost every office. The more important point seemed to be, whether the method itself was a correct one, inasmuch as it was based on the total premium income of an existing office (thus including indiscriminately all classes of assurance) and then applied equally to individual classes. Temporary assurances, endowments, deferred and survivorship assurances had, it was true, been excluded, as being unable to bear their full share of the expenses of management. But no elimination appeared to have been made of the corresponding expenses applicable to those classes, so that the remainder of the assurances were unduly burdened. The total of those special classes was stated to be small, but the actual number was not given.

The author had departed from the usual practice in making the percentage loadings an addition to the pure or net premium, and had taken the proportion of renewal expenses, charged as a percentage of the premium, on the valuation pure premiums. The valuation bases of the offices chosen as the examples were not given, and any variation therein would affect the results. He questioned whether the method of taking the ratio of expenses to the valuation pure premiums was advisable, on the following grounds:—(1) it was the actual premium which had to bear the expenses, (2) it was the usual everyday practice to consider expenses in the light of gross premiums, (3) any change in the valuation basis would, by altering the pure premiums, affect the ratio of expenses to premium income, which would not really be an actual fact. The change in loadings by the change of valuation premiums only affected the respective contributions to surplus from valuation loadings and excess interest.

Besides the primary method of the paper (method A), other methods, or rather variations of the method based on the same details as to expenses, premiums, etc., were mentioned, but some-

what summarily dismissed. Method B did not further divide the portion assumed to represent renewal expenses, but charged the whole as a percentage of the premium income, thereby throwing too great an expense upon the highly-rated policies. Method C charged all the renewal expenses (excluding commission) as a percentage of the sum assured, which had the reverse effect to method B, and told heavily on the low-rated premiums; whilst the fourth method, D, allocated all expenses (except commission) according to the number of contracts, thereby limiting the field to a particular class of business. Method A, which the author had chosen, was a compromise between methods B and C, but he thought that in practice some consideration might be given to method D, and, inasmuch as some portion of renewal expenses were undoubtedly involved per contract, it might be a matter for actuarial discretion in certain individual cases as to whether some allowance should be made.

He submitted that no hard and fast rule could, under any of the methods, be universally applied, and that instead of apportioning the whole of the expenses of an office on a particular basis, and then applying that basis as the foundation of the loadings for all classes, regard should be had to the expenses incurred and likely to be incurred in respect of each distinct class. Further, he submitted that it was a fundamental principle of all business transactions that greater expense might be incurred in respect of business yielding greater profit, and that, when incurred, it should be charged to such business. Now, in life assurance, the two main sources of real profit (excluding loading, which they were discussing) were excess interest and light mortality. The yield, therefore, which was anticipated according to various classes from the two sources mentioned should form some basis for the expense loadings.

If the method of first analysing past expenses and then applying the loading to meet them was to be correctly applied, the analysis should be more exhaustive, and should seek to apportion the expenses not merely between new and renewal, to be charged upon sums assured or premiums, but also between the various classes. Such an analysis, would, he was afraid, be very difficult, and would still tend to be empirical. The better method was to adjust the loadings with a view to present and probable future expenses, and having so fixed them, to endeavour to keep the expenses within the estimates. Moreover, the class of business on the books might differ from that at present being entered into, a further reason for the prospective, rather than the retrospective, view. In that connection he would point out that one effect of the continuous proportionate increase of endowment assurance new business, was that, as the old whole-life business at present on the books gradually passed off, the average rate of premium per-cent increased, and if the actual expenses remained the same, the ratio of expenses to premium income should fall. That, however, did not generally appear to be the case—at least to any extent—so that the expenses might really be changing in amount to an extent greater than the ratios indicated. Turning

to points of detail, he noticed that the author remarked in the second paragraph that the loading for commission required no discussion. Personally, he thought that the question was a very debateable one. It was true that having fixed a certain scale, no discussion was necessary as to how the premium should be loaded therefor; but the commission assumed by Mr. Rietschel in his calculations was 1 per-cent on the sum assured, and $2\frac{1}{2}$ per-cent on renewals. Now, while the majority of ordinary agents were undoubtedly working on those terms, it was well to bear in mind the not uncommon additional expense of over-riding commission, as well as the sometimes liberal terms granted to special agents. He believed, therefore, that to be on the safe side, a larger allowance should, at least with many offices, be made to cover that particular expense.

Whilst the subject of commission was before them, he thought they should also consider it in the light of the very heavy initial expenses, as shown by Mr. Rietschel, and the loss which was thrown upon the office in the case of lapse during early years. A system whereby that loss might be lessened would be to spread the initial commission over the first two or three years, *i.e.*, until the business was really established. The comparison of premiums, as calculated by the author, with those of first-class offices was very interesting, but he thought the tables in which such comparisons were made might be improved by the additional item of the average premiums of those offices. Mr. Rietschel, throughout, gave his own suggested premiums on the one basis only which he had chosen, namely, method A, merely remarking that the differences between the premiums produced by methods A, B, and C were inconsiderable. It would have been interesting to have seen the actual results of those two other methods, especially in their relative effect on the rates for the principal classes of assurance.

MR. J. F. LITTLE said that the paper appeared to him to be of importance as indicating what portion of the with-profit premium rates charged would be required to provide, under existing conditions, for the payment of the sum assured, and what balance would consequently be available for bonus purposes. Incidentally, he might mention that the statement, "It is at present the universal practice to charge definite rates of premium per-cent of the sum assured, regardless of the amount of the policy," was not quite correct, inasmuch as he knew of at least one office that charged a lower premium per-cent for policies of £500 and upwards than for those below that amount. Looking at the apportionment of expenses of management in Table A, as Mr. Rietschel said, the division was entirely empirical, and it seemed to him to be altogether impossible for an actuary to say precisely how much of each particular item of expenditure could be classed one way or the other. Passing to Table C, and also referring to Office No. 2 in the Appendix, the renewal expenses appeared to be stated as a percentage of the net premiums. It was rather curious to note, however, that the author had been able to obtain the net premium income for an office with an

expense rate of 15 per-cent, which one would suppose would not be valued at $3\frac{1}{2}$ per-cent, and therefore would not quote its net premium at that rate. He was rather puzzled, therefore, as to the sum of £330,000, stated to be the average net premium income over two years preceding and following the date of valuation. Also, it did not appear to be proper to charge $2\frac{1}{2}$ per-cent of the net premiums as the renewal commission, when, in point of fact, it was paid on a considerably larger amount, quite 20 per-cent greater, so that the actual commission on the net premiums would be nearer 3 per-cent than $2\frac{1}{2}$ per-cent. If method B were examined, it would be noticed that the percentage of net premiums for renewal expenses, apart from commission was 4·93. On the assumption, which he thought was pretty near the fact, that the net premiums would require a loading of about 20 per-cent on the average, that 4·93 per-cent became just a trifle under 4 per-cent of the office premiums. Adding $2\frac{1}{2}$ per-cent commission, a renewal expenditure of $6\frac{1}{2}$ per-cent of the renewal premiums was arrived at. He was afraid that was too little. He did not think that so low a rate was achieved by any ordinary commission-paying office. Taking the other side of the statement, the expenses for new premiums were 86s. per-cent on the sum assured, an amount that a little surprised him. In order to see whether he had misjudged the skill and ability displayed in the management of British companies, he looked up the figures of one or two of the offices which obtained large amounts of new business. In one case he found that 86s. per-cent. on the new sums assured was a great deal more than the total expenditure for the year: and in several other cases that percentage gave an amount so close to the total that it was obvious those particular offices, at least, were getting their new business at a considerably less cost than 86s. per-cent. The effect of the author's assumption was to increase the premium rates and, to some extent, account for the high rates brought out. For two or three reasons it seemed to him that the premiums erred on the side of excess. In experimenting with some of the author's figures, and transferring about half the new business expenditure, irrespective of the £1 per-cent commission, to the other side, which would bring out a renewal expense rate of about 9 per-cent on the office premiums, and a total new business-expense of £2 13s. per-cent on the sum assured, he found that at the younger ages, at any rate, there would be a reduction of something like 1s. per-cent in the whole life rates, although the loading for renewal commission had been taken on the office premiums.

There was no mention made in the paper of what was rather an important matter, the presumed profit from surrenders and lapses. His own personal experience satisfied him that lapses provided no profit to the office. He had worked in West Australia, where lapses were unfortunately very heavy, and found that the members who allowed their policies to lapse, usually paid from three to twelve months' premiums—very rarely more—before they decided to secede. On the other hand, the phrase "profit from surrenders" was used, rather loosely, to represent

the difference between the reserves held in hand on account of a particular policy, which, as they all knew, was merely an average, and the amount paid as the surrender value. The deduction was based on two grounds, first, that owing to the heavy initial expenses the member who retired before passing out by death or maturity had not contributed to the general fund the proper portion of expenses. The chief deduction, however, was made to counteract selection against the office. When one came to calculate the premiums, using the table the author had used, the OM, it must be remembered that in that table, effect has already been given to the deterioration in the general mortality occasioned by the passing out of the surrenders, and therefore in calculating "cost price" premiums, the last mentioned deduction from the reserve, in the event of surrender, should in some way be involved. He admitted the calculation was not an easy one, but if an actuary were considering the matter from the point of view of his own particular office, he was in a position to make some sort of estimate as to the proportion of surrenders that would occur from time to time, and if he found that the amount was significant, it should be brought into account, and an estimate made of the amount of the reduction of premium that could be allowed on that head.

He desired, in concluding his remarks, to add a personal note. He believed he was the first Australian who had passed through the whole of his examinations in Australia, and who had had the opportunity of addressing his fellow-members in the home of the Institute. In speaking for the first time, he wished to express, on behalf of his fellow-Australians and himself—and there were other Australians present who, he was sure, would agree with him—the deep regard that Australians who had joined their ranks felt for the Institute, for giving them the opportunity, by conducting examinations in the Colonies, of qualifying themselves—not merely in fact but in the eyes of the world—to become actuaries and to exercise their profession; to the not inconsiderable benefit of the life offices conducting business there, and of friendly societies and other bodies who were in need of actuarial advice.

MR. H. E. MELVILLE thought it would have been better if Mr. Rietschel had, in the first place, apportioned the expenses among the various classes of business. That matter was of some importance when one considered that the average sum assured under endowment policies was usually less than under whole-life policies, and that there was a tendency for the average sum assured under with-profit policies to be less than that under non-profit policies. With regard to the various methods of allocating expenses referred to by the author, personally he rather favoured method C. Mr. Rietschel, in dealing with the renewal expenditure, referred first to the minimum expense, and then to the further expense, which he said would be regulated by the size of the premium income and would take the form of a percentage of such income. Although that further expense might be regulated by the premium income, he did not see that that was any reason why it should be allocated to the

individual policies as a percentage of the premiums, and he thought it would be better to treat it as a percentage of the sums assured. In Table A the author suggested in his first item that rents, rates, taxes, etc., should be equally divided between the new business and the renewals. Rents, rates and taxes, however, included branch expenses, and it seemed to him that a great deal of the branch expense was new business expense. Item (10)—policy, receipt and other stamps—the author suggested should be divided equally between the new business and the renewals. This item, however, could be analysed without any great trouble, more accurately, he believed, than Mr. Rietschel suggested. Policy stamps were a new business expense, and their cost was known accurately; the cost of the stamps used on renewal receipts could also usually be ascertained. The balance, perhaps, might be divided between new business and renewal expenses.

MR. C. W. KENCHINGTON said the author suggested that the offices should periodically examine their rates of expenses, in order to enable them to judge how their premium loadings were meeting the expenses. It seemed to him that at no time in the history of an office was it likely to be able so to estimate its expenses that it would be able to tell from its past expenditure what its future experience in that direction was likely to be; at any rate it would certainly be a matter of difficulty. With regard to the rates that might be fixed as a standard at one particular time, it seemed to him to be very doubtful if an office would be in a position of being able to charge one fixed standard premium over any considerable period. There were a few points of detail to which he wished to refer. Mr. Rietschel stated that the survivorship assurance premiums which he submitted were calculated on the basis of the Combined O_{NM} and O^[a,f] Tables at 3½ per-cent interest. It would be of interest to the members if the author had any figures at that rate of interest available to which reference could be made, because so far as he was aware the published rates were at 3 per-cent only, although the premiums based on the 3 per-cent values differed to an inconsiderable extent from those which Mr. Rietschel had given. It seemed to him that in analysing the expenses in the way the author had done it would have been an advantage if he could have indicated, with regard to the three offices investigated in the Appendix, the various proportions of the whole life and the endowment assurance business, because he believed it would be found that the average sums assured in those two classes differed very considerably.

MR. H. E. W. LUTT said that the author, in the course of his paper, had referred to the difference in value of large and small policies to life offices. That difference certainly did exist, which he was able to prove by the following figures abstracted from a comparison of certain average sums assured per policy, and expense ratios. He divided the offices in three classes, namely, those with an average sum assured of (1) over £750; (2) between £400 and £750; and (3) under £400; and obtained the following results:

Group	No. of Offices included	Average sum assured per policy of the group	AVERAGE OF GROUPS	
			Ratio of expenses to premium income	New business expense ratio*
		£	Per-cent	Per-cent
(1)	6	1,100	14	78½
(2)	27	549	15	92
(3)	10	300	16	113

* After allowing $7\frac{1}{2}$ per-cent of the premium income for renewal expenditure.

Those differences indicated the well-known fact that large policies cost less in proportion to the sum assured and premium ; but the differences between the groups were not so large as some of those existing between offices of the same group, and he thought that the point was not of such extreme importance as was apparently suggested in the paper. The tendency had been met in one direction mentioned by Mr. Little, namely, encouraging policies for larger sums assured by somewhat reducing the corresponding premiums. That system was known in this country, and he believed it was also not unusual to allow a slightly larger initial commission on policies for larger sums assured.

One other point which had been raised by Mr. Rietschel was the application of the same process of loading to policies of different classes. The author seemed to imply that insurance companies quoted their term and survivorship rates as a sort of "window-dressing"—that they did not like them, but had to do the business in order to keep their wares before the public. He did not think that was the case, because the expenses attached to policies of that class were so very much less than for ordinary endowment or life assurances. In the first place, they were not advertised and actively sought for, so that there were no branch or inspection expenses specially incurred: indeed, where branch managers were remunerated by an over-riding allowance, it was not unusual to exclude such policies from the account. Secondly, there was generally a minimum policy specified, or else the proposer had to pay the medical fee.

MR. D. C. FRASER said that every actuary must feel the necessity for clearly realising what were the expenses of obtaining new business for his own office, and many of those present must have attempted to make an apportionment such as Mr. Rietschel had brought before them, and probably before reading the paper they had already realised the difficulties and the pitfalls of the question. In the first place, he wished to refer to one or two matters of detail. He was in entire agreement with Mr. Little, whom he congratulated on his very able and penetrating remarks, in his protest against the idea that the ordinary expenses of obtaining new business among British companies could be placed at such a high figure as £4 6s. per cent. In examining the figures for office No. 2, he thought it must

be granted that the apportionment of expenses between the new business and the renewal business was extremely arbitrary. The valuation expenses were said to be nearly £2,000 annually, or for a quinquenium £10,000, and in an office where the valuation cost £10,000, the salaries applicable to renewal business were stated at only £6,000, or less than 1s. per-cent. on the sums assured, about 2s. per-cent. on the funds, or about 18*d.* per policy. It seemed to him that the figure for salaries could not be defended, and that a far larger proportion must be carried to the renewal business. Turning to the general aspects of the paper he felt that his disagreement with the author was of a fundamental character. The author's suggestion was that expenses should be apportioned in a certain manner between new and renewal business, for the purpose of the equitable distribution of profits among various classes of policies. Considering the question from that point of view, it was impossible to overlook the fact that in valuations with a view to distributing profits the ordinary practice of British companies was to calculate reserves upon a net premium basis.

There was another fact of importance which could not be disregarded. A sufficient amount of new business was necessary for the continued existence of a company—necessary to ensure that it should be able to keep its policyholders assured until the termination of their contracts; and therefore the cost of new business up to a certain point might be legitimately held to be a part of the expense incurred in maintaining the existing business on the books. At any rate, so far as new business was required to replace the waste due to claims, the charge might properly be placed upon the whole of the business. So far as new business was required to replace the waste due to surrenders and lapses, the charge might properly be thrown upon the policies which had surrendered or lapsed, and be recovered from the reserves on those policies. That principle gave a limit to the amount which could be allowed for the surrender of a policy, namely, the actual reserve, less the cost to the office of securing a new assurance on a healthy life to replace the surrendered or lapsed policy. If new business were limited to the amount required to maintain the total volume of business, he thought it was clear that there were no special expenses to be charged against the new policies in connection with a distribution of profits. And the argument could be carried a step farther. There was, in practice, an additional margin of profit from surrenders and lapses, over and above the amount required to replace those policies, and that margin supplied the means of maintaining a steady and moderate increase in the total volume of business; and there might be other miscellaneous profits of a minor character available for the same object. An office which kept its expenditure on new business within those limits had, in his opinion, no reason to make any special charge against new policies when apportioning profits. It did not trench upon the normal margins from interest, loading and mortality; it could appropriately make net premium reserves for new policies as well as old; and there were, he believed, many British offices in that position. If an office

exceeded the limit of expenses that he had indicated, the additional expenses could only be provided out of the normal profits from interest, loading and mortality, and must reduce the bonus-giving power of the company, since a bonus could not be given out of money that had been otherwise spent.

With regard to the question of the loading on premiums at different ages and for different classes of whole-life and endowment assurances, the loadings for commission and for profits could be dealt with as indicated by Mr. Rietschel in his paper; as regarded the loading for expenses other than commission, he thought it a fair and reasonable arrangement that the present value of such loading at the date of issue should be a constant amount for each £100 assured. If that were accepted as a fair working principle, then it was easy for an actuary to ascertain the pressure of expenses in his own particular company with very great exactness, and it was not difficult to make a very fair estimate of the burden of expenses for other companies. He did not claim any novelty for those views, he had attempted only to interpret and to express what he believed to be the principles which lay beneath the ordinary and general practice of British companies, and which were latent, if not expressed, in such a paper as that of Mr. Lidstone on the "Distribution of the Divisible Surplus of a Life Assurance Company" (*J.I. A.*, vol. xxxii, p. 73).

MR. L. F. HOVIL said that Mr. Rietschel commenced his paper by analysing the items that went to make up the general expenses of the management of an office, and then experimented with those results on the figures of three offices. The author then obtained the loadings and the percentages on which his premiums were based. Turning to Table A, he thought every one would agree that Mr. Rietschel had allowed the new business to bear too heavy a proportion of the total expenditure. If one began by looking at the figure of 86s., which previous speakers had referred to, a moment's thought convinced him there must be something wrong in Table A. He realised what a difficult thing it was for anybody to analyse the expenses of another office. For instance in the item of "Rents" it was well-known that offices which had the aim of keeping their management expenses very low as a means of attracting business sometimes owned their own premises, and, at the expense of their interest income, did not charge full rents for those offices. If such a thing as that occurred in connection with any apportionment, the figures were immediately thrown out. Taking another item, "Salaries," in the case of some companies he believed salaries would include all the branch managers and inspectors' salaries under a general heading of "Agency Expenses," and so on. Coming to the actual premiums themselves, he was afraid he did not agree that No. 2 office was sufficiently an average to be a general guide as to what without-profit premiums should be. Mr. Rietschel's method of apportionment of expenses was, however, useful for many other purposes. It was useful, for one thing, to show that the seemingly arbitrary additions or constants in the formulas which were in use fairly squared with the additions necessary.

In connection with that point he had looked up Dr. Sprague's old formula, with which the members were all acquainted, which, when reduced to the form in which the author had used it, was very little different from the author's. Dr. Sprague's formula was $1 \cdot 086\pi + 3s. 5d.$ as a constant; while the author's formula was $1 \cdot 090\pi$ and $4s. 3d.$ as a constant. In other words, if premiums were got out based on Dr. Sprague's formula they would practically be the same as Mr. Reitschel's without-profit premiums, less a shilling throughout, as regarded the whole-life premiums at all events. The author had anticipated some criticism as to the use of the $O^{[M]}$ Table rather than the $O^{[NM]}$ for whole-life without-profits. Personally, he was with those who thought it would have been better if he had used the latter Table. He thought there was a certain amount of provident business done in without-profit assurances, but it was provident business done on the lives of small people for very small amounts, people to whom a shilling for two in the rate per-cent was a consideration. The mortality attaching to that class might be nearly up to the $O^{[M]}$, but it was an expensive business, and the opportunity should therefore be taken of charging them the higher rates under the $O^{[NM]}$ Table. The $O^{[M]}$ Table was quite sufficient for endowment assurance without-profit rates. There was a very curious coincidence to which he wished to refer, in connection with the premiums the author had got out for the whole-life with-profit policies. In the year 1835, Mr. Charles Ansell got out some premiums for various offices, with one of which he was connected. He did not know what mortality table and what loadings Mr. Ansell used, but only the rate of interest, namely, 3 per-cent. In spite of all the factors differing, Mr. Ansell's rate for age thirty was £2 10s. 2d. against the author's £2 10s. 1d., and for age forty £3 6s. 3d. against £3 6s., and the premium for age fifty was 4d. per-cent less.

THE PRESIDENT said that in the first place he wished to thank Mr. Little for what he had said as to the use the Institute had been in the Antipodes to those who were struggling, not only to make themselves masters of actuarial science, but to prove to the world that they had made themselves competent. He was sure it was a satisfaction to all the members to feel that the influence of the Institute was so far-reaching as Mr. Little had stated. There were very few points of any importance in the paper which had not been already adequately dealt with, and he would therefore only mention two minor points which were perhaps of some interest. In the discussion which had taken place reference had only been made to loadings to cover expenses; but he would like to point out that in forming a scale of office premiums, even if those premiums were non-profit premiums, it was necessary to bear in mind the fact that it was desired to get some profit, and that it was necessary to have a profit loading even for non-profit business. That was an important point to remember, especially when classes of insurance were being dealt with, such as short-term assurances and contingent survivorship assurances, where it did not pay a company to take the theoretical premiums and put on a loading which barely covered the expense.

A certain risk had to be met—not merely the mathematical risk of the particular policy which was being dealt with, but also the risk that on those small classes of policies they might on the whole lose by adverse fluctuations, and that had to be taken into account.

He wished also to remark that, in considering that part of the office expenses which was usually assigned to the renewal business, a very large part of the expenditure was due to the fact that there was a considerable fund which had to be managed. The work of the directors was largely concerned with the management of the insurance fund; certainly the principal business of the actuary or manager was largely concerned with that question, and that expenditure could not be expressed in the form of a ratio on the renewal premiums: it was rather a ratio on the reserve values of the policies. In other words, it could be best expressed by making a reduction from what might otherwise be considered a safe rate of interest to assume. If it was assumed that the office would make $3\frac{3}{4}$ per-cent, he thought it was a very fair suggestion that $\frac{1}{4}$ per-cent should be taken off that amount to cover the cost of obtaining the interest and managing the funds, and that the premiums should be based on the lower rate. He concluded by asking the members to pass a cordial vote of thanks to Mr. Rietschel for his paper.

The resolution was put and carried with acclamation.

MR. H. J. RIETSCHEL, in reply, said that in writing the essay, he had been painfully conscious of the fact that the apportionment of the expenses was, as he had stated, to a considerable extent empirical, and as he had expected it had been called into question. It must be borne in mind, as Mr. Hovil had pointed out, that he was dealing with offices the detailed analysis of whose expenses was not available, and subsequently he was forced to take the rough-and-ready proportions given in Table A. If he had had the full particulars before him, it was quite possible he would have dealt with the matter somewhat differently, although he thought the ratios given would be substantially confirmed; but the main point which he wished to make was the necessity for bringing the premium loadings into agreement with the expenses of management, and that question had not, as far as he was aware, been previously discussed at the Institute. Mr. Oakley mentioned, in connection with temporary assurances, that he (the author) was wrong in endeavouring to charge the same amount of expenditure to temporary assurances as to other classes of policies, and that the better plan would be to analyse the expenses according to classes of assurance. Personally he failed to see how that was possible. For instance, turning to Table A, he could not understand how one could analyse "rents," and say that so much was due to temporary assurances and so much to endowment assurances, and the same remarks applied to salaries and other items. With regard to the expenses of the procreation of new business, it appeared to him that temporary assurances were obtained as part of the general new business of the office, e.g., as the result of advertising, and there would seem to be no reason, apart from competition, why temporary assurances should not bear their

full proportion of the cost of the advertisements in the same way as the whole-life and endowment assurances. The same argument would apply to the other items of initial expenditure.

Mr. Oakley suggested that the loadings should first be fixed, and that the expenses should then be kept within the estimate. In the paper, he had approached his subject from the point of view of the actual practice of the office in regard to its expenses, and based the expense loadings thereon, and that seemed to him the proper course to adopt. He had been very much interested in Mr. Lutt's examples, which appeared to show that larger policies were *pro rata* cheaper to put on the books than the smaller ones. As Mr. Lutt pointed out, it was certainly inadvisable for an office to obtain the larger policies at the expense of the smaller assurances. Mr. Hovil mentioned that Office No. 2 was not sufficiently representative. He quite agreed with that statement, but wished to point out that the paper had not been written with the idea of producing model premiums, which could not, of course, be identical for all offices, on account of the different rates of expense. He merely took that office for the purpose of illustration. With regard to the apportionment of expenses between new business and renewals, several speakers considered that he had charged considerably too much to new business, as compared with renewals. That was a matter which could only be settled by a more detailed analysis than he was able to make, but looking down the items he certainly found it difficult to believe that he had charged too much expense to the new business, especially when it was borne in mind how large a portion of the energies of an office was nowadays devoted to the procuration of new business.

LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., Barrister-at-Law.

Annuity contracts with married women in Scotland. I AM indebted to Dr. A. E. Sprague, F.I.A., F.F.A., for particulars of a case, decided in Scotland in 1900, which illustrates a danger attendant on contracts with married women in Scotland, and which is of special interest to life assurance companies. The case in question is that of *Dick v. General Life Assurance Company*, 1900, 7 S.L.T., 446, and the facts are as follows: The pursuer, Adam Dick, and his wife were married in 1894. Mrs. Dick became possessed of personal estate to the amount of about £1,300, and on December 15, 1898, she expended £1,129 in the purchase of an annuity of £63 per annum from the defender company. Mrs. Dick died on July 15, 1899.

The present action was then brought by Mr. Dick, as executor of his late wife and as an individual, against the assurance company to set aside the annuity contract and for the repayment of the consideration money on the ground, *inter alia*, that the transaction had taken place without his consent. The Court upheld the pursuer's contention, holding that the purchase of the annuity was void as having been made without the pursuer's consent, and that Mr. Dick, as executor of his wife, was entitled to repayment of the consideration money of £1,129, less the amount of the half year's annuity payment received by Mrs. Dick.

In delivering judgment to this effect, the Lord Ordinary (Lord Kincairney) said: "The transaction is challenged on "the ground that it was carried through by a married "woman without her husband's consent. The case to my "mind, is difficult but not of general importance; for it can- "not occur often that an annuity is sold by an insurance "company to a married woman without her husband's "written consent, when he and she are living together, "and when his written consent can be obtained without "difficulty if he be willing to give it. The objection, it is to "be remarked, is not by a wife pleading that she has not had "the benefit of her husband's curatary, but by the husband "in his own interest. But it appears to me that the plea is "equally open to either, his curatorial power being for his "own benefit as well as for the benefit of his wife.

"The first question is, whether the transaction was within "the power of Mrs. Dick without her husband's consent, "or was beyond her power. . . . The transaction cannot, in "my opinion, be supported at common law. The law "on the subject is thus distinctly stated by Erskine. It "proceeds, he says, 'from the curatorial power of the husband "that all deeds done or granted by a wife without his "'consent are in themselves null, though they should "'relate to her own property.' In this case there was no "conventional exclusion of any of the husband's rights. But "supposing there had been an exclusion of the *jus mariti* ". . . the case would not at common law have been "different, the consent of the husband as his wife's curator "would still have been essential. . . . So far the law "appears to be reasonably clear, and I do not understand the "defenders to place their chief reliance on the common law.

" But they have maintained that the transaction was within " the power of Mrs. Dick in respect of the first and second sub- " sections of section one of the Married Women's Property " (Scotland) Act, 1881. The first sub-section provides that " the movable estate of a woman married after the passing of " the Act shall be vested in her as her separate estate, and " shall not be subject to *jus mariti*. Here the spouses were " married after 1881, and there was no marriage contract. It " is therefore quite clear that the money paid by Mrs. Dick for " her annuity was her separate estate, not subject to *jus mariti*, " so that no question arises under the first sub-section.

" The question depends on the second sub-section which " relates to the right of administration, not to *jus mariti*. It " provides that the husband's right of administration shall be " excluded so far as regards the income of his wife's estate, " but that 'the wife shall not be entitled to assign the pro- " 'spective income thereof, or, unless with the husband's " 'consent, to dispose of such estate.' The material words are " 'the wife shall not be entitled,' 'unless with her husband's " 'consent, to dispose of such estate.' I do not suppose that " it was intended by these words to narrow the rights which " a wife had at common law, but only to affirm the common " law. But the defenders have contended that the rights of " a married woman are greatly enlarged, that the words 'to " 'dispose of such estate' refer only to gratuitous disposal, " and that her right to administer her estate in any other " way is inferentially affirmed. It seems to me impossible to " read the words in that restricted sense. On the contrary, " it is clear that that is not their meaning. For the extent " to which the right of administration is affected is not left " to implication, but is expressly confined to its relation to the " income, leaving the husband's curatorial power otherwise " undiminished, and the concluding words do little more than " express distinctly what would be inferred without them.

" On the whole, on a consideration of the common law, " the recent authorities, and the Act of 1881, I have formed " the opinion that this transaction, whereby Mrs. Dick " exchanged the capital of her fortune for an annuity, cannot " be sustained without proof of the consent of her husband, " or conduct by him equivalent to such consent. I cannot " doubt that he could have prevented the fulfilment of her " intention if he had known of it."

On the question as to the form which the husband's consent should take, it was held that such consent need not, necessarily, be in writing. It was pointed out that "the Statute only requires the consent of the husband, and does not require, as presumably it would have done if that had been intended, his written consent, nor is there any statutory limitation as to the mode of proof." There was, however, in this case, insufficient proof of consent in any form, and in the result the Lord Ordinary found that the transaction between Mrs. Dick and the assurance company was null and void without the husband's consent, express or implied, and that the defenders had failed to prove his knowledge or consent.

Conversion of a friendly society into an assurance company with extended powers. Attention was called recently to the case of *Blythe v. Birtley* ([1910] 1 Ch. 228, 26 T.L.R. 115, 215, *J.I.A.* xliv 290) in which the Court restrained a friendly society from converting into an assurance company with powers extending far beyond those of the friendly society. A sequel to that case has now come before the Courts in the case of *McGlade v. Royal London Mutual Insurance Society, Limited* [1910], 2 Ch. 169. Here the defendant society had been converted from a friendly society into an assurance company under the provisions of section 71 of the Friendly Societies Act, 1896. As a friendly society its powers had been limited to the objects set forth in section 8 of that Act, but the new company took power, under its memorandum of association, to transact practically all forms of insurance, including life, fire, marine, accident, employers' liability, burglary and plate glass insurance. The plaintiff objected to these wide powers, and accordingly brought the present action on behalf of himself and all other members of the defendant society for an injunction to restrain it from exceeding any powers in excess of those previously possessed by the old friendly society, claiming that the case came within the decision of *Blythe v. Birtley*. On the case coming before Eve, J., in March last, he dismissed the application with costs, holding that there was an essential difference between this case and that of *Blythe v. Birtley*, since in the latter case the application was to restrain the officials from carrying out a proposed conversion, whereas in the present

case it was sought to interfere with a company that had been registered as a company since 31 July, 1908. He held, moreover, that in view of the strong words of section 17 of the Companies (Consolidation) Act, 1908, he could not go behind the certificate of incorporation and investigate the authority of those purporting to register the company and consider whether that authority was limited to the registration of another kind of company or extended to the registration of this company. He therefore held that the certificate of incorporation was conclusive, and that the company must be deemed to be one authorized to be registered.

The plaintiff appealed, but the Court of Appeal upheld the decision of Mr. Justice Eve. In delivering judgment to this effect, Cozens-Hardy, M.R., said : "The plaintiff says . . . by his writ, 'The defendant company is a limited company, and I sue it as such; it is not only a limited company, but it is a company of which I am a member, and I sue the company in that character'; and, admitting that he is a member of this limited company, he asks that the limited company may be restrained from carrying out part of the objects found in its registered memorandum of association. I know of no principle or authority which can justify such relief being given in a case like this. It is said that this is inconsistent with the view which this Court took in the recent case of *Blythe v. Birtley*. I do not see that it is in anyway inconsistent with it. I do not desire to qualify or retract a single word that I said in that case, nor do I think that by proper proceedings the members of the old friendly society might not, in one way or another, have presented a plausible case, and very possibly a successful case, for the aid of the Court, even after the certificate of incorporation had been issued. I think it might have been competent for a member to commence an action, suing on behalf of himself and all other the members of the friendly society, asking for a declaration that the special resolution was *ultra vires*, and that all proceedings under it were therefore void. That is entirely inconsistent with the relief claimed in this action; and on that ground, which is substantially the ground on which Eve, J., based his decision, I think that this action is misconceived, and that therefore this appeal fails and must be dismissed with costs."

Buckley, L.J., said : " An exceedingly serious question " I think exists, on which I am not going to express any " opinion, whether the steps which this society has taken " have any validity. It seems to me that that question might " have been raised in more than one way. I apprehend that " this plaintiff could have come here and said, 'I was a member " of the friendly society, and I am so still, because I say " that there have never been carried out proceedings under " the Friendly Societies Act so as to result under section 71, " sub-section 3, in the vacation of the registration of the " friendly society. I sue for a declaration that I am not a " member of the corporation ; that the friendly society could " not covert me into a member of such a corporation ; and I " ask for relief based upon these contentions.' He might " also, probably, have commenced proceedings challenging " the registration *in toto*, not merely challenging his member- " ship of the corporation, but challenging the existence of the " corporation, and have sued for a declaration that the " registration was invalid, and that there was no such corpora- " tion. Probably the same question could have been raised " in a third form. This memorandum of association provides " that 'in the carrying-on of the society's business'—that is " to say, of the business taken over from the friendly society " —' the following fundamental condition shall be observed, " that the policies of the Royal London Friendly Society " immediately before the registration of this society are to " be binding on this society.' I can conceive an action in " which a policyholder of the friendly society might have " sued for a declaration that his contract had not been taken " over by the corporation, again asserting that there was no " such corporation, that they could not carry out this trans- " action, and that he was entitled to continue to pay his " premiums as they from time to time fell due to the society, " and that it was a valid continuing contract with the " society.

" In any one of these forms it seems to me this question " might have been raised. But what is the action with which " we have to deal ? It is an action in which the plaintiff " sues on behalf of himself and all other the members of " the company incorporated under the Companies Acts, . . . " and he sues for a declaration, to put it shortly, that so much " of the business described in the memorandum of association

" as is larger than that of the friendly society is illegal and
 " void, and for an injunction restraining the corporation from
 " carrying on such business. It is an action in which he
 " comes here and affirms the existence of the corporation;
 " says he is a member of the corporation, and seeks to restrain
 " the corporation from carrying out some of its objects.
 " That is an impossible action."

Companies Con-
verted Societies
Act, 1910.

The serious doubts as to the position of converted friendly societies, suggested by the remarks of the judges of the Court of Appeal in the foregoing case, led to the introduction of a bill in the last session of Parliament, for the purpose, as the title expressed it, of removing doubts as to the validity of the conversion of certain societies into companies. This bill, more fortunate than many others, passed into law, under the title of The Companies (Converted Societies) Act, 1910. It provides by section 1 that:—" (1) " where a registered society purporting to act under section " 71 of the Friendly Societies Act, 1896, or any similar " enactment repealed by that Act has, before the passing of " this Act, passed a special resolution for converting the " society into a company under the Companies (Consolidation) " Act, 1908, or any enactment repealed by that Act, and has " been registered under any such Act, the validity of the " special resolution shall not be questioned on the ground " that the objects of the company as set forth in the special " resolution extend beyond those authorised by the rules of " the society at the date of the passing of the resolution, or " that in any other respect the requirements of the said " section or enactment have not been complied with; and for " removing doubts it is hereby declared that section 17 of " the Companies (Consolidated) Act, 1908, relating to the " conclusiveness of certificates of incorporation, shall apply " in such a case as well as in all other cases :

" Provided that, where the business carried on by the " society before conversion included assurance or insurance " business of any description, then, as from the passing of this " Act, the objects of the company shall not extend beyond those " authorised by the rules of the society at the date of the " passing of the special resolution, except so far as may be

"necessary for giving effect to, and for carrying out any assurances, contracts or policies, made, entered into, or issued before the passing of this Act.

(2) "Nothing in this Act shall be construed as prejudicing the right of any such company to alter its memorandum of association with respect to its objects in accordance with the provisions of the Companies (Consolidated) Act, 1908, or as preventing any such company exercising any powers which are merely incidental or auxiliary to the principal business carried on by the company."

Mortgage of life interest including income from leasehold property subject to covenants Position of mortgagee in respect of covenants. Division of life interest.

The case of *In re Loom, Fulford v. Reversionary Interest Society, Limited* [1910] 2 Ch. 230, is one of some interest to those who are concerned with life interests and reversions. The facts are as follows: A testator, one John Loom, left certain property, comprising a leasehold house and some stocks and shares, to his housekeeper, Marian Ross, for life, she paying the ground rent and performing all the covenants and conditions of the lease and keeping the property insured against fire. Marian Ross accepted the legacies, married one Henry Pascoe, and continued to reside in the house. On 28 January 1897, she mortgaged to the Reversionary Interest Society all her life interest to which she was entitled under the will in the house and the dividends of the stocks, and she gave a further charge thereon to the same society in March 1903. Notice of these mortgages was given to the trustees of the will.

By an order for foreclosure made on 14 January 1907, in an action brought by the society against Mrs. Pascoe, it was ordered that she should be absolutely foreclosed from all equity of redemption in the mortgaged property, and notice of this order was given to the trustees. The society did not enforce the foreclosure order in respect of the house, nor in any way enter into possession of it. Mr. and Mrs. Pascoe continued to reside in it, and it became very much out of repair. The premiums for fire insurance, and the ground rent had been paid since 1903 by the trustees out of the dividends on the stocks; and they paid the remainder of the dividends to the society.

On 17 March 1909, the lessors served a notice to repair, under the provisions of the Conveyancing Act, 1881, and as the notice was not complied with, they brought an action against the trustees, claiming possession of the house. Mrs. Pascoe was now an inmate of an asylum as a pauper lunatic, and her husband, who still lived in the house, refused to answer any enquiries. The society declined to do any of the repairs and disclaimed all interest in the house, although they asserted their right to the stock. It was stated that the repairs would cost about £72, and that the house was worth about £300. The trustees, in these circumstances, took out a summons to determine, *inter alia*, whether the society was liable to perform the covenants of the lease; and if not, whether the trustees could do the repairs out of the income of the trust funds, or how they could otherwise be provided for.

On the case coming before Parker, J., he held that no liability rested on the society in regard to the house, and that they were entitled to retain their interest in the stocks and shares in spite of their disclaimer respecting the house. In delivering judgment to this effect, he pointed out that a gift such as that in this case, if the donee accepts it, imposes in equity a personal obligation to fulfil the covenants of the lease, and that having accepted the conditions, the donee becomes personally liable to see that the conditions are performed. He then went on to say: "These gifts in the will are independent gifts, and are not linked together in such a way that one of them cannot be accepted without the other. . . . It seems to me that the society have done nothing, up to the present time, to make them personally liable. The assignment to them did not make them personally liable any more than an equitable assignment of a house would make the assignee liable to the covenants affecting it. It would have been different if after the foreclosure order they had entered into possession, for in that case it might have been argued that they accepted the liability themselves. They did not do anything of that sort, and I cannot see that they are under any liability."

"Then the question arises whether they can assert an interest in the stocks, in which Mrs. Pascoe had a life interest, which were included in the mortgage to them,

" without seeing that the liabilities on the house are discharged.
" Having regard to the fact that the gifts are independent,
" and that the lady is not in the position of a trustee of the
" will, it appears to me that the position is this,—that until
" the assignment of her interest in the stocks the trustees of
" the will might have had an equitable right of set-off in
" respect of the liabilities which accrued under the obligations
" of the donee of the house to keep it in repair; but there
" was nothing to prevent her from assigning her interest in
" the stocks without reference to her liability on the house in
" such a way that the assignee of the dividends on the stocks
" would be free from any liability in respect of the house.
" There was no continuing liability which would oblige him to
" set-off the interest on the stocks against the liabilities on
" the house. Therefore, there being no personal liability on
" the part of the society to perform the covenants of the lease,
" and no such equity as that to which I have referred, the
" society are entitled to retain the interest they have in the
" stocks without being affected by the liability of the lady in
" respect of the house."

Friendly societies
and income tax.
Finance (1909-10)
Act. 1910.

It will be recollect that by the Friendly Societies Act, 1908, the limit of amount for assurances and annuities that could be granted by registered

friendly societies was increased, in the case of assurances from £200 to £300, and in the case of annuities, from £50 to £52 (*J.I.A.*, xliii, 88). It will also be within the knowledge of most of the readers of these Notes that the exemption from income-tax enjoyed by registered friendly societies is given, not by the Friendly Societies Act, but by the Income Tax Acts, and that the limits of amount entitling to the exemption are £200 for assurances, and £30 for annuities, so that even before the Act of 1908, if a society granted annuities up to the full amount allowed by the Friendly Societies Acts, it risked its income-tax exemption. The extension in amount allowed by the Act of 1908, however, brought the matter more prominently into notice, and led to legislation dealing with the matter. By section 70 of the Finance (1909-10) Act, 1910, it is provided that : " The exemption from income tax granted by the Income Tax

“ Acts to a friendly society, and by the Trade Union (“Provident Funds) Act, 1893, to a registered trade union, “ by the rules of which it appears that the sums assured to “ any person by the society or union do not exceed if by way “ of gross sum £200, or if by way of an annuity £30 a year, “ shall extend to any registered friendly society and to any “ registered trade union, if the society or union are restricted “ either by virtue of any Act of Parliament or by their rules “ from assuring to any person any sum exceeding £300 by “ way of gross sum or £52 a year by way of annuity.””

ASSURANCE COMPANIES ACT, 1909.

ORDER OF THE BOARD OF TRADE, MAKING RULES AND REGULATIONS, AND PRESCRIBING FORMS, UNDER THE ABOVE ACT.

RULES RELATING TO DEPOSITS BY ASSURANCE COMPANIES UNDER SECTION 2 OF THE ASSURANCE COMPANIES ACT, 1909.

1. These Rules apply to all Companies to which the Assurance Companies Act, 1909, applies; and in the construction of these Rules, unless and except so far as the context may otherwise require, the following words or phrases shall have the following meanings, that is to say:—

“The Act” means the Assurance Companies Act, 1909.

“Company” means a Company to which the Act applies, and includes an Irish Company as next herein-after defined.

“Irish Company” means a Company to which the Act applies, and which is registered or has its head office in Ireland.

“Assurance business” means all or any of the five classes of assurance business specified in Section 1 of the Act.

“The Court” means the Supreme Court of Judicature in England or in the case of an Irish Company the Supreme Court of Judicature in Ireland.

“The Paymaster-General” means the Paymaster-General for the time being or in the case of an Irish Company the Accountant-General for the time being of the Supreme Court of Judicature in Ireland.

“The Assistant Paymaster-General” means the official or one of the officials acting for the time being as the Assistant or Deputy of the Paymaster-General as hereinbefore defined in relation to business connected with the Court.

“The Bank” means the Bank of England (Law Courts’ Branch), or in the case of an Irish Company the Bank of Ireland or in either case such Bank or Branch of a Bank as may from time to time be appointed to receive and deal with cash and securities under the control of the Paymaster-General on behalf of the Court.

2. Where any Company is required, in pursuance of the Act, to deposit the sum of twenty thousand pounds with the Paymaster-General for the time being for and on behalf of the Court, the Company, or the subscribers of the Memorandum of Association of the Company or any of them, as the case may be (in this Rule referred to as the Depositors), may, in the name of the Company, make application to the Board of Trade for a Warrant, and the Board of Trade may thereupon issue their Warrant to the Depositors for lodgment of such deposit in Court, which Warrant shall be a sufficient authority for the Company or persons therein named to lodge the money therein mentioned at the Bank to the account of the Paymaster-General for and on behalf of the Court, and for the Paymaster-General or the Assistant Paymaster-General to issue directions to the Bank to receive the same, to be placed in the books of the Paymaster-General, to the credit of *ex parte* the Company mentioned in such Warrant, according to the method for the time being in force respecting the lodgment of money.

Provided, that in lieu, wholly or in part, of the lodgment of money, the Depositors may bring into Court as a deposit an equivalent sum of any stocks, funds, or securities in which cash under the control of or subject to the order of the Court may for the time being be invested (the value thereof being taken at a price as near as may be to, but not exceeding, the current market price); and in that case the Board of Trade shall vary their Warrant accordingly, by directing the lodgment of such amount of such stocks, funds, or securities, by the Company or the persons therein named, to the said account of the said Paymaster-General for the credit in his books of *ex parte* the Company mentioned in such Warrant.

3. Where the assurance business by reason whereof the deposit is made is a class of business in respect of which a separate assurance fund is required to be kept (that is to say, is either life assurance business, employers’ liability insurance business or bond investment business), then and in any such case the application to the Board of Trade and the Warrant of the Board of Trade shall specify the particular class of business in respect of which the deposit is being made, and the deposit shall be marked accordingly in the books of the Paymaster-General to a special ledger credit. In all other respects the provisions of the last preceding Rule shall apply to any such separate deposit.

4. Where a lodgment of money or securities has been made under the preceding Rules, the Court may, on the application of the Company, order :—

- (a) Investment in such of the stocks, funds, or securities in which cash under the control of or subject to the order of the Court may for the time being be invested as the applicants desire and the Court thinks fit, and either by way of original investment or by way of variation of investment.
- (b) Payment to the Company of the interest, dividends, or income from time to time accruing due on any stocks, funds, or securities in which the deposit is for the time being invested.
- (c) Transfer or payment in the cases provided for by the Rules following of the deposit and the stocks, funds, or securities for the time being representing the same either from one ledger credit of the Company to another or out of Court.

5. In the subsequent provisions of these Rules the term "the deposit fund" means the money or securities deposited, or the stocks, funds, or securities for the time being representing the same, as the case may be.

6. In any case where it may appear to be just and equitable so to do, and in particular in any of the following cases, namely :—

- (a) Where a Company having carried on or having intended to carry on only a class or classes of assurance business in respect of which a separate assurance fund is not required to be kept (that is to say either fire insurance business or accident insurance business) and having a deposit fund standing to the credit of the Company generally, intends subsequently to carry on a class of assurance business in respect of which a separate assurance fund is required to be kept ;
- (b) where a Company having carried on employers' liability insurance business or bond investment business as the case may be and having a deposit fund standing to a special ledger credit in respect of the class of business in question, has a fund amounting to £40,000 set apart and secured for the satisfaction of the claims of policy holders of that class, and intends subsequently to carry on in the first case bond investment business or in the second case employers' liability insurance business or in either of the said cases life assurance business ;

the Court may on the application of the Company order the deposit fund to be transferred from the general account of the Company to a special ledger credit in respect of a particular class of assurance

business, or from one special ledger credit in respect of one particular class of assurance business to another special ledger credit in respect of another particular class of assurance business, or otherwise to be dealt with as may be just and equitable and not in contravention of any provision of the Act.

7. In any case where it may be just and equitable so to do, and particularly in any of the following cases, namely :—

- (a) Where a Company having carried on or having intended to carry on either fire insurance business, or accident insurance business, or both, and having a deposit fund standing to the credit of the Company generally, makes a further deposit in respect of any other class of assurance business ;
- (b) where a Company has a deposit fund to a special ledger credit in respect of employers' liability business, and the employers' liability fund of the Company set apart and secured for the claims of policy holders of that class amounts to £40,000, and the Company has or makes a further deposit in respect of any other class of assurance business as provided for in Section 33 (e) of the Act ;
- (c) where a Company has a deposit fund to a special ledger credit in respect of bond investment business, and the bond investment fund of the Company set apart and secured for the claims of the policy holders of that class amounts to £40,000, and the Company has or makes a further deposit in respect of any other class of assurance business as provided for in Section 34 (c) of the Act ;
- (d) where a Company has ceased altogether to carry on within the United Kingdom, either assurance business of any class, or the particular class of assurance business to the special ledger credit whereof a deposit fund (not being the sole deposit fund) is standing, and all liabilities in respect of the deposit fund have been satisfied or are otherwise provided for ;

the Court may, on the application of the Company, order the deposit fund to be paid or transferred out of Court and returned to the Company or as it shall direct.

8. The issuing in any case of any Warrant or certificate relating to a deposit or to the deposit fund, or any error in any such Warrant or certificate, or in relation thereto, shall not make the Board of Trade, or the person signing the Warrant or certificate on their behalf, in any manner liable for or in respect of the deposit fund, or the interest or dividends accruing on the same, or any part thereof, respectively.

9. Any application under these Rules to the Court shall be made in such manner as shall from time to time be prescribed by Rules of Court, and until otherwise prescribed in the like manner

in which similar applications under the Life Assurance Companies Acts, 1870 to 1872, and the Employers Liability Insurance Companies Act, 1907, were made immediately prior to the commencement of the Act. Provided always that any application under Rule 6 or Rule 7 shall be served on the Board of Trade.

10. These Rules shall, so far as may be, extend to and authorise applications with regard to deposits already made by existing Companies under the provisions of the Life Assurance Companies Acts, 1870 to 1872, and the Employers Liability Insurance Companies Act, 1907, and for this purpose deposits made under the Life Assurance Companies Acts, 1870 to 1872, and the deposit funds representing the same shall *primâ facie* and in default of reason to the contrary be treated and dealt with as having been made in respect of the life assurance business of the Companies by or on behalf of which such deposits were made, and deposits made under the Employers Liability Insurance Companies Act, 1907, and the deposit funds representing the same shall *primâ facie* and in default of reason to the contrary, be treated and dealt with as having been made in respect of the employers' liability insurance business of the Companies by or on behalf of which such deposits were made.

Where any such deposit as in this Rule mentioned has been made by an Irish Company, the same may be ordered by the Supreme Court of Judicature in England to be transferred from the account of the Paymaster-General of the English Court to a corresponding account of the Accountant-General of the Supreme Court of Judicature in Ireland.

MEMORANDUM.

DEPOSITS BY ASSURANCE COMPANIES.

Investments by Order of the Supreme Court.

By Clause 10 of the Act of 1860 the Lord Chancellors of England and Ireland were empowered to make General Orders as to investments of cash under the control of the Court.

The Order made by the English Court on the 1 February 1861, under the powers of this Act was superseded in 1883 and again in 1888 by the new Rules of the Supreme Court of Judicature. Rule 17 of Order XXII. of the Rules of the Supreme Court, 1883, as amended by subsequent Rules, is as follows :—

Cash under the control of or subject to the order of the Court may be invested in the following stocks, funds, or securities, namely :—

Two-and-a-half per cent. Consolidated Stock :

Two Pounds Fifteen Shillings per cent. Annuities :

Two Pounds Ten Shillings per cent. Annuities :

Local Loans Stock under the National Debt and Local Loans Act. 1887 :

Exchequer Bills :

Bank of England Stock :

India Three-and-a-half per cent. Stock :

India Three per cent. Stock :

Indian Guaranteed Railway Stocks or Shares, provided in each case that such Stocks or Shares shall not be liable to be redeemed within a period of fifteen years from the date of investment:

Stocks of Colonial Governments guaranteed by the Imperial Government, or in respect of which the provisions of the Colonial Stock Act, 1900, and of Section 2 (2) of the Trustee Act, 1893, are for the time being complied with:

Metropolitan Consolidated Stock, Three Pounds Ten Shillings per cent.:

Three per cent. Metropolitan Consolidated Stock:

Two-and-a-half per cent. Metropolitan Consolidated Stock:

Two-and-a-half per cent. London County Consolidated Stock:

Three per cent. London County Consolidated Stock:

Three-and-a-half per cent. London County Council Stock:

Corporation of London Inscribed Two-and-a-half per cent. Debenture Stock:

Corporation of London Inscribed Three per cent. Debenture Stock:

Debenture, Preference, Guaranteed, or Rent-Charge Stocks of railways in Great Britain or Ireland having for ten years next before the date of investment paid a dividend on Ordinary Stock or Shares:

Debenture, Preference, Guaranteed, or Rent-Charge Stocks of railways in Great Britain or Ireland guaranteed by Railway Companies owning railways in Great Britain or Ireland which have for ten years next before the date of investment paid a dividend on Ordinary Stock or Shares:

Nominal Debentures or Nominal Debenture Stock under the Local Loans Act, 1875, or under the Isle of Man Loans Act, 1880, provided in each case that such Debentures or Stock shall not be liable to be redeemed within a period of fifteen years from the date of investment.

RULES RELATING TO THE AUDIT OF ACCOUNTS OF ASSURANCE COMPANIES.

1. The accounts of every assurance company not subject to audit in accordance with the provisions of the Companies (Consolidation) Act, 1908, or of the Companies Clauses Consolidation Act, 1845, relating to audit shall be audited in accordance with the provisions of Section 113 (1) and (2) of the Companies (Consolidation) Act, 1908.

2. No director or officer of the company shall be capable of being appointed an auditor.

3. In the case of a company having a share capital, the auditor or auditors shall be elected annually by the shareholders.

RULES RELATING TO THE CUSTODY, INSPECTION, AND CERTIFICATION OF DOCUMENTS.

1. A copy of every account, balance sheet, abstract, statement, or report required by the Assurance Companies Act, 1909, to be deposited with the Board of Trade shall be kept by the Registrar of Joint Stock Companies, and shall be open to inspection by any person on payment of a fee of one shilling for each inspection ; and any person may procure a copy of any such document or any part thereof on payment of 4*d.* a folio of 7*2* words.

2. The Assistant Registrars are hereby appointed (in addition to the Registrar) for the purpose of certifying documents under Section 21 of the said Act.

*RULES RELATING TO THE QUALIFICATION
OF AN ACTUARY.*

1. Any person signing as actuary valuation returns of life-assurance business, sinking fund or capital redemption insurance business, or bond investment business shall be either,—

- (1) a Fellow of the Institute of Actuaries or of the Faculty of Actuaries ; or
- (2) where application is made by a company and where, in the opinion of the Board of Trade, special circumstances exist, an Associate of the Institute of Actuaries or of the Faculty of Actuaries ; or
- (3) the actuary at the date of making these Rules to a company under the Assurance Companies Act, 1909, having its head office within the United Kingdom or to any closed fund of such a company established in consequence of an amalgamation or transfer ; or
- (4) such other person having actuarial knowledge as the Board of Trade may, on the application of a company, approve.

2. Any person signing as actuary returns with regard to employers' liability business shall be either—

- (1) a Fellow or Associate of the Institute of Actuaries or of the Faculty of Actuaries ; or
- (2) the actuary at the date of making these Rules to a company under the said Act having its head office within the United Kingdom or to any closed fund of such a company established in consequence of an amalgamation or transfer ; or
- (3) such other person as the Board of Trade may, on the application of a company, approve.

[The Rules also include provisions as to Deposits by Underwriters who are members of Lloyd's, as to the security to be rendered

by such Underwriters, and as to the form of Certificate to be furnished on audit of their accounts : also forms of annual statements of account, and as to valuation, to be furnished by such Underwriters, in respect of different classes of assurance business effected by them, based generally upon Schedules 1 to 5 of the Act, in a somewhat abbreviated form.—ED.—J.I.A.]

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, APRIL 1910.

BOARD OF EXAMINERS.

THOMAS G. ACKLAND, *Chairman.*

HENRY J. BAKER, *Hon. Secretary.*

ARTHUR R. BARRAN^F.

JOHN SPENCER.

JOSEPH BURN.

ROBERT R. TILT.

W. PALIN ELDERTON.

HAROLD M. TROUNSER.

GEORGE KING.

ALFRED W. WATSON.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART I).

First Paper.

1. Show that the product of any number of factors, each of which is the sum of two squares, can be expressed as the sum of two squares.

2. Three vessels ply between the same ports. The first sails half a mile per hour faster than the second, and makes the passage in an hour and a half less. The second sails three-quarters of a mile faster than the third, and makes the passage in two and a half hours less. What is the distance between the ports?

3. Prove that if the roots of the equation $ax^2 + 2hx + b = 0$ be real and unequal, the roots of the equation

$$(a+b)(ax^2 + 2hx + b) = 2(ab - h^2)(x^2 + 1)$$

will be imaginary.

4. Prove that the expansion of $(1+x)^n$, where n is a positive fraction, is of the same form as when n is a positive integer.

If $x > 1$ explain the meaning of the equation

$$(1+x)^m \times (1+x)^n = (1+x)^{m+n}.$$

If p be nearly equal to q , so that the third and higher powers of the ratio of their difference to their sum may be neglected, prove that

$$\sqrt[n]{\frac{p}{q}} = \frac{(n-1)q + (n+1)p}{(n+1)q + (n-1)p}$$

5. Expand a^x in ascending powers of x , and prove that the expansion is convergent for all values of x .

Assuming the truth of the logarithmic series, deduce an expansion by means of which a table of common logarithms can conveniently be calculated. Without obtaining numerical results, show how you would use the expansion to obtain the logarithms of the first ten natural numbers.

6. If s_n denotes the sum of the cubes of the first n natural numbers, show that

$$\sum_{n=1}^{n=\infty} \frac{(-1)^n s_n \cdot 2^n}{n} = 0$$

7. If n whole numbers taken at random be multiplied together, find the chance that the last digit of the product is a five.

8. From a very large number of balls, known to be either black or white, any combination being *a priori* equally likely, a ball is drawn and replaced twice, and each drawing gives a white ball: prove that the chance of drawing a white ball at the next draw is $\frac{3}{4}$.

Second Paper.

1. Sum the series

$$(1) \quad \frac{1}{(1+x)(1+x^2)} + \frac{x}{(1+x^2)(1+x^3)} + \dots + \frac{x^{n-1}}{(1+x^n)(1+x^{n+1})}$$

(2) $10 + 9 + 7 + 4 + 0 - 5 \dots$ to 30 terms.

2. From the following values of $f(x)$:

$$f(0) = 7.39, f(5) = 7.13, \text{ and } f(10) = 6.74$$

(1) How would you proceed to find the series $f(0), f(1), f(2), f(3) \dots$?

(2) How would you find a value of x such that $f(10) = 7.00$?

3. If $f(x)$ be a rational integral function of x of the third degree, and $x^{(m)} = x(x-1)(x-2) \dots (x-m+1)$, and $\Delta x=1$, prove that

$$f(x) = f(0) + x\Delta f(0) + \frac{x^{(2)}}{2} \Delta^2 f(0) + \frac{x^{(3)}}{3} \Delta^3 f(0).$$

Use this formula to find the sum of the cubes of the first n natural numbers.

4. Obtain the differential coefficients of (1) $\log \frac{x}{a^x}$ and (2) $e^{\frac{x}{a}}$ with respect to x ; and the second differential coefficient of $\log(a+bx+cx^2)$ with respect to \sqrt{x} .

5. Having given that

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

prove that

$$x^2 \frac{d^{n+2}y}{dx^{n+2}} + (2n+1)x \frac{d^{n+1}y}{dx^{n+1}} + (n^2+1) \frac{dy}{dx^n} = 0$$

6. How would you determine maxima and minima values of a function?

What fraction exceeds its p th power by the greatest number possible?

7. Evaluate the following integrals;

$$(1) \int \frac{dx}{\sqrt{x^2 - a^2}} \quad (2) \int \frac{\log(\log x)}{x} dx \quad (3) \int \frac{x^2 - 1}{x^4 + x^2 + 1} dx$$

8. Prove that

$$\int_{-\frac{1}{2}}^{+\frac{1}{2}} f(x) dx = \frac{1}{2} \left[f(-\frac{1}{2}) + f(\frac{1}{2}) \right] + \frac{1}{24} \left[\Delta f(-\frac{3}{2}) - \Delta f(\frac{1}{2}) \right]$$

approximately; and hence show that $\int_1^3 f(x) dx = 889$, when

$$f(0) = 105, f(1) = 212, f(2) = 421, f(3) = 749, \text{ and } f(4) = 1050$$

Third Paper.

*1. Find, to the nearest pound, the amount on 22 August 1916, at an effective rate of interest of $4\frac{1}{2}$ per-cent, of an immediate annuity of £12 per annum, payable half-yearly for five years, the first payment on 19 October 1910.

2. Find the value of a continuous increasing annuity for n years, the payment at the end of time t being in proportion to that time.

*3. If the present value of a yearly annuity of 1, payable every 3 years throughout a period of 60 years, be half as much again as the value of a similar annuity payable during 30 years only, find approximately the rate of interest per annum.

*4. A loan of £18,323 is to be repaid in five years by an annuity calculated at 3 per-cent, to include principal and interest at 5 per-cent. If the first payment of the annuity is £4,120, and the payments are to increase in geometrical progression, prove that the common ratio is 1·03, and draw up a schedule showing the repayment of the capital, &c.

*5. An issue of £500,000 in debenture bonds of £100 each, bearing interest at $4\frac{1}{2}$ per-cent, payable yearly, is redeemable at 105 by annual drawings spread equally over 5 years, commencing in 3 years' time. Find the price at which a bond of £100 must be bought, so that, after taking into account the chance of its being drawn in any year, the effective yield is 5 per-cent.

*6. Five years ago two loans of £1,000 each were granted at 4 per-cent interest : the one repayable by an annuity for 20 years, to include principal and interest, and the other by equal instalments of £50 a year, with interest on the principal outstanding. It is arranged that a third party is to take over the two loans on a 3 per-cent basis. How much should he pay the original lender ?

*7. A 6 per-cent bond is redeemable on 15 June 1922, at 105. Assuming interest to be payable half-yearly, find what the bond will yield if purchased for 108 on 15 April 1910, by a purchaser who sets up a sinking fund at 5 per-cent convertible half-yearly.

8. Tables are required for finding by inspection the approximate half-yearly rate of interest realized on stock or bonds bearing various rates of interest, payable half-yearly, issued at a premium or discount and redeemable at par in any number of half-years. State (1) how you would construct such tables, and (2) how you would set them out.

* "A Short Collection of Actuarial Tables" will be supplied for use in answering these questions.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE
(PART II).

First Paper.

1. Give the probabilities that of three lives (x), (y) and (z):

- (1) (x) will die in the t th year (a) first ; (b) second ; (c) third ;
- (2) the survivor of (x) and (y) will die in the t th year (a) second ; (b) third.

2. Find expressions for the curtate expectation of life of (x) :

- (1) t years after the death of the survivor of (y) and (z) ; and
- (2) from the failure by the death of (y) of the joint lives (y) and (z) .

3. A bank staff in a stationary condition consists of 500 clerks. They enter at age 20, and retirement is compulsory at age 65, and optional after age 60. Assuming a secession of 2 per-cent per annum for 5 years after entry and a retirement (on pension) of 5 per-cent per annum from age 60 to age 65, give expressions for :

- (1) the number of new clerks required each year;
- (2) the number superannuated each year;
- (3) the total number of pensioners.

4. Explain fully the meaning of the terms "rate of mortality", "force of mortality" and "central death rate."

Show that the difference between the "force of mortality" and the "rate of mortality" at any age is approximately equal to $\frac{1}{4} \left(\frac{d_{-1} - d_{+1}}{l_0} \right)$.

5. Having given the values of A , B , and c , in the formula $\mu_x = A + Bc^x$, show how you would proceed to construct tables of values of l_x , p_x , μ_x and i_x .

6. Explain clearly the meaning of the expression "Uniform Seniority."

Having given the value of $\log c$, show how you would form a complete table of Uniform Seniority for three lives in respect of a mortality table constructed (1) according to Gompertz's law; (2) according to Makeham's first modification of Gompertz's Law.

7. How would you ascertain "the expected deaths" and "expected claims" in one year in respect of policies (a) payable on the first death of two lives; (b) payable on the last death of two lives; and (c) payable only in the event of one life predeceasing another?

Second Paper.

1. Show how you would obtain from a table, following Makeham's law, the single premium for an assurance of 1 payable on the death of (x) should he die first of n lives. What factors would you tabulate to assist in the calculation?

2. Obtain a formula for the value of a complete annuity to (x) payable m times a year, commencing from the death of the survivor of (y) and (z) .

3. Find an expression for the value of an endowment assurance policy effected $n + \frac{k}{m} + \frac{1}{2m}$ years ago on (x) payable at ($x+t$) or previous death, the premium being payable m times a year. How would the formula be modified on the assumption that any unpaid portion of the premium for the year of death is to be deducted from the claim?

4. Show how to compare the policy-values by two tables, if the number living at age 20 is the same in each table, and after that age the rates of mortality are such that the numbers living at ages 21, 22, 23, &c., by the one table are the same as the numbers living at ages 22, 24, 26, &c., by the other.

5. Show how to determine the select 3 per-cent net annual premium (to be doubled after five payments) for a whole-life assurance on (x) with a guaranteed simple reversionary bonus of 30s. per-cent per annum, the representatives of the assured to have the option of leaving the proceeds of the policy on deposit with the company for n years at 5 per-cent per annum.

*6. Find, by the H^M Table, at 3 per-cent interest, the net annual premium at age 30 for an assurance of £1,000, payable on the attainment of age 50 or previous death, an annuity-certain of £50 per annum, to be payable for 20 years, commencing on payment of the £1,000, and a further sum of £2,000 to be paid at the expiration of the term-certain of 20 years.

*7. Find by means of a formula of approximate summation the single premium by the H^M Table for an annuity of £10 payable during the life of (35) after the death of (60), the rate of interest to be taken at $3\frac{1}{2}$ per-cent during the joint existence of the lives and at 3 per-cent thereafter.

* "A Short Collection of Actuarial Tables" will be supplied for use in answering this question.

Third Paper.

1. Show that

$$P_{x^n} + d_t V_{x^n} = P_{x+t-n} (1 - t V_{x^n}) = P_{x+t-1}^1 (1 - t V_{x^n}) + c p_{x+t} (t+1 V_{x^n} - t V_{x^n})$$

and give a verbal interpretation.

2. What is the rationale of the expression $P_{x^n} + P_{y^n} - P_n^-$ as an approximation to the annual premium for a joint-life endowment assurance? In which direction is the error? .

3. Prove that if, at all ages, $l_x = ks^x w^{x^2} g^x$, the value of an annuity on two or more joint lives may be found by substituting an equal number of lives of uniform age and making a suitable change in the rate of interest.

4. The expression $\left(a_x - a_{xy} - \frac{m-1}{2m} A_{xy}^{\frac{1}{m}} \right)$ has been given for the approximate value of a reversionary annuity to (x) after the death of (y) . Explain exactly the nature of the annuity represented by the formula, and obtain a closer approximation.

5. Show concisely how you would calculate a table of last survivor annuities (two lives).

6. How would you classify for valuation purposes whole-life policies by limited premiums? Draft a form of class-book suitable for the valuation of such policies.

7. A company confines its business solely to the issue of deferred and immediate life annuities. Describe fully the method of classification you would adopt in the valuation of these annuities, and give illustrations of the class-books you would use.

Fourth Paper.

1. Describe fully the system of records you would adopt in respect of premiums and interest collected through agents.

2. Draft forms of account books for the collection of premiums which will enable the premiums received during the year to be readily reconciled with the premium income as shown by the Books at the end of the year.

3. A life assurance company purchases Great Indian Peninsula Railway "A" Annuity. What book-keeping entries would you make in respect of the interest included in the purchase-money, and how would you deal with the periodical writing down of the capital out of the annuity payments?

4. What are the principles governing the issue of notes by the Bank of England? The ratio of reserves to liabilities in the Banking Department is taken as $\frac{\text{Notes and Cash}}{\text{Deposits and Bank Post Bills}}$, the assets "Government Securities", and "Other Securities", being excluded from the numerator. A reference to the Issue Department shows, however, that a large proportion of the Note Issue is secured on Government Debt and "Other Securities." Explain the apparent anomaly.

5. Give a list of the principal securities, quoted on the London Stock Exchange, guaranteed by the Indian Government. Point out the difference between an investment in India Rupee Paper and an Indian Sterling Loan.

6. What is the effect on the amount of gold in the Issue Department of the Bank of England when a note issue by that Bank is substituted for the note issue of another Bank—

- (a) If the average circulation of the Bank of England notes is equal to five-sixths of the authorized issue of the Private Bank;
- (b) If the average circulation of the Bank of England notes is equal to one-third of the authorized issue of the Private Bank?

7. Describe the chief characteristics of (a) Inscribed Stock; (b) Registered Stock; and (c) Bearer Bonds.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART III).

First Paper.

1. Give a list of the graduated Mortality Tables which have been constructed from the British Offices data, and for which monetary tables have been published. Explain briefly the principles followed in the treatment of the raw material used for the various tables, and point out the differences, if any, between the tables in this respect. Do not touch upon graduation.

2. Explain fully why, in deducing the Exposed to Risk from the tabulated statistics, different formulas were used for the H^M and the O^M Tables respectively. Give for each of these tables a formula suitable for calculating E_x by a continued process, and illustrate your answer by obtaining E_x for ages 78, 79 and 80, from the following data:—

Age x	E_x	Entered	Died	Withdrew	Existing
77	19,879	353	2,166	99	986
78	...	312	2,018	72	823
79	...	258	1,865	73	713
80	...	217	1,735	43	636
81	...	190	1,550	53	522

3. Deduce Mr. King's central formula, or any other that would be suitable, for finding graduated quinquennial values of the population and the deaths in the construction of mortality tables from census statistics. When the data are in quinquennial intervals for part of the table and in decennial intervals for the remainder, what preparatory process is required, and how is it effected?

How can this method be applied to statistics derived from the records of assured lives?

4. What feature differentiates the calculated "Exposed to Risk of Sickness" in the Manchester Unity Experience, 1893-97, from the corresponding function in previous investigations? What change in principle does the difference produce in the tabulated "Rate of Sickness"? What was the object of the change, and what adjustment of the formula previously in use did it necessitate in the preparation of Commutation Columns?

5. John Finlaison's second method of graduation was equivalent to a summation twice in fives: that is, the graduated value, (u), is $\frac{1}{25} [5]^2 u_{\cdot\cdot}$. Show that this formula introduces an error which involves the second central difference. Ascertain the amount of that second difference error, and deduce a formula analogous to that of Woolhouse whereby it is corrected.

6. Indicate briefly the criteria by which you would judge the relative graduating capacity of different summation formulas. Illustrate your answer by contrasting the graduating power of the two formulas referred to in the previous question, i.e., Finlaison's formula and your deduced modification.

7. How would you construct a graduated Mortality Table directly from the Exposed to Risk and the Deaths on Makeham's hypothesis? How would you proceed in order approximately to eliminate the error introduced by any assumption which the method may require? Mention a recent instance where this method of construction was adopted.

Second Paper.

1. Give an account of the methods followed by Dr. Buchanan in tabulating model office reserves for endowment assurance policies. Explain how, from the results of an O^M valuation of his model office, he approximated to the reserves required upon the $O^{(M)}$ and O^M basis, where the net premiums valued are those of the $O^{(M)}$ Table.

2. What are the arguments for and against the adoption of the combined O^M and $O^{M(5)}$ Tables (O^M alone for the first five years), in a net premium valuation of ordinary whole-life assurances? How would your views be affected if policies less than five years in force were dealt with in the same way as those of longer duration?

3. State what methods have been adopted in practice for valuing joint whole-life policies in groups. Which plan do you prefer, and what are its advantages as compared with other methods?

4. A company grants double benefit policies, which secure an ordinary participating endowment assurance, payable at 50, 55, 60, or 65, or previous death, and an ordinary whole-life paid-up participating policy of like amount commencing at the endowment age, all premiums ceasing at the endowment age.

How would you value these policies in groups during the endowment term: and, as compared with the other policies of the company, how would you assess the bonuses:

- (a) During the endowment term;
- (b) After the endowment has been paid?

5. Explain a method of valuing accurately ordinary whole-life policies in groups according to exact duration.

6. What are the main principles of valuation which are indicated by the schedules required by legislation in the United Kingdom for the valuation returns in respect of employers' liability business?

Third Paper.

1. According to the principles of bonus distribution of a company, the policyholders are not entitled to vested profits until their premiums accumulated at a given rate of interest amount to the sums assured.

By what method would you distribute the profits, and what precautions, if any, as regards valuation or other matters would you consider to be necessary in order that equity may be maintained between policyholders who enter at successive periods of time?

Give full reasons for your answers.

2. What are the main sources of surplus in a life assurance office? How would you estimate the amount due to each of these sources which is included in the surplus realized during the five years between two quinquennial valuations?

3. Describe and criticize any practical methods of distributing bonus to Industrial policyholders.

4. Company A distributes its surplus as a compound reversionary bonus, and company B as a simple reversionary bonus, and they both make their valuations by the same mortality table. Can any arguments be urged in favour of the companies making reserves for existing bonuses at different rates of interest? Give full reasons for your answer.

5. A life assurance company valuing by the O^M Table at 3 per-cent, and earning £3. 18s. 6d. per-cent on its funds, and which, for many years, has distributed a compound reversionary bonus of 35s. per-cent per annum on its whole-life assurances, with a simple reversionary interim bonus of 25s. per-cent per annum, proposes to adopt the discounted bonus system.

State what amount you consider could safely be anticipated, and give reasons for your decision.

Discuss the question as to what effect, if any, should be given to the fact that the bonus dealt with is compound and not simple, and deduce formulas for calculating the necessary scale of premiums, (a) giving effect to the fact that the bonus is compound, and (b) treating the bonus as simple.

6. An office making H^M 3 per-cent net premium valuations wishes to change to the combined O^M and O^{M(5)} Tables at 2½ per-cent. Simple reversionary bonuses are at present allotted, and an average scale of premiums is charged. Discuss the effect on bonus in reference to ordinary whole-life assurances, endowment assurances, and whole-life assurances by limited payments.

Fourth Paper.

1. What bases would you propose to adopt in the calculation of office rates for

- (a) Deferred annuities with return of premiums;
- (b) Deferred annuities without return of premiums?

By what principles would you be guided as to limitation of risk under immediate and deferred annuity contracts?

2. An office which has hitherto granted whole-life assurances only, with bonuses deferred for twenty years, and not declared until the vesting date, opens an endowment assurance section providing for quinquennial declarations of profits. Discuss the terms upon which existing policyholders under both ordinary and limited premium tables may be permitted to transfer to the new section.

3. Explain fully upon what bases as to rate of interest, mortality, and loading you would compute a scale of office annual premiums for convertible term assurances. Give the formulas which you would employ, and show clearly how you would provide for the option of conversion if such option were exerciseable,

- (a) at the end of the term covered by the policy ; or
- (b) not later than five years before the expiration of such term.

4. Explain how you would calculate the office yearly premium for a joint-life non-participating assurance, carrying the option to convert into two single-life policies, each for three-fourths of the original sum assured, at the rates of premium applicable to the ages of the lives assured respectively at the outset.

5. Give an account of the investigation of relative occupational mortality in England and Wales in the years 1900-1902 conducted by the General Register Office. To what extent do you regard the results of this investigation as of value to Life Assurance Companies in settling extra risk premiums? Give reasons for your answer.

6. State your views as to the advisability of offering exceptional terms to under-average lives desiring to purchase life annuities. If it were decided to quote specially favourable terms in particular cases, how would you be guided in fixing your quotations, and how would you deal with these contracts in the valuations of the company?

7. What materials are available for the purpose of determining whether extra premiums should be charged in respect of assurances on the lives of

- (a) Officers in the mercantile marine, and
- (b) Civilians (unacclimatized) proceeding to tropical India?

State generally in regard to each class your opinion as to the extras (if any) which should be charged.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART IV).

First Paper.

1. Define and illustrate the following classes of contracts, namely, (a) illegal; (b) void; (c) voidable. In what circumstances can money paid under such contracts be recovered back?

2. Describe the legal characteristics of an estate *pur autre vie*. What power has the reversioner on such an estate to prevent the concealment of the death of the *cestui que vie*?

3. In what manner are mortgages on the following classes of property usually effected, namely, freeholds : copyholds : registered stock?

4. Describe the necessary elements of a valid will. How can a will be revoked?

5. Discuss the question of the responsibility of an assurance company for the unauthorized acts of its agents. Refer to any recent cases bearing on the subject with which you are acquainted.

6. A policy effected by A on his own life for the benefit of his wife B under the Married Women's Property Act, 1882, is mortgaged to C. The mortgage deed contains only the usual provisions. Subsequently A executes a deed of assignment for the benefit of his creditors, whereupon C applies to the assurance company for the surrender-value of the policy. What discharge should the office require (a) if the policy contains a reservation of a contingent interest to the husband : (b) if there is no such reservation?

7. Give particulars of the returns required under the Employers' Liability Insurance Companies Act, 1907, from companies transacting employers' liability insurance.

8. Give a brief account of the principal alterations made in the law relating to friendly societies by the Friendly Societies Act, 1908. Do these in any way affect the special exemption in respect of income-tax granted to friendly societies in certain circumstances?

Second Paper.

1. An educational authority proposes to establish a pension fund for its staff, which during recent years has been considerably increased, but has now reached, relatively to population, a stationary condition. The staff of each sex consists of two classes, head teachers and assistants, and the salaries in each class are regulated by a scale, all entering the class at the minimum salary and reaching the maximum in 10 years. Vacancies amongst head teachers are filled by promotions of senior assistants under 45 years of age. Assuming that retirement is to take place at the age of 65, that the pension is to be one-half of the final salary, and that the joint contributions of the authority and the teachers are to be calculated as a uniform percentage of salary, how would you proceed to estimate such percentage? No provision need be made for returns at death or withdrawal from the service before the pension age. What point would arise in respect of future admissions to the staff?

2. A local friendly society providing the usual benefits is about to be formed and the draft rules are submitted for your opinion. To what points would you specially direct your attention, and what advice would you give in respect of each? Assume that persons engaged in occupations involving extra risk may apply for admission, but that it is proposed to have one scale of contributions only.

What advice would you give if instructed to report specially upon a proposal to allow surrender-values?

3. A pension society consisting of about 100 members engaged as masters in the mercantile marine provides with other benefits a pension of £20 per annum to a member's widow. No distinction is made between first and subsequent wives, but no wife taken after the member attains age 55 is entitled to this benefit. The pension ceases on the re-marriage of a widow, but is revived in the event of subsequent widowhood. Full records of the operations of the society during a long period are available.

State what information you would require to enable you to value the widows' pension, and show how you would proceed. For the latter purpose assume that you have found that the mortality experience amongst members is characteristic of the occupation.

4. Draft a report to a board of directors advising them as to the desirability of introducing into the company's policies the benefit of extended term assurance, under which the surrender-value of a policy at the date of lapse is applied to keep the policy in force for its full amount as a temporary non-profit assurance.

5. Explain in detail what steps would have to be taken and what accounts would have to be opened by a life office in order to form a separate fund in respect of sinking-fund insurance business, a large number of policies of this class having already been issued by the office in question.

6. Set out fully your views as to the limits within which expenditure in connection with the acquisition of new life assurance business should be confined, indicating what circumstances would affect your conclusions in particular cases.

Third Paper.

1. What would happen if the Bank of England exercised the right, given by the Bank Charter Act, 1844, to issue a certain proportion of notes against silver?

2. Are you in favour of the issue of £1 Bank of England notes? Give reasons for your answer.

3. What was the condition of the rate of exchange between London and New York during the American crisis of 1907? Give some explanation of that condition, and state your own opinion with regard to the matter.

4. Describe briefly how the business of a London bill broker is conducted.

5. An assurance company purchases stock inscribed in the books of the Bank of England. Explain in detail how the transaction is carried out.

6. Would you recommend a life assurance company to invest a portion of its funds in American Railroad currency bonds? Give your reasons.

7. You are required to invest £1,000,000 of life assurance funds to yield about 4 per-cent per annum. What securities would you choose? Set out your proposed investments in groups, stating the proportion of each to the total amount invested. Assume that the whole amount is to be invested within 12 months.

Fourth Paper.

1. Are there any practical reasons why the surrender-value of policies for large amounts should differ materially from the price paid for the same at public auction? Discuss the various points to be considered.

*2. A trust fund consisting of:

- (a) A freehold house in a good neighbourhood, recently let on repairing lease at £100 per annum;
- (b) £2,000 on mortgage at $4\frac{1}{2}$ per-cent;
- (c) Well-secured freehold ground rents, yielding £80 per annum, with reversion in 70 years;

is settled on A, a female aged 65 years, for life, with remainder to B absolutely.

A and B agree, mutually, to determine the trust and divide the fund. Apportion the fund equitably between them.

* "A Short Collection of Actuarial Tables" will be supplied for use in answering this question, but if the rate of interest and table of mortality the student would use in practice are not contained in these tables, he should, in giving his results, state what rate and table he would employ if they were available.

*3. A, a female, aged 8 years, is entitled, provided she attains the age of 21, to the following property absolutely, namely :

£10,000 Consols :

£5,000 India $3\frac{1}{2}$ per-cent Stock :

£3,000 London County Council $3\frac{1}{2}$ per-cent Stock :

£2,000 Birmingham $3\frac{1}{2}$ per-cent Stock ; and

£5,000 on a well-secured mortgage at 4 per-cent.

If A should die before attaining the age of 21, B will, immediately on A's death, become entitled to the property. A is only insurable at second-class rates, an addition of 10s. per-cent to the annual premium being required. Estimate the value of B's interest for purchase.

*4. A, aged 30, is entitled in reversion, if he survive his mother (B), aged 60, to one-fourth of a trust fund invested as follows :

£1,600 on mortgage at 4 per-cent :

£5,000 Pennsylvania Railroad $4\frac{1}{2}$ per-cent gold bonds, 1913 ;

£1,100 Jamaica $3\frac{1}{2}$ per-cent inscribed stock, 1949 :

£1,000 Great Indian Peninsula Railway 3 per-cent guaranteed stock.

The fund is charged with an annuity of £100 to a lady aged 50, and B has power to appoint a life interest to any husband. She has a husband aged 65.

Value A's interest.

5. An advance of £5,000 is to be made on the security of an absolute reversion of ample value, in consideration of a reversionary charge to be paid on the death of the survivor of the life tenants, a male aged 60 and his wife aged 55. The reversioner asks for quotations for the amount of the charge with an option to redeem within the first 5 years, on three months' notice, if, at the time of redemption :

(a) Both life tenants are dead ;

(b) One life tenant only is dead ;

(c) Neither life tenant is dead.

Show how you would determine the amounts of the respective charges, and the redemption amounts during the first 5 years.

* "A Short Collection of Actuarial Tables" will be supplied for use in answering these questions, but if the rate of interest and table of mortality the student would use in practice are not contained in these tables, he should, in giving his results, state what rate and table he would employ if they were available.

PROCEEDINGS OF THE INSTITUTE.—SESSION 1909-1910.

First Ordinary Meeting, 29 November 1909.

The first ordinary meeting of the Institute was held at the Hall of the Institute, on the 29th day of November 1909.

The President (Mr. G. F. HARDY) in the Chair.

A paper entitled "American Railway Securities as Investments for Insurance Companies", was read by the Author, Mr. Hubert Ansell.

The following gentlemen took part in the discussion:—Messrs. G. E. May, H. W. Andras, E. B. Wilkinson, J. D. Watson, W. P. Pulley, W. O. Nash, A. T. Winter, and R. Todhunter.

Second Ordinary Meeting, 20 December 1909.

The President (Mr. G. F. HARDY) in the Chair.

Mr. Austyn James Claude Fyfe, F.F.A., was duly elected an Associate, and Prof. Dr. Ernst Blaschke, Dr. Julius Graf, and Dr. James Klang, were elected Corresponding Members of the Institute.

A paper entitled "On the Mortality of Female Assured Lives, with Graduated Tables deduced from the British Offices' Experience, 1863-1893", was read in abstract by the Author, Mr. C. W. Kenchington.

The following gentlemen took part in the discussion:—Messrs. H. E. W. Lutt, W. Penman, H. P. Calderon, H. J. Rietschel, E. A. Rusher, and the President.

Third Ordinary Meeting, 31 January 1910.

The President (Mr. G. F. HARDY) in the Chair.

A paper entitled "Some points of interest in the operations of Friendly Societies, Railway Benefit Societies and Collecting Societies", was read in abstract by the Author, Mr. A. W. Watson.

The following gentlemen took part in the discussion:—Messrs. V. Marr, H. J. P. Oakley, H. W. Manly, C. W. Kenchington, T. G. Ackland, and the President.

Fourth Ordinary Meeting, 28 February 1910.

The President (Mr. G. F. HARDY) in the Chair.

A paper entitled "Some Notes on the Establishment of the Office of Public Trustee in England", was read in abstract by the Author, Mr. W. C. Sharman.

The following gentlemen took part in the discussion:—Messrs. T. F. Anderson, J. R. Hart, R. R. Tilt, E. K. Allen (a visitor), J. B. Gillison, and S. G. Warner.

Fifth Ordinary Meeting, 21 March 1910.

The President (Mr. G. F. HARDY) in the Chair.

Mr. Robert Oswald Blyth, M.A., F.F.A., was duly elected an Associate of the Institute.

A paper entitled "I. On the Valuation of the Payment on the Death of a Pensioner of the Excess of his Contributions, with or without interest, over his pension payments. II. On a Method of scheduling particulars for the Valuation, in certain cases, of prospective Pensions based on Terminal Salaries", was read in abstract by the Author, Mr. T. Tinner.

The following gentlemen took part in the discussion:—Messrs. E. C. Thomas, G. King, A. W. Watson, H. W. Manly, and the President.

Sixth Ordinary Meeting, 26 April 1910.

The President (Mr. G. F. HARDY) in the Chair.

Mr. Joseph Brotherton Maclean, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "Analysis and Apportionment of the Expenses of a Life Office with a view to ascertaining the Office Premium Loadings", was read in abstract by the Author, Mr. H. J. Rietschel.

The following gentlemen took part in the discussion:—Messrs. H. J. P. Oakley, J. F. Little, H. E. Melville, C. W. Kenchington, H. E. W. Lutt, D. C. Fraser, L. F. Hovil, and the President.

The Sixty-third Annual General Meeting, 6 June 1910.

The President (Mr. G. F. HARDY) in the Chair.

The proceedings of the Annual General Meeting will be found on page 494.

REPORT, 1909-1910.

The Council have the pleasure to report to the Members upon the progress of the Institute during the Session of 1909-1910, the sixty-second year of its existence.

There has been a *decrease* of 35 in the number of members, as compared with the previous year. At the end of the official year in which the Institute was incorporated by Royal Charter the number of Members was 434, while fifteen years later, at 31 March 1900, it was 822. Since that time the numbers have been as follows:

On 31 March 1901,	818,
" 1902,	842,
" 1903,	828,
" 1904,	856,
" 1905,	881,
" 1906,	922,
" 1907,	956,
" 1908,	1,009,
" 1909,	998,
" 1910,	963.

The following schedule shows the additions to, and the changes and losses in the membership which have occurred during the year ending 31 March last :

Schedule of Membership, 31 March 1910.

	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1909 .	254	325	400	19	998
ii. Withdrawals by (1) Death	2	2	1	1 }	63
(2) Resignation or otherwise	6	12	39	... }	
	246	311	360	18	935
iii. Additions to Membership (1) By Election	2	...	3 }	28
(2) By Order of Council	9	...	
(3) By Re-instatement	5	9	...	
	246	318	378	21	963
iv. Transfers (1) By Examination: from Associates to Fellows	11
11
	257	307	378	21	963
(2) By Examination: from Students to Fellows	2
2
	259	307	376	21	963
(3) By Examination: from Students to Associates	28
...	28
	259	335	348	21	963
v. Number of Members in each class on 31 March 1910 .	259	335	348	21	963

There are also 141 candidates admitted as Probationers, and 42 as Students conditionally on their passing Part I of the Examination. These are not included in the above Schedule of Membership.

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of one Corresponding Member, M. A. Thomerean ; two Associates, Messrs. C. R. Ray and J. T. Smith ; and one Student, Mr. D. Williams.

The form of the Revenue Account and Balance Sheet (*see pp. 488-9*), has been revised, as it has been thought desirable to eliminate from the former receipts and expenditure representing capital value of Publications issued by the Institute other than the *Journal*, and in lieu thereof to include in the account the yearly profit or loss on the sales. In order to give effect to this re-arrangement, a monetary value for the stock of Publications, less than the actual cost, has been inserted in the Balance Sheet, and a separate

[Continued on page 490.]

Dr.*Revenue Account for the*

Amount of Funds at the beginning of the year—	£ s. d.	£ s. d.
General Fund (including Stock of Publications, other than <i>Journal</i> , taken at £553 3s. 0d.)	9,414 6 0	
Messenger Legacy Fund	400 3 9	
Brown Prize Fund	294 6 0	
	10,108 15 9	
Subscriptions—		
Fellows	750 15 0	
Associates	667 16 0	
Students	371 14 0	
Probationers	77 14 0	
	1,867 19 0	
One Annual Subscription compounded for	10 10 0	
Fines on Re-instatement	3 3 0	
	1,881 12 0	
Application Fees—		
Associates	8 8 0	
Students	16 5 6	
Probationers	25 4 0	
	49 17 6	
Profit on Sales of Publications	146 0 6	
Dividends and Interest—		
General Fund	279 6 7	
Messenger Legacy Fund	12 0 1	
Brown Prize Fund	8 16 7	
	300 3 3	
	£12,486 9 0	

Publications Account for the

	£ s. d.
Stock (excluding <i>Journal</i>) at the beginning of the year	553 3 0
Balance of expenses on account of G. F. Hardy's Lectures on Construction of Tables	159 13 0
Balance of expenses on account of J. Burn's Lectures on Stock Exchange Investments	317 1 0
Binding Sundry Publications	26 16 10
Profit on Sales	146 0 6
	£1,202 14 4

*Balance Sheet,***LIABILITIES.**

	£ s. d.	£ s. d.	£ s. d.
General Fund	9,736 15 3		
Messenger Legacy Fund	233 9 2		
Accumulated Dividends	178 14 8		
	412 3 10		
Brown Prize Fund	200 0 0		
Accumulated Dividends	103 2 7		
	303 2 7		
Examination Fees for year 1910	99 15 0		
Sundry unpaid Accounts	37 4 0		
	£10,589 0 8		

*year ending 31 March 1910.***£
Cr.**

<i>Journal—</i>	<i>£ s. d.</i>	<i>£ s. d.</i>
Printing of Nos. 236, 237, 238, 239	514 0 10	
Clerical assistance	86 5 0	
	600 5 10	
<i>Less Sales during the year</i>	<i>234 1 8</i>	
<i>Library—</i>		366 4 2
Binding, Purchases, &c.		79 1 7
<i>Meetings</i>		70 18 9
Examination charges	308 3 11	
<i>Less Fees received from Candidates (1909)</i>	<i>242 11 0</i>	
		65 12 11
Tutors for classes in Parts I and II	264 12 0	
<i>Less Fees received from Students</i>	<i>180 12 0</i>	
		84 0 0
<i>Legal Charges</i>		86 11 8
<i>Office Expenditure—</i>		
Rent	600 0 0	
Salaries	331 5 0	
House expenses	75 17 1	
Corporation Duty	14 6 3	
Fire and other Insurances	30 13 3	
Stationery and Printing	160 16 9	
Postage and Telegrams	43 15 8	
Sundries (including bad debts)	25 4 3	
		1,281 18 3
Amount of Funds at the end of the year as per Balance Sheet	10,452 1 8

Examined and found correct, 29 April 1910.

ALBERT G. SCOTT.	Auditors.	
HAROLD DOUGHARTY.		
J. C. WARDROP.		

*year ending 31 March 1910.***£ s. d.**

Sales	528 19 3
Stock (excluding <i>Journal</i>) at the end of the year	673 15 1

Examined and found correct, 29 April 1910.

ALBERT G. SCOTT.	Auditors.	
HAROLD DOUGHARTY.		
J. C. WARDROP.		

*31 March 1910.***ASSETS.****£ s. d.**

£3,000 Natal 3 per-cent Inscribed Stock	2,460 0 0
£1,200 Metropolitan Railway 3½ per-cent Debenture Stock	1,128 0 0
£2,000 Great Eastern Railway 4 per-cent Debenture Stock	2,279 11 3
£1,000 Great Northern Railway Preferred Ordinary Stock	965 0 0
£1,350 Great Western Railway 4½ per-cent Debenture Stock	1,660 10 0
£500 Dominion of Canada 3½ per-cent Registered 1930-50 Stock	498 3 6
Stock of Publications (excluding <i>Journal</i>) in hand	673 15 1
Cash on Current Account and in hand	924 0 10

Examined and found correct, 29 April 1910.

ALBERT G. SCOTT.	Auditors.	
HAROLD DOUGHARTY.		
J. C. WARDROP.		

statement relating to the Publications account is appended. Other items in the Revenue Account, e.g., Examination Fees and Examination Charges, appear on one side only, receipts and payments being set against each other and the balance carried in.

The Annual Subscriptions, together with admission and other fees, amounted to £2,354. 12s. 6d., as compared with £2,394. 0s. 0d. received in the previous year. The net Income and Expenditure for the year were £2,377. 13s. 3d., and £2,034. 7s. 4d. respectively.

The stock in hand of the Institute publications on 31 March was as follows :

No. of Copies	Description of Work
17,418	Parts of <i>Journal</i> .
775	Index to Vols. 1 to 40.
291	<i>Text-Book</i> , Part I (New Edition).
684	„ Part II (Second Edition).
659	Government Joint-Life Annuity Tables.
750	Select Life Tables.
61	A Short Collection of Actuarial Tables.
1,502	Frequency-Curves and Correlation (W. F. Elderton).
136	Messenger Prize Essay (Friendly Societies).
17 <i>in cloth</i> } 2,605 <i>in paper</i> }	Lectures on Finance and Law (Clare and Wood Hill).
1,574	Lectures on the Companies Acts (A. C. Clauson).
1,378	Lectures on the Law of Mortgage (W. G. Hayter).
763	Lectures on the Measurement of Groups and Series (A. L. Bowley).
1,630	Lectures on the Construction of Tables of Mortality, &c. (G. F. Hardy).
1,443	Lectures on Stock Exchange Investments (J. Burn).
338	South African War Mortality (F. Schooling and E. A. Rusher).
366	Barrand's Paper on Life Assurance Law.
1,749	British Offices' Valuation Tables.
672	Transactions of the Second International Congress of Actuaries.
1,163	Examination Questions, 1905-9.

The following papers were submitted at the sessional meetings of the Institute, namely :

29 November 1909.—“ American Railway Securities as Investments for Insurance Companies.”—Mr. Hubert Ansell.

20 December 1909.—“ On the Mortality of Female Assured Lives, with graduated tables deduced from the British Offices' Experience, 1863-1893.”—Mr. C. W. Kenchington.

31 January 1910.—“ Some points of interest in the operations of Friendly Societies, Railway Benefit Societies and Collecting Societies.”—Mr. A. W. Watson.

28 February 1910.—“ Some Notes on the Establishment of the Office of Public Trustee in England.”—Mr. W. C. Sharman.

21 March 1910.—“I.—On the Valuation of the Payment on the Death of a Pensioner of the excess of his Contributions, with or without interest, over his pension payments. II.—On a Method of scheduling particulars for the Valuation, in certain cases, of prospective Pensions based on Terminal Salaries.”—Mr. T. Timmer.

25 April 1910.—“Analysis and Apportionment of the Expenses of a Life Office with a view to ascertaining the Office Premium Loadings.”—Mr. H. J. Rietschel.

For the Examinations held in the United Kingdom and the Colonies on 18, 19, 20, 21, 22 and 23 April 1910, 302 entries were received, namely :

73	for Part	I.
79	I. (§) 3.
75	II.
56	III.
19	IV.

The results will be duly announced. The Council warmly acknowledge the valuable services of the Board of Examiners, and also those of the Honorary Supervisors at centres other than London.

During the year there have been published the Lectures, somewhat extended, entitled “On the Construction of Tables of Mortality and of similar statistical Tables in use by the Actuary”, which were delivered at the Institute during the Session 1904-1905, by Mr. G. F. Hardy. The series of Lectures delivered by Mr. Joseph Burn in 1908-1909, also much amplified, have been issued in book form under the title “Stock Exchange Investments in Theory and Practice, with chapters on the Constitution and Operations of the Bank of England and the National and Local Debts of the United Kingdom.” Considerable help will no doubt have been derived by Students from both these publications, which it is believed have also proved equally useful to the profession at large.

The Assurance Companies Act, 1909, received the Royal Assent on the 3 December last, and comes into operation on the 1 July 1910. The provisions of the original Bill, so far as they affected subjects properly within the cognizance of the Institute, had the very careful and detailed consideration of the Council, who made numerous recommendations to the Board of Trade as to particular portions of the Bill. The Council are glad to be able to report that many of their suggestions have been embodied in the Act.

The Council also had under consideration the provisions of the Finance Bill, 1909, so far as they specially affected Life Assurance Companies, and acting in co-operation with the Committee of the Life Offices’ Association made certain representations to the Chancellor of the Exchequer on the subject, representations which were most courteously received and in considerable measure acceded to.

The Census Bills, Great Britain and Ireland, 1910, have also been under the consideration of the Council, and a Memorandum has been submitted to the Local Government Board on the subject.

The Sixth International Congress of Actuaries was held in Vienna in June 1909, under the auspices of the Mathematisch-Statistischen Vereinigung, the Honorary President of the Congress being His Excellency Dr. Richard Freiherr v. Bienerth, and the acting President Dr. Emanuel Czuber. Three volumes of Reports, Memoirs and Proceedings were issued before the Congress assembled, and a further volume containing a report of the proceedings and discussions was published subsequently.

EXAMINATIONS, 1910.

Examinations were held on the 18th, 19th, 20th, 21st, 22nd and 23rd of April 1910, in the United Kingdom, the Colonies, and India, at London, Liverpool, Edinburgh, Melbourne, Sydney, Montreal, Toronto, Ottawa, Winnipeg, Calcutta, with the following results.

The successful candidates are placed in two classes only, the names being printed in alphabetical order in each class.

PART I.

Seventy-three candidates sent in their names, of whom seventy presented themselves (fifty-two in the United Kingdom, and eighteen in the Colonies), and sixteen passed, namely:—

Class I:

Duttagupta, S.

Class II:

Bedford, F.	Marathay, G. S.
Bill, A. F.	Olifiers, E.
Edwards, A. J. C.	Pequegnat, A. E.
Field, J. M.	Spurgeon, C. B.
Fortington, H. A.	Stockman, G. D.
Horner, B. S.	Trachtenberg, H. L.
McCormack, P. H.	Warwick, R. W.
McLean, P. S.	

PART II.

Seventy-five candidates sent in their names, of whom sixty-seven presented themselves (fifty-four in the United Kingdom, and thirteen in the Colonies), and twenty-two passed, namely:—

Class I:

None.

Class II:

Brown, B. G. H.	Handford, J. J. W.
Brown, P. G.	Hawes, E. E.
Chandler, F. J.	Humphreys, H. L.
Clarke, H. T.	Keevil, N. A. C.
Cook, H. M.	MaeTavish, A. N.
Cooper, J. L.	Marlin, J. H.
Cox, H.	Phillips, E. W.
Drake, C. C. H.	Spiegel, E. W. R.
Edwards, H. A.	Stephenson, H. R.
Emmerson, W. H. R.	Taylor, H. H.
Gawler, O.	Yeldham, W. J.

PART III.

Fifty-six candidates sent in their names, of whom fifty-one presented themselves (forty-five in the United Kingdom, and six in the Colonies), and thirteen passed, namely :—

Class I:

None.

Class II:

Blyth, R. O.	Jamieson, C. W. S.
Clemens, F. B.	King, A. E.
Derrick, V. P. A.	Maclean, J. B.
Doucet, G. D.	Rhodes, F.
Epps, G. S. W.	Simmonds, R. C.
Holness, A. S.	Sneddon, A. W.
Home, N. C. M.	

PART IV.

Nineteen candidates sent in their names, of whom eighteen presented themselves (sixteen in the United Kingdom, and two in the Colonies), and twelve passed, namely :—

Class I:

None.

Class II:

†Burrows, V. A.	†Jefferson, J. A.
†Carter, G. S.	†Kelly, J. J.
†Clinton, L. E.	†Levey, R.
†Edwards, H. H.	†Makepeace, F. L.
†Fippard, R. C.	†Nicholl, C. C.
†Gunningham, S. J.	†Thompson, J. S.

PART IV, § 7.

One candidate, who had passed Part IV of the 1906 Syllabus, presented himself for section 7 of Part IV of the present (1908) Syllabus, and passed :—

Hudson, A. J.

† Those marked (†) have now completed the Examination for the Class of Fellow.

PART I, § 3.

(COMPOUND INTEREST AND ANNUITIES).

Seventy-nine candidates, who had already passed, or been exempted from, Part I of a Syllabus prior to 1908, entered for this section alone, of whom seventy-one presented themselves (fifty-six in the United Kingdom, and fifteen in the Colonies), and twenty-five passed, namely:—

Cox, H.	Pickworth, E. B.
Davies, H. M.	Proddow, W. N.
Denmark, R. J.	Richardson, G. R.
Egleton, H. E.	Shepherdson, H. J.
Finlayson, G. D.	Sturt, A. J.
Hutchings, L. H.	Taylor, H. G. B.
Kubota, T.	Tomlinson, B.
Lewis, D. H.	Turner, J. G.
Mann, F. C.	Tyler, V. W.
Marshall, C. G.	Walters, A. H.
Matheson, D.	Webb, L.
Myers, H. D.	Wisdom, S. H.
Pattison, G. B.	

By Order of the Council,

THOMAS G. ACKLAND,

Chairman of Board of Examiners.

W. P. PHELPS,

L. F. HOVIL,

Joint Honorary Secretaries.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Sixty-third Annual General Meeting of the Institute of Actuaries was held at Staple Inn Hall, Holborn, on Monday evening, 6 June 1910, Mr. George Francis Hardy, President, in the Chair.

The PRESIDENT, at the outset, referred in sympathetic terms to the death of His Majesty King Edward the Seventh, the members present standing during his remarks. (See *J.I.A.*, vol. xliv, p. 303.)

The Report of the Council (given on p. 486) having been taken as read,

The PRESIDENT, in moving "That the Report and Statement of Accounts for the year 1909-10 be and are hereby adopted", said that there was very little to add to the report as to the work during the twelve months. With regard to the number of members, it would be noticed that there was again a slight falling-off from the high-water mark we reached in 1908, due mainly to a decrease in the number of students. He did not think that this slight falling off was necessarily to be regretted, especially if the quality of the students was maintained or even approached in future. He was sorry to say that the results of our examinations, so far as at present reported, were somewhat disappointing, as the percentage of students passed in the early stages of those examinations was still small. In connection

with this subject he might say that a suggestion had reached the Council that some of the second year's students would welcome the opportunity of meeting together in a kind of students' discussion or debating society, for the purpose of ventilating some of the questions which perhaps give them especial difficulty in their work, and possibly to carry on discussions and read papers amongst themselves. This was a matter that the Council were considering sympathetically. They had every wish to encourage any effort on the part of students to improve their efficiency, and desired to afford them all opportunities to better fit themselves for meeting the examiners from year to year.

The Sessional Meetings had been well sustained and most interesting. The discussions which had taken place had been as full as for many years past, and proved that the members had found the papers read to them valuable and suggestive. These papers varied a great deal in character, and included practical and theoretical papers touching the work at many points, and, taken as a whole, they would, he thought, form an addition to our *Journal* which would be of great value to all the members.

The Session had been an eventful one in many ways, as would be seen by some of the references in the report, especially so on account of the passing of the Assurance Companies Act of 1909, which would come into operation in a very few days. This was an Act of great importance, and a great deal of time and attention was devoted to the Bill by the Council of the Institute, when it was before Parliament. He thought they might fairly claim that, in the various recommendations made while the Bill was being considered—many of the most important of which had been embodied in the Act—their sole object was to secure that the Act should be thoroughly effective for the purpose for which it was framed. The President added that only a few days ago Mr. Ryan and himself, as representing the Council, had the pleasure of meeting in conference Mr. Barnes and other officials at the Board of Trade on the question of the definition of an Actuary for the purposes of the Act, and, as a result of that meeting, he thought it would be found that the final proposals of the Board of Trade would be eminently satisfactory to the profession. These proposals would be found to secure that in future all valuations should be made by competent men, and, at the same time, that no really competent man would be permanently shut out merely on technical grounds. Another matter of importance in the course of the Session was the Congress at Vienna, to which he had referred at the commencement of the Session.

The Institute had been very fortunate in not losing any of its Fellows by death during this year, although they had had the misfortune to lose some of the Members. Since the report was issued, they had sustained the loss of their old friend Mr. Holt, an Associate who was so well known to all that it was only fitting that he should say how much they all grieved that he should have been so suddenly taken from them.

He thought that they could look back upon the past Session as one in which good work had been done for the Institute, especially in the classes under the direction of the tutors, in whom the Council have every confidence, and the effect of whose work was bound to be seen.

Mr. TODD formally seconded the resolution for the adoption of the report and accounts, which was carried unanimously.

The ballot for President, Vice-Presidents, Council, Treasurer, and Hon. Secretaries was then proceeded with. Messrs. Kentish and Kenchington being appointed scrutineers. In the result the following gentlemen, recommended by the Council, were all unanimously elected :

President.

GERALD HEMMINGTON RYAN.

Vice-Presidents.

FRANCIS ERNEST COLENSO, M.A.	ERNEST COLQUHOUN.
SAMUEL GEORGE WARNER.	HENRY WALSINGHAM ANDRAS.

Council.

THOMAS GANS ACKLAND.	WILLIAM PEYTON PHELPS, M.A.
*HENRY WALSINGHAM ANDRAS.	EDWARD ARTHUR RUSHER.
*ARTHUR RHYS BARRAND.	GERALD HEMMINGTON RYAN.
ARTHUR DIGBY BESANT, B.A.	FREDERICK SCHOOLING.
HENRY COCKBURN.	JOHN SPENCER.
FRANCIS ERNEST COLENSO, M.A.	EDWARD ROBERT STRAKER.
ERNEST COLQUHOUN.	*HERBERT CECIL THISELTON.
WILLIAM PALIN ELDERTON.	ROBERT RUTHVEN TILT.
JOSEPH ERNEST FAULKS, B.A.	RALPH TODHUNTER, M.A.
DUNCAN CUMMING FRASER, M.A.	HAROLD MOLTKE TROUPER, M.A.
GEORGE FRANCIS HARDY.	SAMUEL GEORGE WARNER.
LEWIS FREDERICK HOVIL.	*ALFRED WILLIAM WATSON.
GEORGE KING.	JAMES DOUGLAS WATSON.
GEORGE JAMES LIDSTONE.	ERNEST WOODS.
*GEOFFREY MARKS.	FRANK BEETRAND WYATT.

Treasurer.

THOMAS GANS ACKLAND.

Honorary Secretaries.

WILLIAM PEYTON PHELPS, M.A.	LEWIS FREDERICK HOVIL.
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* New Members of the Council.

Mr. G. H. RYAN (who was accorded a very hearty reception) said that, on behalf of all the office-bearers whose names were included in the list, he begged to return grateful thanks for the resolution which had just been passed. For himself, the position in which they had placed him unanimously was such as to fulfil and gratify the ambition of any man. When the members chose a comrade from their ranks, and made him head of the profession and guardian of its interests for the time being, they were paying the highest compliment and conferring the greatest distinction in their power; and to the object of their choice the honour was one which had no parallel. He was deeply grateful to them for this signal mark of their esteem and confidence. It was natural at a time like this to look back along the vista of the past, and call to memory the many eminent and distinguished men who had sat in the Presidential Chair of the Institute. He should be the first to disclaim for himself any pretensions to rival or to equal the distinctions of his predecessors. At the same time, the Institute might be served in more ways than one, and he hoped that it might be his happy fortune during his term of office to do some work which would

redound to the credit and success of the Institute, and, when his period of office had passed away, he trusted that it would be held that he had not failed to maintain upon the same high level as his predecessors the honour and dignity and usefulness of the profession.

Mr. B. Woods proposed the re-election of Messrs. Harold Dougharty and J. C. Wardrop, and the election of Mr. H. J. Pearce, as Auditors for the ensuing year.

Mr. A. McDougald seconded the motion, which was carried unanimously.

Mr. R. Cross proposed, "That the cordial thanks of this meeting be accorded to the President, the Vice-Presidents, the Council, Officers, Examiners, and Hon. Supervisors for their services during the past year", and referred to the services rendered by the President and other Officers during the year.

Mr. F. T. M. Byers seconded the resolution, which was carried unanimously.

The President, in briefly acknowledging the vote of thanks, referred especially to the services rendered by Mr. Faulks, the retiring Hon. Secretary, and to his own satisfaction at being succeeded in the Chair by Mr. Ryan, in whose hands the interests of the Institute would be in thoroughly safe keeping.

Mr. Cockburn proposed a vote of thanks to the Auditors, Messrs. A. G. Scott, Harold Dougharty, and J. C. Wardrop, for their services during the past year.

Mr. Marcus N. Adler seconded the motion, which was carried unanimously.

Mr. J. C. Wardrop briefly acknowledged the vote on behalf of himself and his fellow-auditors.

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1909:

Accountants and Auditors, Society of Incorporated
List of Members, &c., 1909-10.

*By whom presented
(when not purchased).*

The Society.

Accountants, Institute of Chartered, in England and Wales.
List of Members, 1910.

The Institute.

Actuarial Society of America.

Transactions, 1909-10. *The Society.*

Containing *inter alia*—

"Some considerations relative to the probable future of the Interest Rate", by W. S. Nichols.

"On Annuity Reserves", by E. McClintock.

"Valuation by Attained Age", by R. Henderson.

"Staff Pension Funds. An Investigation of the Mortality among the Canadian Civil Service", by M. D. Grant.

*By whom presented
(when not purchased).*

Actuarial Society of America—(continued).

Transactions, 1909-10—(continued).

- “ On the rate of permanent disability amongst assured lives and the rate of mortality amongst disabled lives”, by S. H. Pipe.
- “ Life Experience of the Travelers Insurance Company”, by H. J. Messenger.
- “ The Rate of Cessation and Valuation of Renewal Commissions”, by P. C. H. Papps.
- “ Method of making Mortality Investigations by means of Perforated Cards, Sorting and Tabulating Machines, with special reference to the Medico-Actuarial Investigation”, by A. Hunter.
- “ Some uses for the Hollerith Machines”, by H. N. Kaufman.
- “ Reinstatement of Policy on Impaired Risk holding Extended Term Insurance”, by H. Moir.
- “ The total and permanent disability provision in connection with Life Insurance plans”, by F. B. Mead.

Actuaries, Faculty of

Transactions, 1909-10.

The Faculty.

Containing *inter alia*—

- “ Occupation Mortalities ”, by Dr. J. C. Dunlop.
- “ The Mortality Experience of the Gotha Life Office 1829-1896. An account of the Investigation and of its Results”, by G. W. Richmond.

Altenburger (J.).

Noch ein Beitrag zur Theorie und Praxis der mechanischen Ausgleichungs-Methoden. N.D.)

The Author.

American Mathematical Society.

Transactions, 1909-10.

The Society.

American Statistical Association.

Transactions, 1909-10.

The Association.

Assecuranz Jahrbuch.

Herausgegeben von A. Ehrenzweig. 8vo. V.D.
Vols. 10, 14, 25, 30.
„ 11, 12, 13, 15, 17 to 24, 26 to 29.
„ 16.
„ 31.

Dr. L. Spitzer.
Dr. J. Graf.
J. Altenburger.
Purchased.

Australian Mutual Provident Society.

Sixty-first Annual Report, 1910.

The Society.

Austria-Hungary.

Bericht der Arbeiter-Unfall-Versicherungs-Anstalt für das Königreich Böhmen, 1908.)
Versicherungswissenschaftliche Mitteilungen der Mathematisch - Statistische Vereinigung des Österreichisch-ungarischen Verbandes der Privat-)
Versicherungs-Anstalten, 1910.)

The Austrian Government.
The Society.

Bagehot (W.).

Economic Studies. 7th Edit. 8vo. 1908.

Lombard Street. A description of the Money Market.)
New Edit. 8vo. 1910.)

Purchased.

*By whom presented
(where not purchased).*

Bankers.

Journal of the Institute of Bankers.	The Institute.
List of Members, 1910.	
Questions on Banking Practice. (From Vols. I to XXX, inclusive, of the Journal of the Institute of Bankers.) 6th Edit. Svo. 1909.	

Barlow (C.A.M.) and G. L. Gomme.

Notes on the Old Age Pensions Act, 1908, together with the Regulations made thereunder, official circulars and financial instructions by the Treasury. Svo. 1909.	Purchased.

Belgium.

Bulletin de l'Association des Actuaires Belges.	The Association.
Bulletin du Comité central du travail industriel, 1910.	
Bulletin du Comité permanent des Congrès internationaux d'Actuaires, 1909.	
Bulletin du Syndicat des Compagnies d'Assurances-vie populaires opérant en Belgique, 1909-10.	
Caisse Générale d'Epargne et de Retraite. Recueil des Documents administratifs coordonnés. Svo. Brussels. 1909.	F. Hankar.
La Caisse, et ses différents Services. Svo. Brussels. 1910.	

Bellom (M.).

Les Questions Ouvrières et la Science actuarielle. Svo. Paris. 1909.	Purchased.

Berry (Jabez).

Index to Income Tax Acts, being a complete guide to the Statutes relating to Income Tax from 1842 to 1910. Svo. 1910.	Purchased.
[R]	

Besso (M.).

La Previdenza Sociale nel risorgimento (1848-1891). Rome, 1909.	The Author.

"Biometrika."

Volume VII, Parts I, II and III. Containing <i>inter alia</i> —	Purchased.

- “Tables of Γ-Function to seven figures, deduced from Legendre's Tables”, by J. H. Duffell.
- “Tables to facilitate the computation of the Probable Errors of the chief constants of Skew Frequency Distributions”, by A. Rhind.
- “On the distribution of the means of Samples which are not drawn from Random”, by “Student.”
- “A short method of calculating the co-efficient of correlation in the case of integral variates”, by J. A. Harris.
- “On a new method of determining correlation when one variable is given by alternative and the other by multiple categories”, by Karl Pearson.
- “A statistical study in Cancer death-rates”, by Dr. G. D. Maynard.

*By whom presented
(when not purchased).*

Birmingham, Insurance Institute of Transactions, 1908-9.	}	<i>The Insurance Institute.</i>
Containing <i>inter alia</i> —		
“The New Workmen’s Compensation Act”, by R. A. Willes.		
“Mortality Tables, &c.”, by H. J. Pearce.		
“The Judicial Development of the Workmen’s Compensation Act”, by R. A. Willes.		
Blaschke (Dr. E.). Die Prämien und Prämienreserven der Invaliden-Versicherung der Arbeiter. Wien. 1909.	}	<i>The Author.</i>
Blunden (G. H.). Local Taxation and Finance. 2nd Edit. 8vo. N.D.		<i>Purchased.</i>
Bowley (A. L.). An Elementary Manual of Statistics. 8vo. 1910.		<i>Purchased.</i>
Burn (J.). Stock Exchange Investments in Theory and Practice, with chapters on the Constitution and Operations of the Bank of England and the National and Local Debts of the United Kingdom. Being a Course of Lectures delivered at the Institute of Actuaries during the Session 1908-9. 8vo. 1909.		<i>Official.</i>
Cajori (F.). A History of the Logarithmic Slide Rule and allied instruments. 8vo. 1909.	}	<i>Purchased.</i>
Canada. Canadian Annual Financial Review, compiled by W. R. Houston. With Appendix. Ob. 8vo. Toronto, 1909, 1910.	}	<i>Purchased.</i>
Century. of Philosophical Paradoxes, in two Parts. To which is added an appendix containing Answers to the arithmetical problems left unanswered in Hill’s Arithmetic and Alexander’s Algebra. By a Lover of Mathematics. Sm. 8vo. Dublin. 1730. [R]		<i>Purchased.</i>
Cox-Sinclair (E.S.) and T. Hynes. The Taxation of Land Values under the Finance (1909-10) Act, 1910. 8vo. 1910.	}	<i>Purchased.</i>
Denmark. Beretning fra Forsikringsraadet for aaret, 1908. Copenhangen, 1910.	}	<i>Danish Government.</i>
Deutscher Verein für Versicherungs-Wissenschaft. Veröffentlichungen, 1909-10. Zeitschrift für die gesamte Versicherungs-Wissenschaft, 1909-10.		<i>The Society.</i>
De Witt (J.). Researches by Frederick Hendriks relating to the discovery and editing by him in 1851 of the long lost Treatise on Life Annuities written by the Grand Pensionary De Witt in 1671. With many original Notes and published and unpublished Letters and other Documents relating to this and Mr. Hendriks’ researches on the History of Insurance and Life Contingencies. With a collection of Autograph Letters, translations from the Dutch, &c., and portraits and plates. Folio.		<i>Purchased.</i>

*By whom presented
(when not purchased).*

Dodson (James).

The Mathematical Repository containing analytical solutions of Questions. Vol. I. Sm. 8vo. 1748. } } [R] } Purchased.

Drapers' Company Research Memoirs.

III. A second study of the statistics of Pulmonary Tuberculosis. Marital Infection. By E. G. Pope. Edited and revised by Karl Pearson, with an appendix by Ethel M. Elderton.
V. On the inheritance of the Diathesis of Phthisis and Insanity. By Dr. C. Goring. 4to. 1908, 1909.

Dryden (J. F.).

Addresses and Papers on Life Insurance and other subjects. 8vo. Newark, N. J. 1909. } } The Author.

Duffell (J. H.).

Tables of the Γ -Function. 1909. } } The Author.

Duncan (R. H.).

Practical Curve tracing; with chapters on Differentiation and Integration. 8vo. 1900. } } Purchased.

Duncan (W. W.).

Profits *versus* Dividends on Stock Exchange Investments. 8vo. 1900. } } Purchased.

Economic Society (Royal).

Journal of the, 1909-10. } } Purchased.

Edinburgh, Insurance Society of

Transactions, 1906-7, 1907-8. } } The Society.

Containing *inter alia*:

- “ Assignations of Life Policies, and legal decisions thereon ”, by A. H. B. Constable.
- “ The Claims Department of a Life Office ”, by G. W. Richmond.
- “ Valuations under the Employers’ Liability Insurance Companies Act, 1907 ”, by R. M. M. Roddiek.

Elderton (W. Palin and Ethel M.).

Primer of Statistics. Sm. 8vo. 1909. } } Purchased.

Esdale (Marcus).

How Property is valued. 8vo. N.D. } } Purchased.

Eugenics Laboratory Memoirs. 4to. 1907-1910.

II. A first study of the Statistics of Insanity and the inheritance of the insane diathesis. By D. Heron.
IV. On the Measure of the Resemblance of first cousins. By Ethel M. Elderton and Karl Pearson.
V. A first study of the inheritance of vision and of the relative influence of Heredity and Environment on Sight. By Amy Barrington and Karl Pearson.
VIII. The influence of defective physique and unfavourable home environment on the intelligence of school children. By D. Heron.
X. A first study of the influence of parental alcoholism on the physique and ability of the offspring. By Ethel M. Elderton and Karl Pearson.

*By whom presented
when not purchased).*

Firminger (F. L.).

The Workmen's Compensation Acts, 1906 and 1909,
with notes. 2nd Edit. Svo. 1910.

Purchased.

Fisher (Irving).

Report on National Vitality, its wastes and conservation. Svo. Washington. 1909.

H. Cockburn.

FitzGerald (J. V. Vesey) and R. J. Quin.

The New Law Regulating Assurance Companies.
Being the Act of 1909. With Notes. Svo. 1910.

Purchased.

France.

Bulletin de l'Institut des Actuaries français.

The Institute.

Germany.

Sammlung von Versicherungsbedingungen deutscher
Versicherungsanstalten. Unfall und Haftpflicht-
Versicherung. 4to. Berlin. 1910.

The Publishers.

Veröffentlichungen des Kaiserlichen Aufsichtsamts für
Privatversicherung. 4to. Berlin. 1910.
Versicherungs - Statistik über die Reichsaufsicht-
stehenden Unternehmungen. 1902-1908. 4to.
Berlin. 1905-1910.

*K. A. für
Privatversich-
erung.*

Gibson (T.).

The Pitfalls of Speculation. Svo. New York. 1908.

Purchased.

Giffen (Sir R.).

Essays in Finance. 2 Vols. Svo. 1880-1886.

Purchased.

Glover (J. W.).

The Monetary Loss in the United States due to Tuberculosis, based on the Returns of the Twelfth Census of the United States. 1908.

The Author.

A Study of Tuberculosis in the United States, based on the Returns of the Twelfth Census, with some special applications to the State of Michigan. 1909.

Derivation of the United States Mortality Table by Osculatory Interpolation. 1910.

Gotha.

Rechenschaftsbericht der Gothaer Lebensversicherungsbank auf Gegenseitigkeit für das Jahr, 1909.

*The Gotha
Life Office.*

Gover (W. H.).

A concise Treatise on the Law of Capital and Income as between Life-Tenant and Remainderman. 2nd Edit. Svo. 1910.

Purchased.

Hardy (R.P.).

Mortality Experience Investigation for the period 1903-7, and valuation Report of the Liabilities and Assets as at 31st December 1907, of the Great Western Railway Provident Society. Fol. 1910.

The Author.

Report and Valuation for Quinquennium ending 30 June 1909, with Tables of Sicknes and Superannuation Experience of the United Law Clerks' Society. Fol. 1910.

*By whom presented
(when not purchased).*

Head (F. D.).

The Transfer of Stocks, Shares and other Marketable Securities. A Manual of the Law and Practice. }
Svo. 1910. }
} Purchased.

Holland.

Archief voor de Verzekerings-Wetenschap. 1909-10. }
Nationale Levensverzekerings-Bank. Verslag over het }
drei-jahrig tijdvak 1907-9. }
} The Society.
The Company.

Hooper (E. Grant).

The German State Insurance system for providing }
Invalid and Old Age Pensions. 1908. }
} Purchased.

Hutton (C.).

Miscellanea Mathematica, consisting of a large collection of curious mathematical problems and their }
solutions. Sm. Svo. 1755. }
[R] }
} Purchased.

Insurance Institute of Ireland.

Journal of the, 1908-9. }
} The Insurance
Institute.

International Congresses of Actuaries.

Proceedings. The Sixth, Vienna, June 1909. 4 vols. }
Svo. 1909. }
} Purchased.

International Congresses on Diseases of Occupations.

Proceedings. The First, Milan, 1906. Svo. 1906. }
} Purchased.

International Congresses on Workmen's Accidents and Social Insurances.

Proceedings.
The Third, Milan, 1894, 2 vols. Svo.
The Fourth, Brussels, 1897, 1 vol. Svo.
The Fifth, Paris, 1901, 2 vols. Svo.
The Sixth, Dusseldorf, 1901, 1 vol. Svo.
The Seventh, Vienna, 1906, 2 vols. Svo.
The Eighth, Rome, 1908, 3 vols. Svo.
} Purchased.

Italy.

Bollettino della Associazione Italiana per l'incremento della scienza degli Attuari, 1909-10. }
} The Association.

Iversen (L.).

Dodeligheden blandt Forsorgede en Forsikrings- }
Statistisk undersogelse. Svo. Copenhagen. 1910. }
} The Author.

Journal of the Federation of Insurance Institutes.

Volume XII. 1909. }
} The Federation.

Containing *inter alia*—

“Classification of Life Insurance Risks”, by
D. E. Kilgour.

“Insurance Companies as Executors and Trustees”,
by T. F. Anderson.

“Personal and Family history in relation to Life
Insurance”, by Dr. J. Dulberg.

“Progress and Position of Friendly Societies”, by
R. Gordon-Smith.

“Some observations on the comparative Death
Rates of Abstainers and non-Abstainers in
Life Assurance Companies”, by Dr. E.
Duncan.

“Special Policies”, by W. Penman, Jr.

“Twenty years' changes in Life Assurance”, by
S. G. Warner.

“Undesirables (under-average lives)”, by Dr.
T. R. Taylor.

*By whom presented
(when not purchased).*

Lamb (H.).	An Elementary Course of Infinitesimal Calculus. 8vo. Cambridge, 1907.	Purchased.
Leach (R. A.).	Old Age Pensions Act, 1908, annotated. 2nd Edit. 8vo. 1908.	Purchased.
Life Offices' Association.	Memoranda as to the Meetings of the Standing Committee, 1909-10.	The Association.
Loewy (Dr. A.).	Versicherungsmathematik. Sm. 8vo. Leipzig. 1910.	The Author.
London Mathematical Society.	Proceedings, 1909-10.	The Society.
Love (A. E. H.).	Elements of the Differential and Integral Calculus. 8vo. Cambridge. 1909.	Purchased.
Lowenfeld (H.).	Investment practically considered. 8vo. N.D.	Purchased.
Macleod (H. D.).	The Theory and Practice of Banking. 6th Edit. 2 vols. 8vo. 1902.	Purchased.
Manson (E.).	The Debentures and Debenture Stock of Trading and other Companies. 2nd Edit. 8vo. 1910.	Purchased.
Moir (H.).	Agency Arguments and Principles of Life Insurance. 8vo. New York. 1909.	The Author.
Moody (J.).	The Investors' Primer. Svo. New York. 1907.	Purchased.
Moody's Manual of Railroad and Corporation Securities, 1910.		Purchased.
Moore (E. H.), E. J. Wilczynski and Max Mason.	The New Haven Mathematical Colloquium. Lectures delivered before the American Mathematical Society. 8vo. New Haven. 1910.	The Yale-University Press.
Moxon (T. B.).	English Practical Banking. 15th Edit. 8vo. 1910.	Purchased.
New South Wales, Insurance Institute of	Transactions, 1909.	The Insurance Institute.
Containing <i>inter alia</i> —	“Personal Accident Insurance”, by H. E. Aggs.	
Nicholson (J. S.).	A Treatise on Money and Essays on Monetary Problems. 6th Edit. Svo. 1903.	Purchased.
Nunez (A. L.).	Sinopsis para un estudio de la Institucion del Seguro. 8vo. Madrid. 1906.	The Author.
	El Seguro obrero en Espana. Svo. Madrid. 1908.	

*By whom presented
(when not purchased).*

Osborne (R. S.).

Practical Arithmetic. Examples and Exercises. }
Sm. 8vo. 1910. }
} The Author.

Oss (S. F. van).

American Railroads and British Investors. 8vo. 1893. Purchased.

Palmer (Sir F. B.).

Company Law. With an appendix containing the Companies (Consolidation) Act 1908, and other Acts and Rules. 7th Edit. 8vo. 1909. Purchased.

Pageat (Sir John R.).

The Law of Banking. 2nd Edit. 8vo. 1908. Purchased.

Parliamentary Papers.

Acts of Parliament.

Assurance Companies, 1909.	}	Purchased.
Asylum Officers' Superannuation, 1909.		
Finance, 1910.		
Indian Railway Annuities (Sinking Funds), 1909.		
Public Works Loan, 1909.		
Superannuation, 1909.		
Transvaal Railway Service and Pensions, 1908.		
Westminster City Council (Superannuation and Pensions), 1909.	}	
Workmen's Compensation (Anglo-French Convention), 1909.		

Colonies.

Australia, Commonwealth of	}	The Commonwealth Govt.
Official Statistics, 1909-10.		
Official Year Book, 1910.	}	

Canada.

Report of the Superintendent of Insurance for	}	The Government Insurance Dept.
the year 1908.		

New South Wales.

Friendly Societies. Report of the Registrar for 1908.	}	The Government of N.S.W.
Statistical Register for 1908 and previous years.		
Vital Statistics, Report for 1908 and previous years.		
Vital Statistics, Preliminary Report of the Government Statistician for 1909.	}	

New Zealand.

Actuary's Report on the Valuation of the Government Insurance Department, 1908.	}	The Government of N.Z.
Official Year Book, 1909.		

South Australia.

Friendly Societies. Mortality, Sickness and other Tables, based on the Experience of Registered Societies for the period 1895-1904, by H. D. Gouge. 4to. 1909.	}	The Government of S. Australia.
--	---	---------------------------------

Victoria.

Friendly Societies. Thirty - First Annual Report of the Government Statist, 1908.	}	The Government of Victoria.
To which are appended Monetary Tables based on the Victorian Friendly Societies' Experience, 1903-7.		
Statistical Register for 1908.		
Year Book, 1875 to 1886-7, 1888-90, 1902-8, 1908-9.	}	

*By whom presented
(when not purchased).*

Parliamentary Papers—(continued).

Colonies—(continued).

Western Australia.

Friendly Societies. Report of the Proceedings by the Registrar, 1909.

Eighth Annual Return and Report of Proceedings under the Industrial Conciliation and Arbitration Act, 1902. 1909.

Statistical Register for 1907 and previous years.

The Government of W. Australia.

Companies.

Eighteenth Annual Report by the Board of Trade. Fol. 1909.

East India (Railways).

Administration Report on the Railways of India for 1908. Fol. 1909.

Elementary School Teachers (Superannuation) Act, 1898.

Amendment of Superannuation Rules, 1899.

Employers' Liability Insurance Companies.

Statements deposited with the Board of Trade in pursuance of the Act of 1907, and the adaptation of Enactments Order of 1907, during the year ended 31 December 1909. Fol. 1910.

Friendly Societies.

Building Societies.

Fourteenth Annual Report of the Chief Registrar for 1908.

Chief Registrar's Report for 1908. Part A, Part A, Appendix (O), Parts B and C.

Infant Life Protection.

Report of Select Committee, together with Proceedings, Minutes of Evidence, Appendix, and Index. Fol. 1871.

Life Assurance Companies.

Return to the House of Commons of the names, places of business, and object of all Assurance Companies, and a copy of every Account registered by such Companies. In one vol. Fol. 1851-56.

Returns, 1908, 1909.

Old Age Pensions.

Circulars issued by the Local Government Board to County Councils as to the administration of the Act of 1908. Fol. 1908.

Public Accounts.

First to Fifth Reports from the Committee, with Proceedings, Minutes of Evidence, and Appendices. Fol. 1909.

Railway Regulation Acts.

Departmental Committee's Report with reference to the form and scope of the Accounts and Statistical Returns rendered under the Acts. Fol. 1909.

Registrar-General, England.

Seventy-first Annual Report of Births, Deaths and Marriages, 1908.

Purchased.

*By whom presented
(when not purchased).*

Parliamentary Papers—(continued).

Registrar-General. Ireland.

Forty-fifth detailed Annual Report of Births,
Deaths and Marriages, 1908.

Registrar-General. Scotland.

Fifty-third detailed Annual Report of Births,
Deaths and Marriages. (Abstracts of 1907.)

Fifty-fifth Annual Report on Births, Deaths and
Marriages, 1909, and Forty-fifth Annual
Report on Vaccination.

Sinking Funds.

Report of Select Committee on the application of,
in the exercise of borrowing powers (1908),
together with Minutes of Evidence and
Appendix. Fol. 1909.

Superannuation.

Draft Treasury Regulations under Section 3 of the
Bill of 1909. Fol. 1909.

Workmen's Compensation.

Statistics of Proceedings under the Acts of 1897
and 1900, and the Employers' Liability Act,
1880, during the year 1908.

Periodicals.

Accountants' Magazine.

American Bankers' Magazine.

Bankers' Magazine.

Economist.

Insurance Record.

Post Magazine.

Post Magazine Almanack.

Statesman's Year Book.

Purchased.

Purchased.

The Editor.

The Editor.

Purchased.

Helps (E. B.).

An analysis of some phases of American Mortality
Statistics for the nine years 1900-1908. New
York, 1909.

The Author.

Picken (C. H.).

Handbook to Stamp Duties, containing the text of the
Stamp Act, 1891, and of the subsequent Revenue
Acts so far as they relate to Stamp Duties, with a
complete table of all documents liable to Stamp
Duty. 15th Edit. Svo. 1910.

Purchased.

Pickworth (C. N.).

The Slide Rule. A practical manual. 11th Edit.
Svo. 1908.

Purchased.

Plehn (C. C.).

Introduction to Public Finance. 3rd Edit. Svo.
1909.

Purchased.

Price (L. L.).

Money and its relation to Prices. Svo. 1909.

Purchased.

Proctor (R. A.).

Chance and Luck. Svo. 1889.

Purchased.

Rose (T. Kirke).

The Precious Metals, comprising Gold, Silver, and
Platinum. Svo. 1909.

Purchased.

Royal Astronomical Society, Memoirs of the

Volume LIX, Part 4. 1909.

The Society.

*By whom presented
(when not purchased).*

Ruegg (A. H.).	The Employers' Liability Act, 1880, and the Workmen's Compensation Act, 1906. With Notes. 8th Edit. 8vo. 1910.	Purchased.
Russell (H. B.).	International Monetary Conferences, their purpose, character, and results. 8vo. 1908.	Purchased.
Rye (R.A.).	The Libraries of London. A Guide for Students. 2nd Edit. 8vo. 1910.	The Author.
Schooling (W.).	The Assessment of Income Tax. 1910.	The Author.
Scientific and Learned Societies of Great Britain and Ireland.	Official Year Book, 1909.	Purchased.
Sheppard (W. F.).	On the application of the Theory of Errors to cases of normal distribution and normal correlation. 1898. On the calculation of the Double Integral expressing normal correlation. 1900. The calculation of Moments of a Frequency curve. 1907.	The Author.
Sim (J. D. Stuart).	The Law of Friendly Societies and Industrial and Provident Societies. 14th Edit. 8vo. 1909.	Purchased.
Simonson (P. T.).	A treatise on the Law relating to Debentures and Debenture Stock. 3rd Edit. 8vo. 1902.	Purchased.
Spitzer (Dr. L.).	Progetto di Piano di Lavoro per la formazione di una tavola di mortalità di assicurati Italiani. Milan. 1909.	The Author.
Statistical Society (Royal).	Journal of the, 1909-10.	The Society.
Steele (F. E.).	Present-day Banking, its methods, tendencies, and characteristics. 8vo. 1909.	Purchased.
Steffensen (J. F.).	Dodelighedstavelen $D^{M(5)}$	The Author.
Stock Exchange Official Intelligence, 1910.		Purchased.
Stock Exchange Year Book, 1910.		Purchased.
Switzerland.	Rapport du Bureau Fédéral des Assurances sur les Entreprises Privées en Matière d'Assurances en Suisse, 1908.	The Swiss Government.
Toronto, Insurance Institute of	Proceedings, 1908-9. Containing <i>inter alia</i> — “The Fieldman in Life Insurance”, by W. Hamilton.	The Insurance Institute.

*By whom presented
(when not purchased).*

Toronto, Insurance Institute of—(continued).

Proceedings, 1908-9—(continued).

“General Remarks on Old Age Pensions”, by
M. A. Mackenzie.

“Practice of Companies as to Loans on Policies,
Surrender Values, and Settlement of Claims”,
by J. B. McKechnie.

“Adjustment of Personal Accident Claims”, by
E. Williams.

“Classification of Life Insurance Risks”, by D. E.
Kilgour.

Treasury of Human Inheritance.

Parts 1 to 4. 4to. 1909-10.

Purchased.

Trivett (J. B.).

Tuberculosis in New South Wales. A statistical
analysis of the Mortality from Tubercular
Diseases during the years 1876-1908. Svo.
Sydney. 1909.

The Author.

New South Wales Friendly Societies' Experience,
1900-1908. 4to. Sydney. 1910.

Truelove (M. H.).

The Assurance Companies Act, 1909. With the Rules
and Orders made thereunder by the Board of
Trade. With Notes. Svo. 1910.

Purchased.

United States of America.

Official Publications.

Ninth Annual Report of the Bureau of the
Census. Mortality Statistics 1908. La. 4to.
Washington, 1910.

*The U.S.
Government.*

Connecticut.
Forty-fifth Annual Report of the Insurance
Commissioner, 1909.

The Commissioner.

Massachusetts.
Fifty-fourth Annual Report of the Insurance
Commissioner, 1909.

The Commissioner.

New York State.
Fiftieth Annual Report of the Superintendent
of Insurance, 1909.

*The Super-
intendent of
Insurance.*

Victoria, Insurance Institute of

Proceedings, 1909-10.

{ *The Insurance
Institute.*

Webb (C. A.).

Valuation of Real Property. A Guide to the Principles
of Valuation of land and buildings, &c. 8vo.
1909.

Purchased.

Whittall (W. J. H.).

An elementary Lecture on the Theory of Life Assurance.
2nd Edit. Svo. 1909.

Purchased.

Williams (F. A.).

Temporary Annuities and Preliminary Term Policy
Values. American Experience, 4 per cent. Svo.
Mexico. 1910.

The Author.

*By whom presented
(when not purchased).*

Wilshire (A.M.).

An analysis of Williams on the Law of Real Property. }
2nd Edit. 8vo. 1910. }
Purchased.

Withers (A. H.).

The investigation of title to and the completion of }
purchases and mortgages of Life and Reversionary }
Interests in Personality Trust Funds and Policies }
of Life Assurance. 8vo. 1910. }
Purchased.

Yorkshire, Insurance Institute of

Proceedings, 1908-9. }
{ The Insurance
Institute.

Young (T. E.).

A plain guide to Investment and Finance. 2nd Edit. }
8vo. 1909. }
Insurance. A practical exposition for the Student and }
the Business Man. 2nd Edit. 8vo. 1906. }
Purchased.

Young (W. H.).

The Fundamental Theorems of the Differential Calculus. 8vo. 1910. }
} Purchased.

The following additional copies of works already in the Library have also been added :—

Actuarial Society of Edinburgh.

Transactions. Volumes 1 to 4. Purchased.

Birmingham, Insurance Institute of

Transactions, 1894-5, 1895-6, 1896-7, 1899-1900. }
{ The Insurance
Institute.

Catalogue

of Books and Single Papers relating to the subjects of }
the Rate of Mortality, Annuities, and Life }
Assurance, etc, 8vo. 1836. }
L. F. Horil.

De Morgan (A.).

A Budget of Paradoxes. 8vo. 1872. Purchased.

Parliamentary Papers.

Colonies. }
Victoria. }
Year-Book for 1873, 1891-3, 1895-8. }
{ The Victorian
Government.

ERRATA.

Mr. D. C. FRASER calls our attention to the following corrections which should be made in his Actuarial Note "On Formulas for the Force of Mortality" in volume xlivi of the *Journal* :—

Page 426, line 9, for μ_{x+a} read $t \cdot \mu_{x+a}$

$$\text{,, , , } 13, \text{,, } \frac{3}{80} t^2 \text{,, } \frac{3}{800} t^2$$

Ed. J.I.A.

**BRITISH ASSOCIATION FOR THE ADVANCEMENT OF
SCIENCE.—SHEFFIELD, 1910.**

*EXTRACTS FROM ADDRESS TO THE ECONOMIC SCIENCE
AND STATISTICS SECTION.*

*By Sir H. LLEWELLYN SMITH, K.C.B., M.A., B.Sc., F.S.S.,
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(INSURANCE AGAINST UNEMPLOYMENT.)

One of the most significant and important economic tendencies of the present day is the growing recognition of the importance of security and regularity in all operations of industry and commerce. It is, of course, a trite commonplace that the foundation of commerce is security—that safety of person and property and security for the performance of legal obligations are essential conditions of all industrial and commercial development. But it is not of these elementary guarantees that I am speaking, but of the tendency which I see to attach ever greater importance to the certainty and regularity of sequence as distinguished from the mere aggregate volume of business transactions. This tendency is reflected in the enormous development of the method of insurance as a protection against risk.

Nor is this development confined to business transactions properly so-called. A number of the risks and contingencies of human life which cause irregularity and uncertainty in working-class incomes have been brought within the sphere of insurance, whether by voluntary institutions or, as in Germany, by a State system or organisation. And the question of the perfection and further development of the methods of social insurance is absorbing a large amount of the best thought of the day.

All this points to the growing importance attached by social observers to stability and regularity, and the grounds for this attitude are sufficiently obvious, whether we look at the matter from the point of view of the economy of the workman's household, or of the deteriorating effects of irregular habits on physique and character. It may perhaps be suggested that the growing social concern for the maintenance of stability is the counterpart of the growing conviction that with the world-wide development of industry the causes of fluctuations and irregularity are becoming continually more incalculable and their effects more unavoidable by unaided individual effort.

Is this tendency to exalt security as an end a healthy tendency, or ought it to fill us with apprehension?

The ideal of security may not at first sight seem a very heroic aim to put before a country whose economic traditions form a veritable romance of adventure full of the joy of risks encountered and dangers overcome. Some may think with misgiving that the conscious pursuit of a policy of safety implies that we have passed the stage of economic youth and expansion and are entering on the dusk of old age. They may feel as when at Rome we contemplate Aurelian's great wall which for centuries withstood the inroads of barbarians, but the building of which none the less marked the definite close of the period of the fearless and aggressive supremacy of Rome. Are the nations of Europe being invited to enter with the old gods into the fortress of Valhalla, there to await in well-planned security but in growing gloom their inevitable decline? The question is cogent and searching, and modern nations must find the true answer at their peril, for if the two ideals of free adventure and economic security admit of no reconciliation, then the fate of our civilisation is only a matter of time.

But fortunately it is not necessary to admit the essential opposition of these two ideals rightly conceived. For as it seems to me there is a noble as well as an ignoble ideal of adventure, and, corresponding thereto, there is a noble as well as an ignoble ideal of security, and the great problem that lies before us in the future is to distinguish rightly between them and to direct our national policy accordingly.

The first step towards making this distinction is to recognise that ignoble as well as noble results are produced by exposure of risks. If fearless resolution and foresight in encountering and combating danger and risk produced the race of Elizabethan mariners and explorers, and to-day gives us a Shackleton or a Sven Hedin, we know also the craven and panic-stricken population which lives on the slopes of a volcano, exposed every day to incalculable risks against which no precautions can avail.

It is, I think, a definite induction from history and observation that when risk falls outside certain limits as regards magnitude and calculability, when in short it becomes what I may call a gambler's risk, exposure thereto not only ceases to act as a bracing tonic, but produces evil effects of a very serious kind.

It is to the general interest, and it tends to the building up and strengthening of the national character, that everyone should have as strong a motive as possible to guard against risks which can be avoided by reasonable precautions on the part of the individual, and it is also to the general interest that within certain limits the individual should have sufficient resisting power and reserve strength to encounter without the support of his fellows the ordinary minor ups and downs of life which it is not within his power to avoid. What these limits are cannot be laid down dogmatically : they vary widely from nation to nation, from class to class, and from age to age. Vieissitudes which mean famine to the savage pass quite unnoticed in advanced industrial communities, and classes who are accustomed to yearly salaries are unconcerned with fluctuations which bring privation to the weekly wage earner. But within any given nation and class the limits probably change but slowly, and though different schools of social observers will certainly fix the limits at somewhat different points, and there is no doubt a neutral zone within which the relative public advantages and disadvantages of exposure to risk are fairly equally balanced, or at least may be open to legitimate debate, I am disposed to think that the majority of fair-minded men would not differ very widely in the principles governing the demarcation between the spheres of individual and of social protection against economic risk. To take, for example, the risks of unemployment, I think most people would agree that the personal risk of losing employment through bad work, irregular attendance, or drunken habits is one which it is absolutely necessary in the public interest to leave attached in all its force to the individual workman. For the community to guarantee employment to all irrespective of personal effort or efficiency would necessarily impair the national character and lower the national standard. This is, therefore, a risk the direct incidence of which must be borne by the individual, the action of the community being confined to such indirect measures as may strengthen the power of the individual to meet the risk, as, for example, by technical and general training.

On the other hand I think that most people would agree that in a country like the United Kingdom at the present time, the incalculable risk of a prolonged depression of trade, due perhaps to some financial catastrophe thousands of miles away, is one the exposure to which of the individual workman does little but harm. Such a risk is too much beyond his powers of foresight, and also too great in magnitude in proportion to his reasonable opportunities of making provision, to exercise any appreciable effect in stimulating self-help, while the liability to see all his savings swept away in a few weeks by cyclical fluctuations in employment which he can do nothing to avoid is a demoralising risk acting on his character precisely like the liability to earthquake or other cataclysm, and discouraging to a marked extent the accumulation of savings and the development and maintenance of habits of providence.

Between these two extremes, the risk due to personal inefficiency and that resulting from a world-wide depression of trade, lie intermediate classes of risks about which there might be more difference of opinion, and the incidence of which probably acts on national character in very different ways in countries at different stages of development.

I propose presently to examine more closely some of these classes of risks. At the moment, however, I am only concerned to illustrate my general proposition that neither free adventure nor economic security suffices singly as an ideal of economic conduct without careful discrimination, and that the criterion

for such discrimination is the effect of exposure to each class of risk in building up or degrading the national character.

In suggesting that the attention of economists is being directed and will continue to be directed in an increasing degree to the ends of economic conduct as distinct from a mere analysis and description of existing conditions, I have taken a single example, the pursuit of economic security as an objective, and have drawn a vital distinction between the classes of economic risks exposure to which tends to the building up or to the degradation of the national character. And as regards these risks I have taken a single illustration, that of unemployment, partly because the evils resulting therefrom have been very much in our thoughts during the last few years, partly because their analysis affords good illustrations of almost every class of economic risk.

I might go on to take other examples, but I think that it may perhaps serve a more useful purpose if during the time that remains to me I follow up in further detail the particular illustration which I have chosen, and inquire specifically how far the risks of unemployment are risks which it is expedient in the public interest that each individual should be left to meet unaided, or how far they are from the social point of view 'insurable risks' which can properly be met by combined action.

We shall find that the reply to the proposition is by no means a simple one, that it will differ to a large extent for different trades, and that probably it will also differ widely for different countries.

At the outset it is to be noted that I use the term 'insurable risk' for the purpose of this inquiry in a much narrower sense than that which it bears in the ordinary language of the insurance world. Broadly speaking, if the term be used in its widest sense there are no risks that are not insurable except those which are the result of the direct wilful act of the insured person. Thus you can insure against fire but not arson, against death but not suicide. And even with regard to acts which are voluntary the modern tendency is to take a very broad view, and to narrow the classes of cases excluded. Thus most life assurance companies will pay on death, even if due to suicide, provided that the policy was taken out sufficiently long before the death to make it fairly certain that suicide was not in contemplation at the time.

As I am now using the term 'insurable,' however, I mean not merely a risk in respect of which you could get some company or underwriter to quote you a premium, but a risk for which some sort of social insurance is a practicable and appropriate remedy—bearing in mind the critical distinction already drawn between different classes of risks.

Moreover, by 'insurable risk' I do not mean a risk which can be fully covered by insurance, but one the consequences of which may be mitigated by a payment which nevertheless falls far short of complete indemnity. It hardly needs demonstration that full indemnity against the risks of unemployment could not be offered without disastrous results, inasmuch as a large section of persons regard idleness as in itself more attractive than work. The universal practice of organisations, voluntary or public, which insure against sickness, accident, or unemployment, is to make the benefit payable much less than the full rate of wages, and in all that follows this condition is assumed.

For the purpose of the present inquiry the causes of unemployment group themselves naturally under three heads—periodic fluctuations, local and industrial displacements, and personal causes.

Of these I have already touched on the first group in discussing cyclical and seasonal fluctuations of employment. Seasonal changes are, of course, the direct result of cosmical causes, and whether or not cyclical fluctuations are ultimately psychological or (as Jevons thought) cosmical phenomena, there can be no doubt that for our present purpose we may regard them as ultimate facts beyond the control of the individual. These two elements in unemployment are pre-eminently insurable elements, since, being due to recurrent oscillations and not to progressive changes, they can only be met by some method, either individual or collective, of spreading the earnings of good periods over good and bad alike, and not by any remedy which aims at altering the permanent relation between the demand for labour and the supply. Moreover, of the two alternative methods, collective insurance is more appropriate for the purpose than

individual providencee, because while the oscillations are fairly well defined, their intensity and (in the case of cyclical fluctuations) their wave length are affected by many uncertain elements, climatic, financial, industrial, and political, which are incapable of exact prediction, and (what is even more important) the personal incidence of the unemployment due to the oscillations is uncertain.

The next group of causes includes changes in industrial processes or methods or in the local distribution of industries, or in the character of industrial demand. How far are these classes of risks properly insurable?

As regards local distribution, the answer depends on the scope of the insurance scheme. No purely local fund can, of course, compensate a workman for the shifting of his industry to other districts, without incurring ruinous expense besides impairing the mobility of labour. If, however, the insurance scheme be national in scope and be worked in conjunction with systematic machinery for notifying to the workman the existence of vacancies in other districts, the risk of unemployment due to local displacement is clearly an 'insurable' risk. As no national scheme could embrace a wider area than the United Kingdom, the above argument does not apply with its full force to the risk of displacement of industry by foreign competition, and this case needs separate treatment. It is undoubtedly a risk beyond the individual's control, and it has, therefore, one of the essential marks of an insurable risk; and if the scheme embrace a large group of trades of sufficient variety to insure each other against the risk of some particular branch being attacked by foreign competition there is no reason why this class of risk should throw an excessive burden on a national fund. The only question to be considered is, therefore, whether the insurance of British workmen in an industry liable to be transferred by competition to a foreign country will operate prejudicially by checking industrial mobility, there being obviously not the same opportunity for the workman to follow the work as in the case of local redistribution of industry within the limits of the insuring country.

In this respect the case we are now considering is on all fours with that of a trade decaying through a permanent change of industrial demand, or an alteration of industrial processes. If there is appreciable mobility of labour between the decaying trade and other healthy branches embraced within the scope of the insurance scheme, and if its magnitude is small as compared with the total area of industry covered by the scheme, then the risk is fairly insurable. If, however, these conditions are not fulfilled, the case of the permanently decaying trade may present a real though by no means insuperable difficulty which will have to be carefully borne in mind by those responsible for devising and working any unemployment insurance scheme.

The conclusion seems to be that the extent to which the risk of unemployment due to industrial and local displacement is properly insurable depends partly on a wise choice being made of the group of trades and of the geographical area to be embraced by the scheme, partly on the judicious limitation of the benefits payable thereunder. Our analysis points to the necessity of a large area, both geographical and industrial, and further suggests that the groups of trades included should be such as are unlikely as a whole to undergo wholesale and rapid displacement, and within which any decay to be apprehended is likely to be only local and partial and not on a scale too great to be compensated by the expansion of other branches of trade within the insured group.

There remain the risks due to personal causes. Of these we have already ruled out the risks due to the wilful act of the workman, and to these we must now add the personal risk attributable to exceptional deficiencies, physical, mental, or moral. These are not properly trade risks, the burden of which ought to fall in a special degree on those following a particular industry, and if they were allowed to do so they would ruin any scheme of insurance based on the trade group. There is still, however, one important class of personal risk to which all are liable, and which is in the main beyond the control of the individual, viz. the increasing liability to unemployment due to advancing years. I do not intend to trench on the important but quite separate problems of national provision for old age and invalidity as such. I am solely referring to the ascertained statistical fact that the chance of unemployment is a function of

age, and that beyond a certain age the risk is materially increased. For example, among a body of nearly eight thousand engineers whose industrial records were analysed for the purpose, I found that whereas the average number of working days lost in the year by the whole body was fifteen, that for members below the age of forty-five was less than twelve, while for members between the ages of forty-five and fifty-five it was twenty, and for members between fifty-five and sixty-five, thirty-three. (Above sixty-five the figures are affected by superannuation.) The question we have to ask is, how far this class of risk is insurable?

The answer depends again on the scope of the scheme. A voluntary scheme which workmen are free to join and leave at their pleasure cannot deal satisfactorily with a risk of this kind, especially as no scheme of graduated contributions according to age is likely to be administratively feasible. Trade unions which give unemployment benefit are in an exceptional position, because they exist primarily for trade protection purposes, and hence have a hold on their members which no voluntary insurance scheme pure and simple could possess. Generally speaking, personal unemployment due to advancing years is insurable, and only insurable, under a scheme which applies compulsorily throughout the whole period of the workman's industrial life.

It results from our analysis that some of the risks of unemployment are properly insurable and others are not, and the next step is to ascertain broadly the relative importance of the insurable and non-insurable elements. Now an examination of the available statistics indicates clearly that at all events as regards certain large groups of trades in which unemployment is acute—namely, the building, engineering, and shipbuilding trades—the insurable element in the risk of unemployment predominates largely over the non-insurable element.

The method of statistical proof of this proposition may be indicated as follows:—

1. The percentage of unemployment in these trades—taking an average of good and bad years together—has not varied very widely during the period of fifty years during which the statistics have been collected (the average for the first decade of the period was 5·6; for the second, 4·5; for the third, 6·8; for the fourth, 5·2; and for the fifth, 7·2. The average for the whole period was 5·9). As the period of oscillation is not exactly ten years, part even of the differences shown above is accounted for by the presence of an excessive proportion of good or bad years in particular decades. Thus we may fairly say that the element of unemployment due to progressive expansion or contraction of the demand for labour has been relatively small.

2. The percentage of unemployment found during the seven best years of the cycles has averaged 2·4, and in only two out of these seven years has it diverged by more than unity from this average.

3. The variation between the worst and the best years of the various cycles has averaged 8·5 per-cent—*i.e.*, more than three times the average percentage of unemployment in good years.

Now, broadly speaking, if we neglect any progressive changes in the total demand for labour, which are evidently slight as compared with the intensity of the periodic fluctuations in that demand, we may say that the percentage who are unemployed in years of good employment gives a maximum limit which the voluntary or non-insurable risk cannot exceed, since it also includes a number of minor accidental risks which are properly insurable—*e.g.*, the risk of unemployment through a fire or other accidental stoppage of work, or through defects in the local redistribution of work and labour. Moreover, through the method of averaging employment over the year, the risk of seasonal want of employment is included, and this is mainly an insurable risk.

We may further regard the difference between unemployment in a good and bad year as giving a minimum measure of the insurable element in unemployment, since this difference is wholly the result of changes in the demand for labour, and is independent alike of the choice of the individual and of the gradual progressive changes, if any, that affect the total field of employment. Hence, as this difference is much greater than the minimum percentage in a good year, we may regard our proposition as being proved.

But at this point it is necessary to forestall and reply to an objection that will certainly be taken to the proposition just laid down. It will be pointed out that the experience of all relief works and of all schemes for the relief of distress due to unemployment establishes clearly that the great majority of the unemployed, or at least those who seek relief from distress, are very markedly inferior both as regards their industrial capacity and their physical and moral qualifications to the average employed workmen in the same trade. It is possible in a large number—probably in the majority—of these cases to trace clearly the operation of the personal defects which have contributed to unemployment—bad time-keeping, drink, slovenly work, and so forth—and those who are most familiar with the personal side of the problem are, I think, likely to put the personal or non-insurable element in the risk of unemployment very much higher than I have done in relation to the involuntary insurable element.

But in this criticism there is, I think, confusion of thought. Of course, if fifty men out of every thousand are out of work, those fifty individuals are likely to be less eligible than any other fifty taken at random. We might, if so disposed, construct a geometrical curve like those used in expounding the doctrines of utility and rent, in which the number of workmen employed is expressed by abscissæ and the degrees of efficiency by ordinates. Then it will appear at a glance that in a time of good trade the efficiency of the 'marginal' labourer—that is, of the worst man who just manages to retain his employment—is necessarily less than when the total demand for labour has shrunk from any cause. In the latter case the workmen discharged will for the most part be in the less eligible section; and this state of things is quite independent of the true cause of the shrinkage in the demand for labour, so that while the personal defects of A may be the decisive reason why he is selected for unemployment instead of B, it does not follow that these defects are a principal or even a contributory cause of his unemployment.

It is a very complex and difficult question, only to be determined in any given case with full regard to all the circumstances, to what degree the increase or decrease of the personal efficiency of the labourer conduces to an increase or decrease in the total demand for labour, or to what degree it merely enables him to shift the burden of unemployment on to someone else. Broadly speaking there is no doubt that the total demand for labour is to a material extent dependent on its average efficiency. For example, a quite new demand for labour would be created if it were possible to level up all the feeble-minded and the physically and morally defective members of the community to the normal level. The abnormal defects of these persons (the true unemployables) are the *vera causa* of their unemployment, which does not in the main result from any deficiency in industrial demand, but from the fact that their services are so worthless relatively to that of the normal workman, that to all intents and purposes they may be regarded as an industrially useless surplus. Their unemployment is, therefore, emphatically not an 'insurable risk,' and they would need to be excluded from the scope of any scheme of insurance as rigorously as exceptionally bad lives are excluded from life and sickness insurance.

But if we put aside the comparatively small section of abnormals, there is ground for asserting that at all events within the great groups of trades to which I have already referred the influence of variations in efficiency among ordinary normal workmen on the total demand for labour at any given time, though by no means negligible, is not nearly so powerful as that of variations in industrial conditions which are beyond the control of the individual workman.

If, then, the insurable elements in unemployment in these trades largely predominate over the uninsurable elements, it would be comparatively simple to devise an appropriate scheme for dealing with the evil, if every separate case of unemployment could be readily assigned to its appropriate category, so that the benefits of the scheme should be exclusively available in the case of unemployment falling within the insurable category, just as a policy of marine insurance excludes in terms losses due to a number of specified causes. But in actual practice I need hardly say that any such separation of causes can only be made to a very limited extent. In the real world of industry the various elements that contribute to unemployment are inextricably intermixed. We

can imagine the case of a carpenter who with equal truth might ascribe his unemployment to the competition of structural steel, to the general trade depression, to the severity of the winter, to local overbuilding, or to the defects in his own training.

There are few, but only a few, of the causes of unemployment which can be definitely distinguished and excluded in terms from the benefit of an insurance scheme, such, for example, as holidays, strikes and lock-outs, voluntary leaving of a situation, sickness, and crime. If, then, it is necessary, as it certainly is for the success of a scheme, that it should discriminate against unemployment due either to exceptional defects or to causes within the control of the individual, this discrimination must be effected automatically in the course of the working of the scheme itself rather than by any rule professing to exclude ineligible cases from its scope.

The crucial question from a practical point of view is, therefore, whether it is possible to devise a scheme of insurance which, while nominally covering unemployment due to all causes other than those which can be definitely excluded, shall automatically discriminate as between the classes of unemployment for which insurance is or is not an appropriate remedy.

We can advance a step towards answering this crucial question by enumerating some of the essential characteristics of any unemployment insurance scheme which seem to follow directly or by necessary implication from the conditions of the problem as here laid down.

1. The scheme must be compulsory : otherwise the bad personal risks against which we must always be on our guard would be certain to predominate.

2. The scheme must be contributory, for only by exacting rigorously as a necessary qualification for benefit that a sufficient number of weeks' contribution shall have been paid by each recipient can we possibly hope to put limits on the exceptionally bad risks.

3. With the same object in view there must be a maximum limit to the amount of benefit which can be drawn, both absolutely and in relation to the amount of contribution paid ; or, in other words, we must in some way or other secure that the number of weeks for which a workman contributes should bear some relation to his claim upon the fund. Armed with this double weapon of a maximum limit to benefit and of a minimum contribution, the operation of the scheme itself will automatically exclude the loafer.

4. The scheme must avoid encouraging unemployment, and for this purpose it is essential that the rate of unemployment benefit payable shall be relatively low. It would be fatal to any scheme to offer compensation for unemployment at a rate approximating to that of ordinary wages.

5. For the same reason it is essential to enlist the interest of all those engaged in the insured trades, whether as employers or as workmen, in reducing unemployment, by associating them with the scheme both as regards contribution and management.

6. As it appears on examination that some trades are more suitable to be dealt with by insurance than others, either because the unemployment in these trades contains a large insurable element, or because it takes the form of total discharge rather than short time, or for other reasons, it follows that, for the scheme to have the best chance of success, it should be based upon the trade group, and should at the outset be partial in operation.

7. The group of trades to which the scheme is to be applied must, however, be a large one, and must extend throughout the United Kingdom, as it is essential that industrial mobility as between occupations and districts should not be unduly checked.

8. A State subvention and guarantee will be necessary, in addition to contributions from the trades affected, in order to give the necessary stability and security, and also in order to justify the amount of State control that will be necessary.

9. The scheme must aim at encouraging the regular employer and workman, and discriminating against casual engagements. Otherwise it will be subject to the criticism of placing an undue burden on the regular for the benefit of the irregular members of the trade.

10. The scheme must not act as a discouragement to voluntary provision

for unemployment, and for that purpose some well-devised plan of co-operation is essential between the State organisation and the voluntary associations which at present provide unemployment benefit for their members.

Our analysis, therefore, leads us step by step to the contemplation of a national contributory scheme of insurance universal in its operation within the limits of a large group of trades—a group so far as possible self-contained and carefully selected as favourable for the experiment, the funds being derived from compulsory contributions from all those engaged in these trades, with a subsidy and guarantee from the State, and the rules relating to benefit being so devised as to discriminate effectively against unemployment which is mainly due to personal defects, while giving a substantial allowance to those whose unemployment results from industrial causes beyond the control of the individual.

Is such a scheme practicable?

This is a question partly actuarial, partly administrative, and partly political, and it is, of course, quite impossible to discuss it adequately on an occasion such as this.

I may, however, say that so far as can be judged from such data as exist (and those data are admittedly imperfect and rest on a somewhat narrow basis), a scheme framed on the lines I have indicated is actuarially possible, at least for such a group of trades as building, engineering, and shipbuilding—that is to say, a reasonable scale of contributions will yield benefits substantial in amount and of sufficient duration to cover the bulk of the unemployment ordinarily met with in these trades.

The administrative difficulties of such a scheme are, of course, great, but none of these difficulties is, I think, insuperable if there be a general desire that the experiment should be made. Certainly the experience of the few foreign schemes which have broken down creates no presumption against success, for the failures have been quite clearly attributable to causes which would not operate in the case of a national scheme such as is now under discussion, especially if it were worked, as it naturally would be, in close connection with the new Labour Exchanges.

Perhaps the most difficult administrative problem would be the adjustment of the scheme, so that while its benefits are not confined to workmen for whom provision is made by voluntary associations, it would yet operate so as to encourage the work of these associations, and not to undermine and destroy them, either by competition or detailed control. The problem, however, though difficult, is one for which a solution can assuredly be found if it be the general desire that a scheme shall be brought into operation.

The remaining question is one of high policy. What importance do we as a nation attach to the policy of promoting industrial security by collective action? And what sacrifices are those interested prepared to make for such object, and, in particular, to minimise the irregularity of working-class incomes so far as affected by irregular demand for labour? The final answer will depend not only on the general view taken of the relations of the individual and the State, and of the scope and limits of political action, but also on the relative weight attached to this particular object as compared with other objects which also have claims on public funds and energy.

THE LIFE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Returns for 1909, published in 1910.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	320,537,275	37,783,759	358,321,034
Adjustments in connection with the transfer of Sinking Funds and other Assurances from Life Funds, and the addition to the official list of the Royal London Mutual Insurance Society	+ 1,375,235	- 525,050	+ 850,185
	321,912,510	37,258,709	359,171,219
Premiums	27,937,702	13,336,561	41,274,263
Consideration for Annuities	2,101,084	5,013	2,106,097
Interest and Dividends (less Tax)	12,341,281	1,306,701	13,647,982
Increase in value of Investments	128,379	...	128,379
Fines, Fees, &c.	15,923	1,091	17,014
Miscellaneous	109,975	101,292	211,267
	£361,546,854	£52,009,367	£416,556,221
OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	22,113,564	5,300,300	27,413,864
Cash Bonuses and Reduction of Premiums	1,309,653	733	1,310,386
Surrenders	2,195,323	212,524	2,407,847
Annuities	2,357,918	5,287	2,363,205
Commission	1,594,502	3,358,528	4,953,030
Expenses of Management	2,321,362	2,413,706	4,735,068
Bad Debts	2,492	325	2,817
Decrease in value of Investments	1,228,950	9,386	1,238,336
Interest on Capital and Dividends and Bonuses to Shareholders	461,225	626,873	1,088,098
Miscellaneous	483,823	192,087	675,910
Balance* at the end of the Year	330,478,042	39,889,618	370,367,600
	£361,546,854	£52,009,367	£416,556,221

* This Balance includes the whole of the Life and Annuity Funds (£365,489,219), and, in addition, the Capital, &c., of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets.

LIABILITIES	Ordinary Companies	Industrial Companies*	TOTAL
	£	£	
Paid-up Capital (including sundry Shareholders' Balances) . . .	12,157,250	2,045,487	14,202,737
Life and Annuity Funds . . .	325,885,514	39,603,705	365,489,219
Employers' Liability Funds of Companies transacting Life Business . . .	662,198	2,000	664,198
Fire Funds of Companies transacting Life Business . . .	15,999,509	...	15,999,509
Leasehold Redemption Funds . . .	2,116,395	9,028	2,125,423
Marine Funds of Companies transacting Life Business . . .	1,515,925	...	1,515,925
Accident and General Funds . . .	2,807,728	12,000	2,819,728
Reserve Funds	5,409,441	1,392,446	6,801,887
Other Funds	3,458,180	294,785	3,752,915
Profit and Loss Balances . . .	4,940,416	6,710	4,947,126
Depreciation and Investment Balances	4,096,269	14,190	4,110,459
Outstanding Claims	5,272,909	8,538	5,281,447
Outstanding Accounts	1,060,558	25,955	1,095,513
Temporary Loans	414,470	79,330	493,800
	£385,805,712	£43,494,174	£429,299,886

ASSETS	Ordinary Companies	Industrial Companies*	TOTAL
	£	£	
Mortgages	97,630,298	4,712,369	102,342,667
Loans on Policies	22,485,645	217,345	22,702,990
,, Rates	37,437,178	12,195,582	49,632,760
British Government Securities . . .	5,880,246	2,138,792	8,019,038
Indian and Colonial Government Securities	17,195,406	1,958,565	19,153,971
Foreign Government Securities . . .	14,013,396	1,071,561	15,084,957
Debentures	85,572,008	6,227,715	91,799,723
Shares and Stocks	42,912,852	1,579,954	44,792,806
Land and House Property and Ground Rents	31,906,504	10,768,924	42,675,428
Life Interests and Reversions . . .	10,161,791	5,193	10,166,984
Loans on Personal Security . . .	2,114,893	3,464	2,118,357
Agents' Balances and Outstanding Premiums	7,567,600	782,669	8,350,359
Outstanding Interest	3,694,375	363,755	4,058,130
Cash, Deposits, Stamps, &c. . .	6,460,363	526,384	6,986,747
Deficiencies, Establishment Expenses, &c. . . .	226,813	641,902	868,715
Miscellaneous	546,254	...	546,254
	£385,805,712	£43,494,174	£429,299,886

* In the case of one or two Companies, transacting both Ordinary and Industrial business, but not returning separate Balance Sheets, the Liabilities and Assets, given in the above columns, under the heading "Industrial Companies", are those appertaining to the *combined* operations of such companies.—[ED. J.I.A.]

INCREASE (+) or DECREASE (-) in the Chief Items of this Year's
SUMMARY as compared with the corresponding Items for the
previous Year.

	Ordinary Companies	Industrial Companies
INCOME.	£	£
Premiums	+ 1,074,820	+ 239,452
Consideration for Annuities	+ 112,147	+ 1,136
Interest and Dividends (less Tax) . . .	+ 480,061	+ 91,646
 OUTGO.		
Claims	+ 2,965,646	+ 122,963
Annuities	+ 75,541	- 406
Surrenders	+ 203,859	+ 53,088
Commission	+ 175,585	+ 45,740
Expenses of Management	+ 135,538	+ 50,787
Net Decrease in value of Investments, &c.	1,100,571	9,386
 LIABILITIES.		
Paid-up Capital (including sundry Shareholders' Balances)	- 709,441	- 323,867
Life and Annuity Funds	+ 10,468,675	+ 2,435,927
 ASSETS.		
Mortgages (including Loans on Rates) . . .	+ 2,327,141	+ 782,931
Life Interests and Reversions	- 476,868	- 1,444
Loans on Policies	+ 1,294,482	+ 26,425
British Government Securities	- 148,701	- 22,556
Indian and Colonial Government Securities	- 1,393,267	+ 310,596
Foreign Government Securities	+ 1,211,834	+ 337,271
Debentures	+ 6,140,435	+ 699,103
Shares and Stocks	+ 1,662,484	+ 281,108
Land and House Property and Ground		
Rents	+ 1,412,500	+ 298,698
Loans on Personal Security	+ 303,549	- 958

MARY OF THE ASSURANCES IN FORCE, as shown by the last Returns of the Companies.
ORDINARY BUSINESS.

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur- ances Amount	Net Amount
	No.	Amount	No.	Amount	No.	Amount		
ASSURANCES.		£		£		£		£
le Term of Life	793,642	375,300,805	143,204	71,358,354	936,846	446,659,159	28,321,144	418,338,015
ted number of								
emiums	63,394	38,782,111	17,493	7,708,911	80,887	46,491,022	2,169,426	44,321,596
owments	557,036	414,082,916	160,697	79,067,265	1,017,733	493,150,181	30,490,570	462,659,611
owment Assur-	1,538	350,314	25,579	6,080,106	27,117	6,430,420	20,010	6,410,410
ees								
st Lives	1,481,656	234,038,156	155,410	32,277,427	1,636,466	266,315,583	3,805,048	262,510,535
Survivor	15,639	3,080,316	2,721	970,600	18,360	4,050,916	234,096	3,816,820
ellaneous	740	638,068	1,338	1,661,060	2,078	2,299,128	380,822	1,918,306
	7,838	2,995,850	36,610	34,811,483	44,448	37,807,333	7,478,556	30,328,777
	2,368,847	655,185,620	382,355	154,867,941	2,746,202	810,053,561	42,409,102	767,644,459
ANNUITIES.								
ediate	43,907	2,371,064	60,401	2,310,663
rred	18,368	475,440	29,974	445,466
	62,275	2,846,504	90,375	2,756,129

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur- ances Amount	Net Amount
	No.	Amount	No.	Amount	No.	Amount		
ASSURANCES.		£		£		£		£
le Term of Life	59	5,638	25,147,974	249,673,347	25,148,033	249,678,985	1,670	249,677,315
ted number of								
emiums	294	5,793	294	5,793	...	5,793
owments	59	5,638	25,148,268	249,679,140	25,148,327	249,684,778	1,670	249,683,108
owment Assur-	2,189,643	21,034,953	2,189,613	21,034,953	...	21,034,953
ees								
st Lives, &c.	23	2,312	756,109	7,996,109	756,132	7,998,421	88	7,998,333
	447,423	7,091,605	447,423	7,091,605	400	7,091,205
	82	7,950	28,541,443	285,801,807	28,541,525	285,809,757	2,158	285,807,599
ANNUITIES.								
ediate	51	1,761	...	1,761
rred	7	132	...	132
	58	1,893	...	1,893

The above figures are based on Returns deposited, for the most part, during the past five years, and therefore, merely an approximation to the amount of contracts in force at the present time. The s of the Colonial and Foreign Companies have been excluded, as their Returns do not separately extent of business in the United Kingdom.

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